THERMOMAGNETIC WAVES IN A SOLID BODY

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It is demonstrated that waves of a new type may exist at sufficiently low temperatures in several metals and semimetals with a temperature gradient ∇T. The possibility of existence of such waves, named thermomagnetic waves, was previously predicted for a plasma. It is found that the waves become unstable in the presence of a magnetic field. In a weak magnetic field the instability is of a convective nature; in a strong field it is absolute. The features of the waves and their instability in conductors with carriers of one or both signs are considered.

A new type of wave, called thermomagnetic[1], can arise in an inhomogeneous plasma with a temperature gradient. It was shown in [2] that these waves can grow in the presence of a magnetic field. On the other hand, the instability of an inhomogeneous plasma was investigated by many in the absence of collisions, or more accurately, under conditions when the oscillation frequency is much higher than the collision frequency (see the review[3]). A case pertaining to the other extreme, in the sense that the frequency of the oscillations is much lower than the collision frequency, was considered in [1,2].

It is shown in this paper that such thermomagnetic waves (TM waves) are possible in many metals and semimetals at sufficiently low temperatures. In the case of Bi and Cu they can be observed at temperatures on the order of 20—30° K and lower. In the presence of a magnetic field these waves can grow, and in a weak magnetic field, when the electron Larmor frequency Ω is much lower than the collision frequency $1/\tau$, the instability has a convective character, becoming absolute in a strong field. In the case when the carriers have only one sign, the increase in the thermal emf, for example as a result of the dragging of the electrons by the phonons or as a result of singularities in electron scattering, can change radically both the critical temperature gradient and the critical magnetic field, and also the increment of the oscillations in the presence of instability. When the carriers of opposite sign are equal in number, the thermal emf in a strong magnetic field, together with the increment, can increase strongly. In a strong magnetic field, TM waves go over into waves with a quadratic spectrum when the temperature gradient approaches zero; such waves were considered in [4] for a plasma and in [5] for metals.

1. THERMOMAGNETIC WAVES FOR A SINGLE TYPE OF CARRIER (ELECTRONS OR HOLES)

If there is only one type of carrier in a solid (electrons or holes), then the electric current density in the presence of temperature gradient is determined by the expression

$$\mathbf{j} = \sigma\mathbf{E}^* + \sigma_1[\mathbf{E}^*\mathbf{H}] + \sigma_2\mathbf{H}(\mathbf{E}^*\mathbf{H}) - \beta\nabla T - \beta_1[\nabla T\cdot\mathbf{H}] - \beta_2\mathbf{H}(\mathbf{H}\nabla T). \tag{1.1*}$$

Here $\mathbf{E}^* = \mathbf{E} - e^{-1}\nabla\zeta$ ($e < 0$ for electrons) and ζ—chemical potential.

Therefore

$$\mathbf{E} = e^{-1}\nabla\zeta + \eta\mathbf{j} + \eta_1[\mathbf{jH}] + \eta_2\mathbf{H}(\mathbf{jH}) + \alpha\nabla T + \alpha_1[\nabla T\cdot\mathbf{H}] + \alpha_2\mathbf{H}(\mathbf{H}\nabla T), \tag{1.2}$$

where

$$\eta = \frac{\sigma}{\sigma^2 + (\sigma_1 H)^2}, \quad \eta_1 = -\frac{\sigma_1}{\sigma^2 + (\sigma_1 H)^2},$$

$$\eta_2 = \frac{\sigma_1^2 - \sigma\sigma_2}{[\sigma^2 + (\sigma_1 H)^2](\sigma + \sigma_2 H^2)}, \quad \alpha = \frac{\beta\sigma + \beta_1\sigma_1 H^2}{\sigma^2 + (\sigma_1 H)^2},$$

$$\alpha_1 = \frac{\beta_1\sigma - \sigma_1\beta}{\sigma^2 + (\sigma_1 H)^2},$$

$$\alpha_2 = \frac{1}{[\sigma^2 + (\sigma_1 H)^2](\sigma + \sigma_2 H^2)}[\beta(\sigma_1^2 - \sigma\sigma_2) - \beta_1\sigma_1(\sigma + \sigma_2 H^2) + \beta_2(\sigma^2 + (\sigma_1 H)^2)].$$

[EH] = E × H*.

