WEAK INTERACTIONS IN COLLIDING BEAMS OF ELECTRONS

B. L. IOFFE, L. B. OKUN', and A. P. RUDIK

Institute of Theoretical and Experimental Physics

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Possible experiments are discussed for the study of the weak interaction in electron-electron and electron-positron colliding beams at energies of the order of 100—1000 BeV, for which the weak interaction becomes a strong one. The main results of this work are given in a table and are those for processes 9—12, which occur in the second order of perturbation theory with respect to the weak interaction and are described by the diagrams of Fig. 2.

IT is well known that cross sections caused by the weak interaction increase with increasing energy. In the interaction between leptons and nucleons this increase is evidently "cut off" at energies of the order of 1 BeV by the form-factors caused by the strong interaction. In the interaction between leptons and leptons this increase should continue up to $E \sim 10^3$ BeV (where E is the energy of each of the colliding particles in their center-of-mass system), provided that a form-factor of the weak interaction itself does not begin to manifest itself at smaller energies. Such a form-factor could be caused, for example, by the intermediate W meson, if it exists. An experimental study of the weak interaction under conditions in which it is strong would be extremely valuable, since it would provide a possibility for obtaining information about the dependence of the weak-interaction vertex on the energy and the momentum transfer.

In this paper we calculate the cross sections of a number of inelastic processes caused by the weak interaction, and discuss the possibilities for experimental observation of these processes. We shall start from a point four-fermion structure of the weak interaction (assuming that there is no W meson), and shall assume that the standard "square of the charged current" scheme holds. The table lists a series of reactions caused by the weak interaction. Some of them (1—5), which occur in the interaction of neutrinos and photons with electrons, have been treated earlier and discussed as conceptual experiments by a number of authors.[2-6] Unfortunately, energies of the order of 100-1000 BeV are practically unattainable in processes 1—5. In fact, in order to have an energy $E \sim 10^3$ BeV in the center-of-mass system, neutrinos (or photons) with laboratory energies of the order of 10^9 BeV would be required.

We would like to emphasize that there are much more realistic prospects of studying weak interactions at energies $\sim 10^3$ BeV with colliding electron beams.

In this paper we consider the behavior at energies $10^2 - 10^3$ BeV of a number of weak processes which can occur in e^+e^- collisions (processes 6—11) and in e^-e^- collisions (process 12).[1)] Let us briefly discuss these processes. Unfortunately it is practically impossible to observe process 6, which occurs in first order in G.

The photons produced in process 7 are essentially like bremsstrahlung and are emitted along the directions of the momenta of the colliding electron and positron. Therefore it is hard to distinguish them from the photons that accompany the Møller scattering of electrons by positrons. The possibility of distinguishing the photons from process 7 that emerge at large angles requires special treatment and will not concern us here.

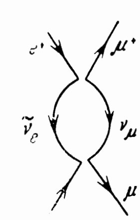

FIG. 1

Process 8 occurs in second order in G. The diagram for this process is shown in Fig. 1. The cross section for the process contains an unknown cut-off parameter Λ. Processes 9—12 are free

1)Some estimates of two-particle weak processes in colliding electron and positron beams at energies of the order of several BeV have been given in [8-9].

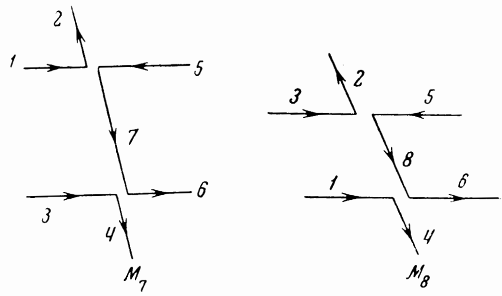

FIG. 2

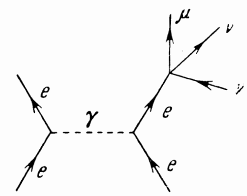

FIG. 3

from this kind of uncertainty, since the diagrams for these processes are of second order in G and do not contain closed loops.[2)] This enables us to make definite predictions of their cross sections for $E \gtrsim 100$ BeV (of course within the framework of the scheme considered here). Figure 2 shows the diagrams for process 12. Two analogous diagrams describe process 9. There is only one diagram for process 10, and the same is true of process 11. It follows at once from dimensional considerations that the cross sections for processes 9—12 are proportional to G^4s^3. It is a laborious task, however, to calculate the coefficients. The details of the calculations are given in Appendices A, B, and C.

