MASS DIFFERENCE OF MIRROR NUCLEI

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The change in the ground-state energy of a nucleus due to replacement of a neutron by a proton is considered by the interacting quasiparticle method. An equation is obtained for the mass difference of mirror nuclei. Comparison with experiment is carried out. The constant $f_- = 0.4$, which describes scalar interaction of quasiparticles on the Fermi surface, is found.

THE change in energy of the ground state of mirror nuclei due to the substitution of a proton for a neutron (mass difference of mirror nuclei) was measured with good accuracy[1] by determining the energy of the β decay of mirror nuclei. Calculations of the mass differences of mirror nuclei were made in many papers (for example, [2,3]), but in all these papers the interaction between the quasiparticles (residual interaction) was not taken into account. The most detailed calculation was made by Sood and Green[2], who used the wave functions calculated for a well with a diffuse edge. The parameters of the well were chosen to agree with the binding energy of the last neutron and the last proton, and thus varied from nucleus to nucleus.

The method of interacting quasiparticles, developed by Migdal[4], makes possible a correct approach to this problem[5]. To calculate the nuclear phenomena connected with small excitation energies, one introduces in this theory certain quantities that are constant over all the nuclei and for all phenomena.

A constant describing the scalar interaction between the quasiparticles on the Fermi surface, $f_- = f_0^{pp} - f_0^{np}$, is introduced in the equation for the mass difference of the mirror nuclei. Comparison with experiment yields $f_- = 0.4$, which is approximately in agreement with the value obtained from the isotopic shift, $f_- = 0.6$, and does not contradict energy symmetry[5].

It must be noted that in earlier investigations no account was taken of the difference in magnitude between the spin-orbit splittings of proton and neutron levels. Comparison with experiment has shown that the spin-orbit splitting of the protons exceeds the spin-orbit splitting of the neutrons by a factor $\alpha(2l + 1)$, where $\alpha \cong 0.1$ MeV.

EQUATION FOR THE MASS DIFFERENCE OF MIRROR NUCLEI

The energy change due to the addition of a particle in the state λ_0 is of the form[5]

$$\Delta E = a\delta\Sigma^{\lambda_0} = a^2\Gamma_{\lambda'\lambda'}^{\lambda_0\lambda_0}\delta_0\rho_{\lambda'} = a^2\sum_{\lambda'}\Gamma_{\lambda'\lambda'}^{\lambda_0\lambda_0}(\tilde{n}_{\lambda'} - n_{\lambda'}), \quad (1)$$

where a—renormalization of the single-particle Green's function, $\bar{n}_{\lambda'}$, $n_{\lambda'}$—occupation numbers in the new and old nuclei, respectively. Let us determine the energy change due to replacing a neutron by a proton

$$E^Q = ad\delta\Sigma^{\lambda_0} = a\delta\Sigma^{\lambda_0,p} - a\delta\Sigma^{\lambda_0,n} = a^2\Gamma'^{\lambda_0\lambda_0}_{\lambda'\lambda'}\delta_0\rho_{\lambda'}, \quad (2)$$

where Γ' —change in amplitude due to the account of the Coulomb interaction in first order[1)] in e^2. The quantity Γ' satisfies the equation

$$\Gamma' = \Gamma^Q + \Gamma^Q A\Gamma^0 + \Gamma^\omega A\Gamma'. \quad (3)$$

Then, using (2) and (3), we obtain

$$E^Q = a^2\Gamma^Q(\delta\rho)^0 + a^2\Gamma^\omega AE^Q; \ (\delta\rho)^0 = \delta_0\rho + A\Gamma^0\delta_0\rho, \quad (4)$$

where Γ^0 —amplitude of interaction of particles without account of the Coulomb interaction, satisfying the equation

$$\Gamma^0 = \Gamma^\omega + \Gamma^\omega A\Gamma^0. \quad (5)$$

Γ^ω —amplitude of interaction between the quasiparticles on the Fermi surface, and Γ^Q —analogous quantity which takes into account only the Coulomb interaction

$$\Gamma^Q = \frac{e^2\delta(t_1 - t_2)}{x_1 - x_2}\{\delta(x - x_1)\delta(x' - x_2) - \delta(x' - x_1)\delta(x - x_2)\}. \quad (6)$$

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1)Terms of order e^4 make a contribution <1% and are neglected.