The heat flux is

$$\mathbf{q} = (\alpha T - \zeta/e)\mathbf{j} + \alpha_1 T[\mathbf{jH}] + \alpha_2 T\mathbf{H}(\mathbf{jH}) - \varkappa\nabla T - \varkappa_1[\nabla T\cdot\mathbf{H}] - \varkappa_2\mathbf{H}(\mathbf{H}\nabla T) \tag{1.3}$$

(in [2] the coefficients β and α were denoted respectively by α and Λ).

We have made use of the Onsager relations. The thermal conductivity can be anisotropic, as is the case for example when electronic thermal conductivity predominates, so that it becomes necessary then to assume that all three coefficients κ, κ_1, and κ_2 differ from zero; on the other hand, if phonon thermal conductivity predominates, then we can assume the thermal conductivity to be isotropic and put $\kappa_1 H = \kappa_2 H^2 = 0$. The electric field of the oscillations is not purely longitudinal, and we have for the frequency $\omega \ll ck$ and $\omega \ll 1/\tau$; then the displacement current $(4\pi)^{-1}\partial E/\partial t$ and the charge-density oscillations can be neglected. In this case the equations of the problem take the form*

$$\partial\mathbf{H}/\partial t = -c\,\mathrm{rot}\,\mathbf{E},\quad \mathrm{rot}\,\mathbf{H} = 4\pi c^{-1}\mathbf{j},\quad \mathrm{div}\,\mathbf{H} = 0, \tag{1.4}$$

$$TdS/dt = -\mathrm{div}\,\mathbf{q} + (\mathbf{jE}), \tag{1.5}$$

where S—entropy per unit volume.

We separate the constant and oscillating parts of the quantities **H**, **E**, and T, for example

$$\mathbf{H} = \mathbf{H}_0 + \mathbf{H}'\exp\{i(\mathbf{kr} - \omega t)\}$$

(we shall henceforth leave out the subscript zero) and linearize over the primed quantities. We must take account here of the dependence of the kinetic coefficients on T and H^2. We confine ourselves to the case $kL \gg 1$, where $L = T/|\nabla T|$ is the characteristic scale of the inhomogeneity. Then, for example,

$$\frac{\partial\eta}{\partial T}[\nabla T\cdot\mathbf{j}'] \ll \eta\,\mathrm{rot}\,\mathbf{j}',$$

and consequently we can neglect in the calculation of curl **E** the temperature dependence of the kinetic coefficients. The effect of ∇T on the oscillation spectrum becomes appreciable if the thermomagnetic terms in (1.2) prevail over the galvanomagnetic terms. In the "classical" theory of metals we have $\alpha \sim T/e\zeta$. This quantity increases by several orders of magnitude, reaching values $1/e$ under the influence of the dragging of the electrons by the phonons [6], and in a few other cases. For example, it was observed in [7] that small impurities of Sn and Fe in Cu increase α by as much as two orders of magnitude. We shall therefore write in the general case $\alpha = \alpha_0/e$. When $\Omega\tau \gg 1$, the quantity $\alpha(H)$ can increase relative to $\alpha(0)$ even under the influence of magnetic scattering [8], and $\alpha_2 H^2 = \alpha(0) - \alpha(H)$.

When $\Omega\tau \ll 1$, the maximum galvanomagnetic term $\eta\mathbf{j}' = (c/4\pi)\eta$ curl $\mathbf{H}'$ can be small compared with the maximum thermomagnetic term $\alpha_1[\nabla T \times \mathbf{H}']$ if

$$(\zeta/T)(c/\omega_0 l)^2 kL/\alpha_0 \ll 1. \tag{1.6}$$

Here $\omega_0 = \sqrt{4\pi ne^2/m}$ —plasma frequency, and l—electron mean free path.