We have calculated the differential cross sections in two different ways. The first is the usual technique of four-component spinors and projection operators, and is briefly described in Appendix A. The second is the two-component spinor technique[1,10,11] and is described in Appendix C. This method is based on the fact that at high energies the leptons are involved in the weak interaction through only two components. Therefore, if at the start we go over to the two-component way of writing the spinors, then the only one among the matrix elements for the various polarizations of the initial and final particles that is different from zero is the one with each lepton polarized in the direction opposite to its momentum and each antilepton polarized along its momentum. The calculation of the differential cross section accordingly reduces to the calculation of this one matrix element, and there is no need to average over the polarizations of the leptons (that is, to take traces). The integration of the differential cross sections is contained in Appendix B.

As can be seen from the table and the expressions for the total cross sections σ, the numerical coefficients of G^4s^3 are small ($\sim 10^{-5}$). The result of this is that in spite of the rapid increase with energy the cross sections for the four-particle processes 9—12 at energies E of the order of 100 BeV are still much smaller than that for the two-particle process 6.

It is interesting to note that if the increase of the weak interactions ceases at energies of the order of several hundred BeV owing to a "dispersion" of the vertices, it may turn out that inelastic processes of the type of 9—12 will never become important.

To find the values of the energy at which a dispersion of the weak-interaction vertices must necessarily become apparent, we consider the two-particle weak processes 1—4, 6 from the point of view of the unitarity of the S matrix.

A simple analysis (see [12]) shows that in processes 1 and 3 the only nonvanishing wave is that with J = 0, and in processes 2 and 4, the wave with J = 1. In fact, in the relativistic limit $E \gg m$ the leptons are involved in weak interactions through only two components. Particles (electrons, neutrinos) have left-handed helicity, and antiparticles (positrons, antineutrinos) have right-handed helicity. Since the spin state is fixed in this way and, as is well known, the V – A amplitude is antisymmetric under the interchange of the two initial (or of the two final) particles, it follows that in collisions of two particles the only contribution to the amplitude is that of the wave with J = 0, and for particle-antiparticle collisions the only contribution is that with J = 1.

This can also be verified directly by substitution of these values of the helicities in the general expression for a helicity amplitude [see Eq. (31) and Table 1 of the paper by Jacob and Wick[12]].

The results are

$$d\sigma = |f|^2 d\Omega,$$
$$f = T^0(s) / \sqrt{s} \quad \text{for} \quad J = 0,$$
$$f = {}^3/_2 T^1(s) s^{-1/2}(1 + \cos\vartheta) \quad \text{for} \quad J = 1,$$
$$iT = e^{2i\delta} - 1, \quad |T| \leqslant 2 \quad \text{for elastic scattering},$$
$$T = e^{2i\delta}, \quad |T| \leqslant 1 \quad \text{for an inelastic process}.$$

The restrictions on the magnitude of T are due to the requirement that the S matrix be unitary. By

2)As V. B. Berestetskii has remarked, there are also contributions to processes 9–12 from diagrams of order Ge^2 (of the type of Fig. 3), and at energies $E \lesssim 100$ BeV the contributions of these diagrams are important and exceed those of the diagrams of Fig. 2.