Table I

λ^0	A	E^Q_{exp}, MeV	E^Q_1, MeV	$\delta E^0 = E^Q_{exp} - E^Q_1$	δE^Q_{theor}
$1p\,^3/_2$	7	1.646±0.002	1,613	0.03	0.07
$1p\,^3/_2$	9	2.032±0.006	2.119	—0.09	0,10
$1p\,^3/_2$	11	2.761±0,003	2,644	0,12	0.13
$1p\,^1/_2$	13	3.066±0,005	2,580	0.43	0.17
$1p\,^1/_2$	15	3.539±0.006	3,073	0.47	0,23
$1d\,^5/_2$	17	3,550±0.006	3.614	—0.06	0,09
$1d\,^5/_2$	19	4,027±0,008	4.012	+0,01	0.12
$1d\,^5/_2$	21	4.266±0.006	4.367	—0.10	0,14
$1d\,^5/_2$	23	4.841±0,010	4.708	+0,13	0.16
$1d\,^5/_2$	25	5,062±0.008	5.039	0,02	0.20
$1d\,^5/_2$	27	5.584±0.010	5.368	0,22	0,23
$2s\,^1/_2$	29	5.749±0.010	5,494	0.26	0,42
$2s\,^1/_2$	31	6.220±0.060	5.87	0,35	0,45
$1d\,^3/_2$	33	6.360±0,030	5.79	0.57	0,28
$1d\,^3/_2$	35	6.760±0.040	6,12	0.64	0,34
$1d\,^3/_2$	37	6,920±0,110	6,43	0.49	0,37
$1d\,^3/_2$	39	7.294±0.030	6.7	0.59	0,41
$1f\,^7/_2$	41	6.740±0.050	7.164	—0.42	0,19

δE^Q_{theor} – correction for interaction without account of the term $\pm\,\alpha(2l+1)$; E^Q_{exp} – experimental difference of the Coulomb energies; E^Q_1 – difference of nucleon energies in a well with smeared edge, without account of interaction [2].

We note that the quantities Γ^ω, Γ^Q, Γ^0, and A, which enter in (4) and (5), are matrices in the isotopic variables. Thus, to determine the mass difference of mirror nuclei it is necessary to solve Eqs. (4) and (5).

Numerical calculations show that the system (4) and (5) can be solved by iteration. Performing the first iteration, we obtain

$$E^Q = \Gamma^Q(\delta_0\rho + A\Gamma^\omega\delta_0\rho) + \Gamma^\omega A\Gamma^Q\delta_0\rho. \qquad (7)$$

Recognizing that $\Gamma^Q A\Gamma^\omega\delta^0\rho = \Gamma^\omega A\Gamma^Q\delta_0\rho$, we obtain

$$E^Q = \Gamma^Q\delta_0\rho + 2\Gamma^\omega A\Gamma^Q\delta_0\rho. \qquad (8)$$

The quantity

$$\Gamma^{Q\lambda_0\lambda_0}_{\lambda_1\lambda_1}\delta_0\rho_{\lambda_1} = \sum_{\lambda_1}|\varphi_{\lambda_0}(\mathbf{r}_1)|^2|\varphi_{\lambda_1}(\mathbf{r}_2)|^2\frac{e^2}{|\mathbf{r}_1-\mathbf{r}_2|}dv_1\,dv_2$$

$$-\sum_{\lambda_1}\varphi_{\lambda_0}(\mathbf{r}_1)\varphi^*_{\lambda_1}(\mathbf{r}_1)\frac{e^2}{|\mathbf{r}_1-\mathbf{r}_2|}\varphi_{\lambda_0}(\mathbf{r}_2)\varphi^*_{\lambda_1}(\mathbf{r}_2)\,dv_1dv_2 \qquad (9)$$

is the difference between the Coulomb energies of the mirror nuclei, without account of the interaction between the quasiparticles, and was calculated by Sood and Green[2]. It must be noted that the second term in (9) is smaller than the first by $Z^{2/3}$. The first term, on the other hand, is the energy of the particle in the state λ_0 in the Coulomb field of the nucleus. Consequently, the interaction must be taken into account only in the first term of (9).