In the case when $\Omega\tau \ll 1$, the maximum galvanomagnetic term $\eta_1\mathbf{j}' \times \mathbf{H}$ is small compared with the maximum thermomagnetic terms of order $\alpha_2 H'(H\nabla T)$ only if

$$kLH^2/4\pi nT\alpha_0 \ll 1. \tag{1.7}$$

When $\Omega\tau \ll 1$, and the terms containing η_1, η_2, and α_2 can be neglected, and the oscillation spectrum is of the form

$$\omega = -c\alpha_1(\mathbf{k}\nabla T) - i(c^2k^2/4\pi)\eta. \tag{1.8}$$

If (1.6) is satisfied, the attenuation of the frequency (1.8) is small. The changes in the temperature T' and in the magnetic field $\mathbf{H}'$ in TM-oscillations, in accordance with (1.2), add to the alternating part of the magnetic field contributions of the form $\nabla T \times \mathbf{H}' + \nabla T' \times \mathbf{H}$, the ratio of which $kLT'H/TH' \ll 1$ in accordance with (1.5), if (1.6) is satisfied, owing to the smallness of the contribution made by the electrons to the specific heat down to the lowest temperatures of interest to us. Consequently the TM waves are isothermal when $\Omega\tau \ll 1$. Weakly damped waves (1.8) can exist also in the absence of a constant magnetic field, $\mathbf{H} = 0$. Waves of this type in a plasma were considered earlier [1].

In the inverse extreme case $\Omega\tau \gg 1$, we can neglect terms containing η, η_2, and α_1, but we must take into account the dependence of the kinetic coefficients on H^2. We must distinguish between two cases: A) anisotropic thermal conductivity, which is realized for example when electronic thermal conductivity predominates, and B) isotropic thermal conductivity, which can be realized only when phonon thermal conductivity prevails.

The most essential terms containing ∇T and $\nabla T'$ in (1.2) with $\Omega\tau \gg 1$ are of the form $\alpha_2\mathbf{H}(\mathbf{H}'\cdot\nabla T) + \alpha_2\mathbf{H}(\mathbf{H}\cdot\nabla T')$ and their ratio is $kLT'H/TH'$. In case A the ratio is of the order of unity, and for TM waves the most significant terms in the equation for thermal conductivity

*rot = curl.

(1.5) are those containing κ_2, since $\omega C \ll \kappa_2 k^2 H^2$ for TM waves (C—thermal conductivity per unit volume). From (1.5) we have, with allowance for (1.3)

$$iT'(\mathbf{kH}) + (\mathbf{H}'\nabla T) + 2(\mathbf{HH}')(\mathbf{H}\nabla T)\partial \ln \varkappa_2/\partial H^2 = 0. \quad (1.9)$$

Substituting (1.9) in (1.4) we obtain

$$\omega\left[\omega - 2\frac{\partial}{\partial H^2}\left(\ln\frac{\alpha_2}{\varkappa_2}\right)c\alpha_2[\mathbf{kH}]^2(\mathbf{H}\nabla T)\right] - (ck)^2\left[\frac{c\eta_1}{4\pi}(\mathbf{kH}) - i\alpha_2(\mathbf{H}\nabla T)\right]^2 = 0. \quad (1.10)$$

Inasmuch as $H^2 \frac{\partial}{\partial H^2} \ln(\alpha_2/\kappa_2) \ll 1$ when $\Omega\tau \gg 1$, we can rewrite (1.10) in the form

$$\omega = \pm ck\left[\frac{c\eta_1}{4\pi}(\mathbf{kH}) - i\alpha_2(\mathbf{H}\nabla T)\right]. \quad (1.11)$$

The two branches of (1.11) correspond to waves with right- and left-hand circular polarization. The left-polarized wave increases if $\alpha_2(\mathbf{H}\cdot\nabla T) < 0$ and the right-polarized wave if $\alpha_2(\mathbf{H}\cdot\nabla T) > 0$. Waves (1.11) without ∇T in the plasma were found by Ginzburg[4], and were investigated in solids by Kaner and Skobov[5]. The waves (1.11) were obtained in [2] for a fully-ionized plasma with $\nabla T \neq 0$, in the case when inequality (1.7) is valid. It was shown in the same paper that when (1.7) is satisfied, the growth of (1.11) is a manifestation of absolute instability.