Serial No.	Process	σ	σ_{max}	E_0, BeV	$\sigma_{100}\times 10^{34}$, cm²	Reference
1	$\nu_e e^- \to \nu_e e^-$	$\frac{G^2 s}{\pi}$	$\frac{8\pi}{s}$	440	6.5	[2-4]
2	$\bar{\nu}_e e^- \to \bar{\nu}_e e^-$	$\frac{G^2 s}{3\pi}$	$\frac{24\pi}{s}$	720	2.2	[2-4]
3	$\nu_\mu e^- \to \nu_e \mu^-$	$\frac{G^2 s}{\pi}$	$\frac{2\pi}{s}$	310	6.5	[2-4]
4	$\bar{\nu}_e e^- \to \bar{\nu}_\mu \mu^-$	$\frac{G^2 s}{3\pi}$	$\frac{6\pi}{s}$	540	2.2	[2-4]
5	$\gamma e^- \to \mu^- \bar{\nu}_\mu \nu_e$	$\frac{\alpha G^2 s}{36\pi^2} \ln \frac{s}{m_\mu^2}$			$6\cdot 10^{-3}$	[5-7]
6	$e^+e^- \to \nu_e \bar{\nu}_e$	$\frac{G^2 s}{6\pi}$	$\frac{3\pi}{s}$	540	1.1	
7	$e^+e^- \to \nu_e \bar{\nu}_e \gamma$	$\frac{\alpha G^2 s}{6\pi^2} \ln \frac{s}{m_e^2} \ln \frac{s}{4\omega_{min}^2}$			~ 0.1	
8	$e^+e^- \to \mu^+\mu^-$	$\frac{G^2 s}{6\pi} \frac{G^2 \Lambda^4}{(2\pi)^4}$	$\sigma_{el} = \frac{4\pi\alpha}{3s}$	$E_0' = 160$	$1.1\cdot 10^{-2}$	[8, 9, 13]
9	$e^+e^- \to e^+\mu^-\nu_e\bar{\nu}_\mu$	$\frac{G^4 s^3}{(2\pi)^5 45}\left[\frac{7}{24} + 10 - \pi^2\right]$			$4.2\cdot 10^{-6}$	
10	$e^+e^- \to \mu^+\mu^-\nu_\mu\bar{\nu}_\mu$	$\frac{G^4 s^3}{(2\pi)^5 45}\frac{1}{6}$			$1.6\cdot 10^{-6}$	
11	$e^+e^- \to \mu^+\mu^-\nu_e\bar{\nu}_e$	$\frac{G^4 s^3}{(2\pi)^5 45}\frac{1}{8}$			$1.2\cdot 10^{-6}$	
12	$e^-e^- \to \mu^- e^- \bar{\nu}_\mu \nu_e$	$\frac{G^4 s^3}{(2\pi)^5 45}\frac{1+10-\pi^2}{2}$			$5.5\cdot 10^{-6}$	

Remarks. σ is calculated in first order in G for 1–4, in first order in G and e for 5, 7, and in second order in G for 8–12. $G = 10^{-5}/m_p^2$, $e^2 = \alpha = 1/137$.

σ_{max} is the maximum value of the cross section for the two-particle reaction occurring in the channel with total angular momentum J = 0 (processes 1 and 3) or J = 1 (processes 2, 4, 6). E_0 is the energy at which $\sigma_{max} \times \sigma_{100}$ is the value of σ at energy E = 100 BeV, where E is the energy of each of the colliding particles in the c.m.s.; $s = 4E^2$.

For process 7 σ is calculated on the assumption that $\ln (s/m_e^2) \gg 1$. ω_{min} is the minimum photon energy detected in the experiment. σ_{100} is calculated for the case $4\omega_{min}^2/s = 10^{-2}$.

For process 8 σ_{el} is the cross section for the electromagnetic conversion $e^-e^+ \to \mu^-\mu^+$ (see [14]). E_0 is the energy at which $\sigma = \sigma_{el}$. The quantities E_0' and σ_{100} for process 8 are calculated on the assumption that $G\Lambda^2/(2\pi)^2 = 0.1$, where Λ is the cut-off parameter.

using these restrictions we easily get the maximum values σ_{max} of the cross sections, which are given in the table. In accordance with the procedure for obtaining the cross sections σ, the values σ_{max} are obtained by averaging over the initial polarizations of the electron (and positron), while there is of course no such averaging over states of the neutrinos. The table also gives the energy E_0 at which increase of the cross section comes into conflict with the unitarity condition. For $E \gtrsim E_0$ further terms of the perturbation-theory expression for the scattering amplitude become important; the effective interaction is no longer a point interaction, and states with higher orbital angular momenta come into play. We see that the energy E_0 is different for different vertices and lies in the range 300–700 BeV. At energies of the order of 500 BeV the cross sections for processes 9–12 are of the order of 10^{-36} cm², and thus are still extremely small ($\sim 10^{-3}$) as compared with the cross sections for the two-particle processes (1–4, 6). Possibly the smallness of processes of the type of 9–12 means that the dispersion of the weak interaction is mainly due to two-particle virtual states (in both the s channel and the t channel).

Measurements of cross sections of the order of 10^{-36} cm² at E ~ 500 BeV are about three or four orders of magnitude beyond present experimental possibilities both as to energy and as to beam intensity.[3)] The suggested experiments, however,

[3)]As is well known, experimental colliding-beam apparatus is being constructed at a number of laboratories (Novosibirsk, Stanford, Frascatti, and so on). The possibilities of these installations can be judged, for example, from the following data. Experiments are now being begun with the Frascatti storage ring at energy E = 250 MeV and effective intensity 6×10^{30} cm^{-2} hr^{-1} (counting rate 6 events per hour at cross section 10^{-30} cm²). In 1965 a storage ring with energy 750 MeV and intensity 10^{33} cm^{-2} hr^{-1} is to come into operation. There is a project for construction at Brookhaven of a 70 BeV electron accelerator (in this connection see the survey lectures [13]).

are not absolutely unrealistic, unlike experiments with neutrino beams at energies of 10^9 BeV. Of course if the W meson is observed experimentally, it may be that energies much smaller than those we have been considering will suffice for the study of the dynamics of the weak interaction.

