ENERGY DEPENDENCE OF THE LIFETIME OF QUASISTATIONARY STATES

A. I. BAZ'

Submitted to JETP editor May 16, 1964

J. Exptl. Theoret. Phys. (U.S.S.R.) 47, 1874-1876 (November, 1964)

A formula is obtained for the dependence of the lifetime of a quasistationary state on the precise value of the energy.

THE problem treated in this note arises in the following way. Suppose that in the scattering of particles there is a resonance with width Γ, located at energy E_0. We want to know the lifetime T of the quasistationary state as a function of the energy E of the scattered particle. There are several approaches to this question.

1. One approach is based on the work of Wigner.[1] Let us consider a wave packet, scattered by a potential of radius R. For $r > R$ we have

$$\chi_k(r)+\chi_{k+dk}(r)=\left[\exp\left\{-ikr-\frac{iEt}{\hbar}\right\}+\exp\left\{-i(k+dk)r-\frac{i(E+dE)t}{\hbar}\right\}\right]-\left[\exp\left\{ikr-\frac{iEt}{\hbar}+2i\delta(k)\right\}+\exp\left\{i(k+dk)r-\frac{i(E+dE)}{\hbar}t+2i\delta(k+dk)\right\}\right],$$

where δ is the scattering phase. The center of gravity of the incident packet is determined by the condition that the phases of the two terms in the incident wave be equal (in which case the amplitude of the wave function is a maximum). This gives for the law of motion of the packet incident on the scatterer:

$$\hbar\frac{dk}{dE}r+t\equiv\frac{r}{v}+t=0.$$

For the scattered packet we get similarly

$$\frac{r}{v}-t+\frac{2}{v}\frac{d\delta}{dk}=0.$$

From these formulas we see that the incident wave reaches the edge of the potential at the time $T_1 = -R/v$, while the scattered wave passes this point at the time $T_2 = R/v + (2/v)d\delta/dk$. The difference of these times

$$T(E)\equiv T_2-T_1=\frac{2}{v}\left(\frac{d\delta}{dk}+R\right)$$

is the lifetime of the state formed in the scattering. Near resonance,

$$\delta=\tan^{-1}[\Gamma/(E_0-E)],$$

and we get, neglecting the time of passage R/v,

$$T(E)=\frac{2\hbar\Gamma}{(E-E_0)^2+\Gamma^2}\equiv T(E_0)\frac{\Gamma^2}{(E-E_0)^2+\Gamma^2}.$$

The lifetime reaches a maximum $T(E_0) = 2\hbar/\Gamma$ for $E = E_0$, and falls off according to the Lorentz law as one moves away from the center of the resonance.

2. A second approach is based on the following model.

Let us consider a potential with a barrier, within which and in some small neighborhood of which there is a magnetic field H directed along the z axis. Suppose that we scatter particles of spin $^1/_2$, polarized along the y axis. The equation has the form

$$\varphi''+(k^2-v)\varphi=-\frac{2m\mu\sigma_zH}{\hbar^2}\varphi\equiv-\alpha\begin{pmatrix}1&0\\0&-1\end{pmatrix}\varphi,$$

$$\alpha=\frac{2m\mu H}{\hbar^2},$$

where φ stands for the column vector $\varphi = \begin{pmatrix}\varphi_1\\\varphi_2\end{pmatrix}$ and μ is the magnetic moment.

We shall assume that $H\mu \to 0$, so that we can use perturbation theory. The zeroth approximation gives

$$\varphi^{(0)}=\chi_k(r)\begin{pmatrix}i\\1\end{pmatrix},\qquad \chi_k\sim\sqrt{\frac{2}{\pi}}\sin(kr+\delta).$$

In the next order of perturbation theory we have

$$\varphi^{(1)}=\chi_k(r)\begin{pmatrix}i\\1\end{pmatrix}+\frac{\alpha}{k}\sqrt{\frac{\pi}{2}}e^{i(kr+\delta)}\int_0^R\chi_k^2(r)\,dr\begin{pmatrix}i\\-1\end{pmatrix}.$$

We have stopped the integration at the radius R of the potential, since near resonance χ_k^2 is very large inside the barrier, and the integration over the whole region where the magnetic field is located can be replaced by an integration over the interior of the sphere $r < R$.

The total wave function for the outgoing wave has the form

$$e^{i(kr+\delta)}\frac{1}{i\sqrt{2\pi}}\begin{pmatrix} i & -\beta \\ 1 & -i\beta \end{pmatrix} \qquad \beta=\frac{\pi\alpha}{k}\int_0^R \chi_k^2(r)\,dr.$$

This wave function corresponds to a rotation of the spin through an angle 2β around the z axis. Near resonance

$$\int_0^R \chi_k^2(r)\,dr=\frac{\hbar}{\pi}\sqrt{\frac{2E_0}{m}}\frac{\Gamma}{(E-E_0)^2+\Gamma^2}.$$

We obtain the angle of rotation

$$\vartheta=2\beta=2\mu H\frac{2\Gamma}{(E-E_0)^2+\Gamma^2}.$$

Dividing this expression by the Larmor frequency, we again get for the lifetime the familiar formula

$$T(E)=\frac{\vartheta}{2\mu H/\hbar}=\frac{2\hbar}{\Gamma}\frac{\Gamma^2}{(E-E_0)^2+\Gamma^2}.$$

It is a pleasant duty to express my gratitude to Ya. B. Zel'dovich for numerous discussions.

---

[1] E. P. Wigner, Phys. Rev. 98, 145 (1955).

Translated by M. Hamermesh
271