We proceed to TM waves in case B, with $\Omega\tau \ll 1$. Here $kLT'H/TH' \ll 1$, and therefore the TM waves are isothermal, as in the case when $\Omega\tau \ll 1$. The dispersion equation is of the form

$$\omega[\omega - c\alpha_2(\mathbf{k}[\mathbf{H}\nabla T])] + \frac{c^2k^2}{4\pi}\left[\frac{ic\eta_1}{4\pi}(\mathbf{kH}) + \alpha_2(\mathbf{H}\nabla T)\right] \times\left[\frac{ic}{4\pi}\eta_1(\mathbf{kH}) + \alpha_2\left(2(\mathbf{H}\nabla T) - \frac{1}{k^2}(\mathbf{kH})(\mathbf{H}\nabla T)\right) + \frac{2}{k^2}[\mathbf{kH}]^2(\mathbf{k}\nabla T)\frac{\partial\alpha_2}{\partial H^2}\right] = 0. \quad (1.12)$$

If condition (1.7) is satisfied, so that the TM terms predominate, then

$$\omega_{1,2} = \frac{1}{2}c\alpha_2(\mathbf{k}[\mathbf{H}\nabla T]) \pm\frac{1}{2}c\alpha_2\left\{(\mathbf{k}[\mathbf{H}\nabla T])^2 - 4(\mathbf{H}\nabla T)\left[2(\mathbf{H}\nabla T) - \frac{1}{k^2}(\mathbf{kH})(\mathbf{k}\nabla T) + \frac{2}{k^2}[\mathbf{kH}]^2(\mathbf{H}\nabla T)\frac{\partial\ln\alpha_2}{\partial H^2}\right]\right\}^{1/2} = 0, \quad (1.13)$$

or, if we recognize that when $\Omega\tau \gg 1$ we have $\partial\alpha_2/\partial H^2 = -\alpha_2/H^2$, we can rewrite (1.13) in the form

$$\omega_{1,2} = \frac{1}{2}c\alpha_2(\mathbf{k}[\mathbf{H}\nabla T]) \pm \frac{1}{2}c\alpha_2\left\{(\mathbf{k}[\mathbf{H}\nabla T])^2 - 4(\mathbf{H}\nabla T)(\mathbf{kH})\left[\frac{2}{H^2}(\mathbf{kH})(\mathbf{H}\nabla T) - k\nabla T\right]\right\}^{1/2}. \quad (1.14)$$

This expression shows that the TM waves with $\Omega\tau \gg 1$ can grow at certain angles between the vectors **k**, **H** and ∇T. The increment increases as the vectors **k**, **H**, and ∇T approach coplanarity. In the latter case

$$\omega_{1,2} = \pm ic\alpha_2\{(\mathbf{H}\nabla T)(\mathbf{kH})[2H^{-2}(\mathbf{kH})(\mathbf{H}\nabla T) - \mathbf{k}\nabla T]\}^{1/2}. \quad (1.15)$$

Expression (1.14) is much more complicated than (1.11) in case A of anisotropic thermal conductivity, and calls for an analysis. The frequencies (1.14) were investigated in [2] for a fully ionized plasma and in [9] for a partially ionized plasma. It was shown in [2] that the increment is a maximum when all three vectors **k**, **H** and ∇T have the same direction. Then

$$\omega = \pm ic\alpha_2 k(\mathbf{H}\nabla T) \quad (1.16)$$

corresponds to an aperiodic growth. In case B, like in case A, the TM waves are absolutely unstable. Thus, in spite of the difference between (1.14) and (1.11), the orders of magnitude of the frequencies are the same when inequality (1.7) is satisfied.