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APPENDIX A

CALCULATION OF THE MATRIX ELEMENTS

Let us consider the two diagrams M_7 and M_8 (Fig. 2) which describe the weak interaction of leptons in second-order perturbation theory. The correspondence between the indices of the diagrams in Fig. 2 and the particles involved in processes 9—12 is as follows:

process 9

$$\begin{matrix} e^+ & e^- & \rightarrow & e^+ & \mu^- & \nu_e & \bar{\nu}_\mu \\ 4 & 1 & & 3 & 2 & 6 & 5 \end{matrix}$$

process 10

$$\begin{matrix} e^+ & e^- & \rightarrow & \mu^+ & \mu^- & \nu_\mu & \bar{\nu}_\mu \\ 4 & 1 & & 3 & 2 & 6 & 5 \end{matrix}$$

process 11

$$\begin{matrix} e^+ & e^- & \rightarrow & \mu^+ & \mu^- & \nu_e & \bar{\nu}_e \\ 2 & 3 & & 1 & 4 & 6 & 5 \end{matrix}$$

process 12

$$\begin{matrix} e^- & e^- & \rightarrow & e^- & \mu^- & \nu_e & \bar{\nu}_\mu \\ 1 & 3 & & 4 & 2 & 6 & 5 \end{matrix}$$

Processes 9 and 12 are described by the difference of diagrams M_7 and M_8; processes 10, 11 are described by diagram M_7 alone.

The matrix elements M_7 and M_8 (for which we use the same symbols as for the diagrams of Fig. 2) can be written out in the following way:

$$M_7 = {}^1/_2 G^2 \bar{u}_2 O_i u_1 \cdot \bar{u}_6 O_k \hat{p}_7 p_7^{-2} O_i u_5 \cdot \bar{u}_4 O_k u_3,$$

$$M_8 = {}^1/_2 G^2 \bar{u}_2 O_i u_3 \cdot \bar{u}_6 O_k \hat{p}_8 p_8^{-2} O_i u_5 \cdot \bar{u}_4 O_k u_1, \qquad (A.1)$$

where $O_i = \gamma_i(1 + \gamma_5)$, and the masses of all particles have been set equal to zero.

In the calculation of the squares of the matrix elements it is convenient to introduce the following notations (cf. [4], pages 36 and 70)

$$\chi_{ilkm} = t_{ilkm} + i\varepsilon_{ilkm},$$

$$t_{ilkm} = \delta_{ik}\delta_{lm} + \delta_{im}\delta_{lk} - \delta_{il}\delta_{km} \qquad (A.2)$$

and to use the relation

$$\gamma_i\gamma_k\gamma_l = [t_{ilkm} + i\varepsilon_{ilkm}\gamma_5]\gamma_m,$$
$$\gamma_i\gamma_k\gamma_l(1+\gamma_5) = \chi^*_{ilkm}\gamma_m(1+\gamma_5) = \chi_{likm}\gamma_m(1+\gamma_5),$$
$$\gamma_i\gamma_k\gamma_l(1-\gamma_5) = \chi_{ilkm}\gamma_m(1-\gamma_5). \qquad (A.3)$$

In the approximation in which the masses of all leptons are set equal to zero we get for the squares of the matrix elements:[4)]

$$|M_7|^2 = \frac{2^{13}G^4}{p_7^4}(p_1p_5)(p_4p_6)[2(p_2p_7)(p_3p_7) - p_7^2(p_2p_3)] \equiv 2^{13}G^4|\mathfrak{M}_7|^2,$$
$$|M_8|^2 = \frac{2^{13}G^4}{p_8^4}(p_3p_5)(p_4p_6)[2(p_1p_8)(p_2p_8) - p_8^2(p_1p_2)] \equiv 2^{13}G^4|\mathfrak{M}_8|^2,$$
$$M_7M_8^* + M_7^*M_8 = -\frac{2^{13}G^4}{p_7^2p_8^2}\mathrm{Re}[\chi_{\alpha\beta\gamma\nu}\chi_{\alpha'\beta'\gamma'\nu}]$$
$$\times (p_4p_6)(p_{1\alpha}p_{3\beta}p_{5\gamma}p_{8\alpha'}p_{7\beta'}p_{2\gamma'} \equiv -2^{13}G^4\mathfrak{M}_{78}, \qquad (A.4)$$