COMPARISON WITH EXPERIMENT

Let us write down expression (8) in the representation of the function φ_λ:

$$E^Q = \sum_{\lambda_1}\Gamma^{Q\lambda_0\lambda_0}_{\lambda_1\lambda_1}\delta_0\rho_{\lambda_1} + 2\sum_{\lambda\lambda'\lambda_1}\left(\frac{dn}{d\varepsilon_0}\right)^{-1}\langle\lambda_0\lambda_0|f_-|\lambda\lambda'\rangle\frac{n_\lambda-n_{\lambda'}}{\varepsilon_\lambda-\varepsilon_{\lambda'}}\Gamma^{Q\lambda\lambda'}_{\lambda_1\lambda_1}\delta_0\rho_{\lambda_1} \qquad (10)$$

where $f_- = f_{nn} - f_{np}$. In writing down (7) we made use of the isotopic invariance $f_{nn} = f_{pp}$, $f_{np} = f_{pn}$. In fact f_- depends on the coordinate r and assumes different values inside and outside the nucleus. It is natural to use for f_- the interpolation formula

$$f_- = f_-^{nuc} + (f_-^{nuc} - f_-^{vac})\frac{n(r)-n(0)}{n(0)},$$

where n(r)—density of nuclear matter.

Equation (10) was solved numerically in the oscillator model. For the scalar symmetry vertices

Table II

Averaging over the states	Interval of A	$\overline{\delta E^Q_{theor}}$, MeV	$\overline{E^Q_{exp} - E^Q_1}$, MeV
$1p\,^3/_2$; $1p\,^1/_2$	7÷15	0,14	0.19
$1d\,^5/_2$; $1d\,^3/_2$	17÷27 33÷39	0.23	0.25
$2s\,^1/_2$	29—31	0.43	0.30
$1p\,^3/_2$; $1p\,^1/_2$; $1d\,^5/_2$; $1d\,^3/_2$; $2s\,^1/_2$; $1f\,^7/_2$	7—41	23	0,20

in the oscillator model, the effective selection rules are with respect to $\lambda(nl)$ and $\lambda'(n'l')$, with $n = n' + k$ and $l = l'$. Except for the nuclei with A = 29 and 31, it was possible to confine oneself to the first iteration. The solution is changed little when f_-^{vac} varies from f_-^{nuc} to zero. Tables I and II list values corresponding to $f_-^{vac} = 0$.

In comparing theory with experiment it is necessary to take into consideration the difference in the magnitude of the spin-orbit splittings of the neutron and proton levels. The mirror-nuclei mass difference for an odd particle in the states $j = l \pm s$ is equal to

$$\Delta M = E^Q \pm {}^1\!/_2\alpha(2l+1),\ E^Q = E_1^Q + E_2^Q.$$

Here E_1^Q is the difference between the Coulomb Green's functions without interaction, as calculated by Sood and Green, and E_2^Q is the correction to this difference.

The calculation results are summarized in Tables I and II. Table II lists the average values of the correction due to the interaction, for nuclei with the same orbital angular momentum of the added particle, and also the average value for all nuclei ($f_- = 0.4$). These average quantities do not include the difference in the spin-orbit coupling.

Comparison of individual nuclei was made with account of the term $\alpha(2l+1)/2$, with α chosen to be 0.1 MeV (see the diagram). The ordinates of the diagram represent the difference in the experimental values of the β-decay energy[1] and of the Coulomb energy E_1^Q.

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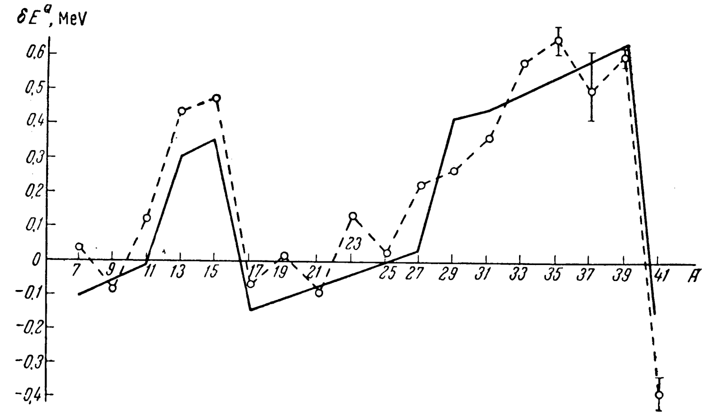

The dashed lines join the experimental points; continuous curve – theoretical with account of the spin-orbit term.

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