In the opposite extreme case $H^2kL/4\pi nT\alpha_0 \gg 1$, when not thermomagnetic but galvanomagnetic terms predominate in (1.2), the spectrum in case B with $\Omega\tau \gg 1$ is of the form

$$\omega_{1,2} = \pm ck\left\{\frac{c\eta_1}{4\pi}(\mathbf{kH}) - i\alpha_2\left[\frac{3}{2}(\mathbf{H}\nabla T) - \frac{1}{2k^2}(\mathbf{kH})(\mathbf{k}\nabla T) + \frac{1}{k^2}[\mathbf{kH}]^2(\mathbf{H}\nabla T)\frac{\partial}{\partial H^2}(\ln\alpha_2)\right]\right\} + \frac{1}{2}c\alpha_2(\mathbf{k}[\mathbf{H}\nabla T]) + \frac{ic^2}{4\pi}(\eta k^2 + \eta_2[\mathbf{kH}]^2). \quad (1.17)$$

This expression corresponds to weakly damped or weakly growing waves. Growth is possible only when the temperature gradient exceeds a minimum value, determined from the condition

$$|\alpha_2\nabla T|H = c\eta k/4\pi.$$

When $\nabla T = 0$ the spectrum (1.17) for case B coincides with (1.11) [we have disregarded the weak damping in (1.11)]. For $\Omega\tau \gg 1$ we have $\eta_1 = 1/nec$.

2. CONDITION FOR THE OCCURRENCE OF INSTABILITY

Let us investigate the case when the vectors H and ∇T are parallel, and the growth is least dependent on the presence or absence of isotropy in the thermal conductivity. We start with case B of Sec. 1. The dispersion equation in an arbitrary magnetic field is of the form

$$\left[\omega + c\alpha_1(\mathbf{k}\nabla T) + \frac{ic^2k^2}{4\pi}\eta\right] \times \left\{\omega + c\alpha_1(\mathbf{k}\nabla T) + \frac{ic^2k^2}{4\pi}\eta + \frac{ic^2}{4\pi}\eta_2[\mathbf{kH}]^2\right\} - c^2\left[\frac{c}{4\pi}\eta_1(\mathbf{kH}) + \alpha_2(\mathbf{H}\nabla T)\right] \times \left[\frac{ck^2}{4\pi}\eta_1(\mathbf{kH}) + \alpha_2(\mathbf{kH})^2(\mathbf{H}\nabla T)\frac{1}{H^2}\right] = 0. \qquad (2.1)$$

In the derivation of (2.1) we made use of the relation $\alpha + \alpha_2 H^2 = \alpha(0)$, where $\alpha(0)$—value of α in the absence of a magnetic field. If the component $k_{\|}$ is specified (along ∇T; in the experiments this corresponds to specifying the thickness of the sample in the ∇T direction), then it is of interest to determine the minimum magnetic field $\mathbf{H}$ at which instability sets in first, under the condition that k can be varied arbitrarily. For $\Omega\tau \gg 1$ this instability was already obtained by us in (1.16). If we put $\omega = \omega_r + i\omega_i$, then (2.1) has for $\mathbf{H} = 0$ only solutions with $\omega_i < 0$, corresponding to damping. Let us find the magnetic field for which $\omega_i = 0$. Using (2.1) and the condition $\omega_i = 0$, we can eliminate ω and obtain an equation for $k_{\perp}^2 = 0$:

$$\begin{aligned}&(2+\gamma_2)^2(1+\gamma_2)(k_{\perp}/k_{\|})^8 + (2+\gamma_2)[(1+\gamma_2)(4+\gamma_2) \\ &\quad + (1+\gamma_1^2)(2+\gamma_2)](k_{\perp}/k_{\|})^6 \\ &\quad + [(2+\gamma_2)(4+\gamma_2)(1+\gamma_1^2) + 2(4+\gamma_2)(1+\gamma_2) \\ &\quad - (u/v_m k_{\|})^2(\gamma_1^2 + (2+\gamma_2)^2](k_{\perp}/k_{\|})^4 \\ &\quad + [2(4+\gamma_2)(1+\gamma_1^2) + 4(1+\gamma_2) \\ &\quad - 2(u/v_m k_{\|})^2(2\gamma_1^2 + \gamma_2 \\ &\quad + 2)](k_{\perp}/k_{\|})^2 + 4(1+\gamma_1^2)(1 - u^2/v_m^2 k_{\|}^2) = 0,\end{aligned} \qquad (2.2)$$

where we put

$$\gamma_1 = \frac{\eta_1 H}{\eta}, \quad \gamma_2 = \frac{\eta_2 H^2}{\eta}, \quad v_m = \frac{c^2\eta}{4\pi}, \quad u = c\alpha_2(\mathbf{H}\nabla T)$$

(ν_m is a quantity with the dimension of velocity).