where

$$\mathrm{Re}[\chi_{\alpha\beta\gamma\nu}\chi_{\alpha'\beta'\gamma'\nu}] = t_{\alpha\beta\gamma\nu}t_{\alpha'\beta'\gamma'\nu} - \varepsilon_{\alpha\beta\gamma\nu}\varepsilon_{\alpha'\beta'\gamma'\nu},$$
$$|M|^2 = |M_7|^2 + |M_8|^2 + M_7M_8^* + M_7^*M_8$$
$$= 2^{13}G^4|\mathfrak{M}|^2 = 2^{13}G^4[|\mathfrak{M}_7|^2 + |\mathfrak{M}_8|^2 - \mathfrak{M}_{78}]. \qquad (A.5)$$

The expression for the total cross section for the process described by the diagrams 7 and 8 is as follows (the process is regarded as occurring according to the scheme $1 + 3 \rightarrow 2 + 4 + 5 + 6$)[5)]:

$$\sigma(2, 4, 5, 6) = \frac{G^4}{4\pi^8 s}\int \frac{d\mathbf{p}_2}{E_2}\frac{d\mathbf{p}_4}{E_4}\frac{d\mathbf{p}_5}{E_5}\frac{d\mathbf{p}_6}{E_6} \qquad (A.6)$$
$$\times |\mathfrak{M}|^2\delta^4(p_1 + p_3 - p_2 - p_4 - p_5 - p_6) = \sigma_7 + \sigma_8 - \sigma_{78},$$

where $s = 2p_1p_3 = 4E^2$.

APPENDIX B

CALCULATION OF THE TOTAL CROSS SECTIONS

Process 12:

$$\begin{matrix} e^- & e^- & \rightarrow & \mu^- & e^- & \bar{\nu}_\mu & \nu_e. \\ 1 & 3 & & 2 & 4 & 5 & 6 \end{matrix}$$

It is easy to see that for this process $\sigma_7(\mu^- e^- \bar{\nu}_\mu \nu_e) = \sigma_8(\mu^- e^- \bar{\nu}_\mu \nu_e)$ (the squares of the matrix elements, $|\mathfrak{M}_7|^2$ and $|\mathfrak{M}_8|^2$, differ only by the interchange $p_1 \rightleftarrows p_3$). The calculation of σ_7 is made as follows. We introduce the variables $Q = p_2 + p_5$ and $R = p_4 + p_6$; then $Q + R = p_1 + p_3$, $p_7 = p_1 - Q = -p_3 + R$, and $p_8 = p_3 - Q = -p_1 + R$.

4)The sign in the expression for $M_7M_8^* + M_7M_8$ allows for the fact that the difference of M_7 and M_8 is used.

5)The average over spins of the initial leptons has been taken.

By using the relation

$$\int q_{1\alpha}q_{2\beta}\frac{d\mathbf{q}_1}{\omega_1}\frac{d\mathbf{q}_2}{\omega_2}\delta^4[q_1+q_2-q]=\frac{\pi}{6}[q^2\delta_{\alpha\beta}+2q_\alpha q_\beta], \quad \text{(B.1)}$$

we transform the expression for σ_7 to the form

$$\sigma_7(\mu^-e^-\bar{\nu}_\mu\nu_e)=\frac{G^4}{16\pi^8E^2}\int\frac{d^4Q}{p_7^4}\pi R^2[2(p_3p_7)p_{7\alpha}-p_7^2p_{3\alpha}]p_{1\beta}\frac{\pi}{6}\times[Q^2\delta_{\alpha\beta}+2Q_\alpha Q_\beta]. \quad \text{(B.2)}$$

When we use the relations

$$p_1p_7={}^1\!/_2(p_7^2-Q^2)=-p_1Q,\quad p_7Q=-{}^1\!/_2(Q^2+p_7^2),$$
$$p_3p_7={}^1\!/_2(R^2-p_7^2)=p_3R,\quad p_3Q=p_1p_3+{}^1\!/_2(p_7^2-R^2), \quad \text{(B.3)}$$

we have

$$\sigma_7(\mu^-e^-\bar{\nu}_\mu\nu_e)=\frac{G^4}{96\pi^6E^2}\int\frac{d^4Q}{p_7^4}\{-Q^4R^4-2(QR)p_7^2Q^2R^2+p_7^4[(p_1p_3)-Q^2)^{R2}\}. \quad \text{(B.4)}$$