When $\mathbf{H} = 0$ we have $\gamma_1 = \gamma_2 = u = 0$, and the equation has no positive real roots. According to the Descartes theorem, the equation can have a positive root if there is at least one alternation of sign among the coefficients. Inasmuch as $\eta + \eta_2 H^2 = \eta_0 > 0$ (η_0—value of η when $\mathbf{H} = 0$), the coefficients of $k_{\perp}^8$ and $k_{\perp}^6$ are always positive. The coefficients that can reverse sign are those of $k_{\perp}^4$, $k_{\perp}^2$, or the free term. The free term can reverse sign at a minimum field given by

$$\alpha_2(\mathbf{H}\nabla T_{cr}) = c\eta k_{\|}/4\pi, \qquad (2.3)$$

and in this case, in accordance with the theorem mentioned, instability must set in; then $k_{\perp}^2 = 0$.

If we vary ∇T at a fixed magnetic field and fixed $k_{\|}$ (sample thickness), then the minimum ∇T at which instability sets in again satisfies the condition (2.3).

In the presence of only one type of carrier, η depends little on the magnetic field and the dependence is approximately the same for both $\Omega\tau \ll 1$ and $\Omega\tau \gg 1$ (1.2); α_2 is practically independent of H when $\Omega\tau \ll 1$, and for $\Omega\tau \gg 1$ we have $\alpha_2 \propto 1/H^2$. Therefore, in accordance with (2.3), $\nabla T_{cr} \propto H^{-1}$ when $\Omega\tau \ll 1$ and $\nabla T_{cr} \propto H$ when $\Omega\tau \gg 1$. Weak magnetic fields facilitate the occurrence of instability, while strong ones hinder it. It follows from these estimates that, for specified ∇T and $k_{\|}$, (2.3) has either two solutions or none. The instability can exist in a finite interval of magnetic fields from H_{min} to H_{max}, where H_{min} and H_{max} are solutions of (2.3).

In case A, when the thermal conductivity is anisotropic, the determination of the linear field for arbitrary value of $\Omega\tau$ necessitates rather cumbersome calculations. In the limiting case $\Omega\tau \ll 1$ the result is the same as for case B.

Thus we find the minimal ∇T at which instability sets in when $\Omega\tau \gg 1$. In this case the dispersion equation takes the form

$$\left(\omega + i\frac{c^2k^2}{4\pi}\eta\right)^2 + \frac{ic^2}{4\pi}k_{\perp}^2\left(\eta_2 + \frac{\alpha_2^2 T}{\varkappa_2}\right)\left(\omega + \frac{ic^2k^2}{4\pi}\eta\right) + c^2\left[\frac{ic}{4\pi}\eta_1(\mathbf{kH}) + \alpha_2(\mathbf{H}\nabla T)\right]^2 = 0. \qquad (2.4)$$

When $\nabla T = 0$ the equation has only solutions with $\omega_i < 0$. The value of ∇T_{cr} at which $\omega_i = 0$ again satisfies the condition (2.3).