In the center-of-mass system of the colliding electrons the four-vectors p_1, p_3, p_7, Q, R take the following forms:

$$p_1=\{E,\mathbf{E}\},\quad p_3=\{E,-\mathbf{E}\},$$
$$Q=\{\omega_1,\mathbf{q}\},\quad R=\{\omega_2,-\mathbf{q}\},$$
$$p_7=\{E-\omega_1,\ \mathbf{E}-\mathbf{q}\},\quad \omega_1+\omega_2=2E$$

and $p_7^2=-QR+2Eqx$, where x is the cosine of the angle between the vectors **E** and **q**. The integration over the angle φ gives the factor 2π; the integration over x is between the limits $-1\le x\le+1$; the integration over q is between the limits $0\le q\le E$; and that over ω_1 is between the limits $q\le\omega_1\le 2E-q$.

We now give the integrals over the angular and energy variables which are encountered in our further calculations (L stands for the expression

$$L=\ln\frac{(\omega_1+q)(\omega_2+q)}{(\omega_1-q)(\omega_2-q)}\Big).$$

Setting

$$I(f)=\int_{-1}^{+1}f\,dx,$$

we have

$$I(p_7^4)={}^2\!/_3[3(QR)^2+4E^2q^2],\quad I(p_7^2)=-2(QR),$$
$$I(1)=2,\ I(p_7^{-2})=-L/2Eq,\quad I(p_7^{-4})=2/Q^2R^2 \quad \text{(B.5)}$$

and, setting

$$J(\varphi)=\int_0^E dq\int_q^{2E-q}q^2d\omega_1\varphi,$$

we have

$$J(Q^2R^2)=\frac{2}{45}E^8,\quad J(Q^2)=J(R^2)=\frac{1}{9}E^6,\quad J(q^2)=\frac{1}{15}E^6,$$
$$J(Q^4)=\frac{2}{15}E^8,\quad J\Big[\frac{Q^2R^2(QR)L}{Eq}\Big]=\frac{4}{15}E^8,\ J[(QR)^2]=\frac{14}{45}E^8,$$
$$J\Big\{\frac{\omega_2^2[(QR)-4E^2]L}{q}\Big\}=-\frac{263}{90}E^7,$$
$$J\Big[\frac{\omega_2^3\omega_1}{q(QR)}L\Big]=\Big(\frac{\pi^2}{15}+\frac{1}{8}\Big)E^5. \quad \text{(B.6)}$$

The method for calculating the last integral requires particular attention. Here it is convenient to change the procedure even before doing the integration over x, and to choose as variables of integration the components of the four-vector $p_7=\{\omega,\mathbf{p}\}$. In these variables the integral $\int d^4Q$ is converted into the integral

$$\int d^4Q=\int_{-E}^{E}d\omega\int_{|\omega|}^{2E-|\omega|}p^2dp\int_{-1}^{\bar{x}}dx',\quad \bar{x}=\frac{\omega^2-p^2-2E|\omega|}{2Ep}$$

where x′ is the cosine of the angle between the vectors **p** and **E**. The specific integrals for which this method must be used are

$$F_1=E^4\int\frac{d^4Q}{p_7^2p_8^2},\quad F_2=\int\frac{\omega^4d^4Q}{p_7^2p_8^2}. \quad \text{(B.7)}$$

After we introduce the variables $z=\omega+p$, $y=\omega-p$ and perform the integration over x′

$$\int_{-1}^{\bar{x}}\frac{dx'}{p_8^2}=\frac{1}{4Epzy}\ln\frac{y+2E}{y-2E} \quad \text{(B.8)}$$

the first of these integrals is reduced to the form[6)]

$$F_1=\frac{E^3}{8}\Big\{\int_{-2E}^{0}\frac{dy}{y}\ln\frac{y+2E}{y-2E}\int_{-y}^{2E}dz-\int_0^{2E}\frac{dz}{z}\int_{-z}^{0}dy\ln\frac{y+2E}{y-2E}\Big\}. \quad \text{(B.9)}$$

The integrals that appear here are elementary or are given in tables.[7)]

The integral F_2 is calculated in an analogous way, except that the orders of integrations over y and z have to be changed in a suitable way. The results are

$$F_1=\frac{\pi^3}{12}E^4,\quad F_2=\pi E^4\Big[\frac{\pi^2}{60}-\frac{1}{8}\Big]. \quad \text{(B.10)}$$

Using (B.5) and (B.6), we get as the final expres-

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6)We have here made use of the fact that the integrand is an even function of ω.