3. THERMOMAGNETIC WAVES IN CONDUCTORS WITH ELECTRONS AND HOLES

Oscillations in semi-metals and metals are always quasineutral, and regardless of the ratio of electrons to holes we have $\nabla\zeta_+ + \nabla\zeta_- = 0$ ($\zeta_{\pm}$ = chemical potentials). Therefore the density of the electric current can be written in the form

$$\begin{aligned}\mathbf{j} &= (\sigma_+ + \sigma_-)\mathbf{E}^* + (\sigma_{1+} + \sigma_{1-})[\mathbf{E}^*\mathbf{H}] \\ &\quad + (\sigma_{2+} + \sigma_{2-})\mathbf{H}(\mathbf{E}^*\mathbf{H}) - (\beta_+ + \beta_-)\nabla T \\ &\quad - (\beta_{1+} + \beta_{1-})[\nabla T\cdot\mathbf{H}] - (\beta_{2+} + \beta_{2-})\mathbf{H}(\mathbf{H}\cdot\nabla T),\end{aligned} \qquad (3.1)$$

where $\mathbf{E}^* = \mathbf{E} - e^{-1}\nabla\zeta_+$.

From this we express $\mathbf{E}$ in terms of $\mathbf{j}$ and ∇T:

$$\begin{aligned}\mathbf{E} &= e^{-1}\nabla\zeta_+ + \eta^*\mathbf{j} + \eta_1^*[\mathbf{jH}] + \rho_2^*\mathbf{H}(\mathbf{jH}) \\ &\quad + \alpha^*\nabla T + \alpha_1^*[\nabla T\cdot\mathbf{H}] + \alpha_2^*\mathbf{H}(\mathbf{H}\nabla T).\end{aligned} \qquad (3.2)$$

The quantities η^*, η_1^*, η_2^*, α^*, α_1^*, and α_2^* are expressed in terms of $\sigma_+ + \sigma_-$, $\beta_+ + \beta_-$, etc. in accordance with (1.2), provided we replace there σ by $\sigma_+ + \sigma_-$, β by $\beta_+ + \beta_-$, etc. If the thermal conductivity is isotropic, then the oscillations are isothermal, so that the inequality $kLT'H/TH' \ll 1$ is valid as before. The anisotropy can be appreciable when $\Omega\tau \gg 1$, but even then the order of magnitude of the frequencies and the character of the resultant instabilities remain the same. We therefore confine ourselves to the case of isotropy.

In weak magnetic fields ($\Omega\tau \ll 1$) we obtain from (1.4) and (3.2) the oscillation spectrum

$$\omega = -ic^2k^2\eta^* / 4\pi - c\alpha_1^* (\mathbf{k}\nabla T). \tag{3.3}$$

The waves attenuate weakly if condition (1.7) is satisfied.

If the mobility of carriers of one sign, say the mobility μ_- of the electrons, is much larger than the mobility of the carriers of the opposite sign, then a case when $\Omega_-\tau_- \gg 1 \gg \Omega_+\tau_+$ is possible. In this case $\sigma_{1-}H \gg \sigma_+ + \sigma_-$ and $\beta_{1-}H \gg \beta_+ + \beta_-$. Therefore

$$\eta_1^* H \approx \eta_{1-}H \gg \max(\eta^*, \eta_2^* H^2),$$

$$\alpha^* \approx \alpha_-, \quad \alpha_2^* \approx \alpha_{2-} \gg \alpha_1^*,$$

where η_{1-}, α_-, and α_{2-} are expressed by formulas (1.2) for carriers of one sign (electrons). The dispersion equation is the same as for a conductor with a one type of carrier. Thus, when $\Omega\tau \ll 1$ the quantities η and α_1 are simply replaced by η^* and α_1^*, and when $\Omega_-\tau_- \gg 1 \gg \Omega_+\tau_+$ the holes do not participate at all in the oscillations, and the frequency of the oscillations is the same as in an electronic conductor.

In a strong magnetic field, when $\Omega_-\tau_- \gg 1$ and $\Omega_+\tau_+ \gg 1$, the situation for conductors with equal numbers of electrons and holes is essentially different than in Sec. 1 when the Hall mobility is very small. In this case

$$\sigma_{1+} + \sigma_{1-} \sim \sigma_{1+} / (\Omega_+\tau_+)^2 + \sigma_{1-} / \Omega_-\tau_-)^2 \ll \max(\sigma_{1+}, \sigma_{1-})$$

and the terms with η_1^* and α_2^* in (3.2) are no longer maximal for $\Omega\tau \gg 1$, as was the case in a conductor with carriers of the same sign (Sec. 1). The largest terms of (2.2) will be those containing η^*, η_2^*, and α_1^*. The oscillation frequencies are of the form

$$\omega_1 = -c\alpha_1^*(\mathbf{k}\nabla T) - ic^2k^2\eta^* / 4\pi,$$
$$\omega_2 = -c\alpha_1^*(\mathbf{k}\nabla T) - ic^2(\eta^* k^2 + \eta_2^*[\mathbf{kH}]^2) / 4\pi. \tag{3.4}$$