7)See, for example, [15], formulas 4.291 (1,2). We note, by the way, that the expressions for the indefinite integrals 2.729 (1,3) are incorrect [in the coefficient of ln (a + bx) the a should be changed to −a].

sion for $\sigma_7(\mu^- e^- \bar{\nu}_\mu \nu_e)$:

$$\sigma_7(\mu^- e^- \bar{\nu}_\mu \nu_e) = G^4 E^6 / 90\pi^5. \quad \text{(B.11)}$$

The expression for $\sigma_{78}(\mu^- e^- \bar{\nu}_\mu \nu_e)$ is of the form

$$\sigma_{78} = \frac{G^4}{16\pi^8 E^2} \int \frac{dp_2 dp_4 dp_5 dp_6}{E_2 E_4 E_5 E_6} \frac{p_{1\alpha} p_{3\beta} p_{5\gamma} p_{8\alpha'} p_{7\beta'} p_{2\gamma'}}{p_7^2 p_8^2}$$

$$\times \mathrm{Re}[\chi_{\alpha\beta\gamma\nu} \chi_{\alpha'\beta'\gamma'\nu}]\, \delta^4 [p_1 + p_3 - p_2 - p_4 - p_5 - p_6]. \quad \text{(B.12)}$$

Introducing the quantities Q and R as before, we transform σ_{78} to the form

$$\sigma_{78}(\mu^- e^- \bar{\nu}_\mu \nu_e) = \frac{G^4}{16\pi^8 E^2} \int d^4 Q \frac{1}{p_7^2 p_8^2} \frac{\pi^2}{6} R^2 [Q^2 \delta_{\gamma\gamma'} + 2 Q_\gamma Q_{\gamma'}]$$

$$\times p_{1\alpha} p_{3\beta} (p_3 - Q)_{\alpha'} (p_1 - Q)_{\beta'} [t_{\alpha\beta\gamma\nu} t_{\alpha'\beta'\gamma'\nu} - \varepsilon_{\alpha\beta\gamma\nu} \varepsilon_{\alpha'\beta'\gamma'\nu}]. \quad \text{(B.13)}$$

Summation over α, β, γ, α', β', γ' leads to the expression

$$\sigma_{78}(\mu^- e^- \bar{\nu}_\mu \nu_e) = \frac{G^4}{16\pi^8 E^2} \frac{\pi^2}{6} \int d^4 Q \frac{R^2}{p_7^2 p_8^2} \{8Q^2 (p_1 Q)(p_3 Q)$$

$$+ 8Q^2 (p_1 p_3)^2 - 4Q^2 (p_1 p_3)(p_1 Q) - 4Q^2 (p_1 p_3)(p_3 Q)$$

$$- 2Q^4 (p_1 p_3) - 8 (p_1 p_3)(p_1 Q)(p_3 Q)\}. \quad \text{(B.14)}$$

After the integration over the angle variables we have

$$\sigma_{78}(\mu^- e^- \bar{\nu}_\mu \nu_e) = \frac{G^4}{16\pi^8 E^2} \frac{2\pi^3}{6} \int_0^E d\omega_1 \int_{\omega_1}^{2E-\omega_1} q^2 dq$$

$$\times \left\{ -\frac{(QR) Q^2 R^2}{Eq} L - 8E^2 R^2 + 4Q^2 R^2 + \frac{4E}{q} \omega_2^2 \right.$$

$$\left. \times [(QR) - 4E^2] L + \frac{16 E^3 \omega_2^3 \omega_1}{q (QR)} L \right\}. \quad \text{(B.15)}$$

The values of the integrals which appear here are given in (B.6), and the final result for σ_{78} is

$$\sigma_{78}(\mu^- e^- \bar{\nu}_\mu \nu_e) = \frac{G^4 E^6}{45\pi^5} (\pi^2 - 10).$$

Process 9:

$$\begin{matrix} e^- e^+ \to \mu^- & e^+ & \bar{\nu}_\mu & \nu_e \\ 1\;4 \quad\; 2 & 3 & 5 & 6 \end{matrix}$$