The waves propagate with frequency ω_1 when the vectors **k**, **H** and **H′** lie in a single plane. For waves with frequency ω_2 we have $(\mathbf{k}\cdot\mathbf{H}\times\mathbf{H}') = 0$ and $(\mathbf{H}\cdot\mathbf{H}') = 0$, with H′ perpendicular to the plane in which the vectors k and H lie. Both waves are damped, unlike the case of carriers of one sign (Sec. 1). The waves attenuate weakly if $H^2kL/4\pi nT\alpha_0 \ll 1$. For the waves to grow it is necessary to impose a more stringent condition. It is necessary that the term of (3.2) containing α_2^* and causing the growth be larger than the terms with η^* and η_2^*; this is equivalent to $H^2\Omega\tau kL/4\pi nT\alpha_0 \ll 1$. Under this condition

$$\omega_{1,2} = -c\alpha_1^*(\mathbf{k}\nabla T) + \tfrac{1}{2}c\alpha_2^*(\mathbf{k}[\mathbf{H}\nabla T])$$
$$\pm \tfrac{1}{2}c\alpha_2^*\{(\mathbf{k}[\mathbf{H}\nabla T])^2 - 4(\mathbf{H}\nabla T)[2(\mathbf{H}\nabla T)$$
$$- k^{-2}(\mathbf{kH})(\mathbf{k}\nabla T) + 2k^{-2}(\mathbf{H}\nabla T)[\mathbf{kH}]^2\partial\ln\alpha_2^* / \partial H^2]\}^{1/2}. \tag{3.5}$$

One of the frequencies (3.5) grows weakly. The growth is maximal when $\mathbf{k} \parallel \mathbf{H} \parallel \nabla T$. The minimum field or the minimum temperature gradient at which the growth begins is determined from the condition

$$\alpha_2^*(\mathbf{H}\nabla T_{\mathrm{cr}}) = c\eta^* k_\parallel / 4\pi. \tag{3.6}$$

When $\Omega_+\tau_+ \gg 1$ and $\Omega_-\tau_- \gg 1$ we have $\eta^* \propto H^2$ and $\alpha_2^* \propto 1/H^2$. Therefore $\nabla T_{\mathrm{cr}} \propto H^3$. A strong magnetic field hinders the start of the instability to an even greater degree than for conductors with a single type of carrier. For carriers of a single type the instability, as in Sec. 2, exists only in a finite magnetic-field interval for specified ∇T and $k_\parallel$.

In semi-metals, in which $n_+ \neq n_-$, the thermal emf $\alpha^*(H)$ in a strong magnetic field can differ strongly from $\alpha^*(0)$—the thermal emf in the absence of a magnetic field in the case when the dragging effect is appreciable[6]. Thus, when

$$(H / c)^2\mu_+^2\mu_-^2(n_+ - n_-)^2 \gg (n_+\mu_+ + n_-\mu_-)^2$$

we have

$$\alpha^*(H) = \frac{4}{3e}\frac{n_+ + n_-}{n_+ - n_-}, \quad \alpha^*(0) = \frac{4}{3e},$$

$$\alpha_2^* H^2 = \alpha^*(0) - \alpha^*(H), \quad \alpha_1^* = \frac{8c}{3eH^2}\frac{n_+ + n_-}{(n_+ - n_-)^2}\frac{\mu_+ + \mu_-}{\mu_+\mu_-}$$

The frequency is determined from (3.5), and it is possible to have $\omega_i > \omega_r$, while the growth can in this case be much larger than for carriers of one sign.

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