For this process $\sigma_7(\mu^- e^+ \bar{\nu}_\mu \nu_e) \neq \sigma_8(\mu^- e^+ \bar{\nu}_\mu \nu_e)$. We first consider the expression for σ_7:

$$\sigma_7(\mu^- e^+ \bar{\nu}_\mu \nu_e) = \frac{G^4}{16\pi^8 E^2} \int \frac{dp_2 dp_3 dp_5 dp_6}{E_2 E_3 E_5 E_6} \frac{(p_1 p_5)(p_4 p_6)}{p_7^4}$$

$$\times [2 (p_2 p_7)(p_3 p_7) - p_7^2 (p_2 p_3)]$$

$$\times \delta^4 [p_1 + p_4 - p_2 - p_3 - p_5 - p_6].$$

We introduce the variables

$$Q = p_2 + p_5 = p_1 - p_7, \quad R' = p_3 + p_6 = p_4 + p_7;$$

$$Q + R' = p_1 + p_4$$

and make $\sigma_7(\mu^- e^+ \bar{\nu}_\mu \nu_e)$ into an integral over Q:

$$\sigma_7(\mu^- e^+ \bar{\nu}_\mu \nu_e) = \frac{G^4}{16\pi^8 E^2} \frac{\pi^2}{36} \int \frac{d^4 Q}{p_7^4} p_{1\alpha} p_{4\alpha'} (Q^2 \delta_{\alpha\beta} + 2 Q_\alpha Q_\beta)$$

$$\times (R'^2 \delta_{\alpha'\beta'} + 2 R'_{\alpha'} R'_{\beta'}) (2 p_{7\beta} p_{7\beta'} - p_7^2 \delta_{\beta\beta'})$$

$$= \frac{G^4}{16\pi^8 E^2} \frac{\pi^2}{36} \int \frac{d^4 Q}{p_7^4} \{ -2 Q^4 R'^4 - 4 p_7^2 Q^2 R'^2 (QR')$$

$$+ p_7^4 \left[2 (p_1 p_4)(Q^2 + R'^2) - \frac{5}{2} Q^2 R'^2 \right]$$

$$- p_7^6 (QR') - \frac{1}{2} p_7^8 \}.$$

Using (B.5) and (B.6), we finally get

$$\sigma_7(\mu^- e^+ \bar{\nu}_\mu \nu_e) = G^4 E^6 / 135\pi^5. \quad \text{(B.16)}$$

The expression for $\sigma_8(\mu^- e^+ \nu_\mu \nu_e)$ is

$$\sigma_8(\mu^- e^+ \bar{\nu}_\mu \nu_e) = \frac{G^4}{16\pi^8 E^2} \int \frac{dp_2 dp_3 dp_5 dp_6}{E_2 E_3 E_5 E_6} \frac{(p_3 p_5)(p_4 p_6)}{p_8^4}$$

$$\times [2 (p_2 p_8)(p_1 p_8) - p_8^2 (p_1 p_2)]$$

$$\times \delta^4 (p_1 + p_4 - p_2 - p_3 - p_5 - p_6). \quad \text{(B.17)}$$

Using the fact that $p_3 + p_5 = p_8 - p_2$ and integrating over dp_5 and dp_8, we get

$$\sigma_8(\mu^- e^+ \bar{\nu}_\mu \nu_e) = \frac{G^4}{16\pi^8 E^2} \pi \int \frac{dp_2 dp_6}{E_2 E_6} \frac{(p_8 - p_2)^2}{p_8^4} [2 (p_2 p_8)(p_1 p_8)$$

$$- p_8^2 (p_1 p_2)]. \quad \text{(B.18)}$$

Let us consider the integral

$$\int (p_8 - p_2)^2 p_{2\alpha} \frac{dp_2}{E_2}. \quad \text{(B.19)}$$

It must obviously be of the form $A p_8^4 p_{8\alpha}$. We find the constant A by calculating the integral in a system in which $\mathbf{p}_8 = 0$ (the limits of integration are $0 \leq E_2 \leq E_8/2$): $A = \pi/24$. Then

$$\sigma_8(\mu^- e^+ \bar{\nu}_\mu \nu_e) = \frac{G^4}{16\pi^8 E^2} \frac{\pi^2}{24} \int \frac{dp_6}{E_6} (p_4 p_8)(p_1 p_8) p_8^2. \quad \text{(B.20)}$$

Using the fact that $p_8 = p_1 + p_4 - p_6 \equiv P - p_6$, we calculate the remaining integral:

$$\int p_8^2 p_{6\alpha} p_{8\beta} \frac{dp_6}{E_6} = \frac{\pi P^4}{240} [6 P_\alpha P_\beta + P^2 \delta_{\alpha\beta}] \quad \text{(B.21)}$$

and we finally get

$$\sigma_8(\mu^- e^+ \bar{\nu}_\mu \nu_e) = G^4 E^6 / 180\pi^5. \quad \text{(B.22)}$$

Let us now consider the expression for $\sigma_{78}(\mu^- e^+ \bar{\nu}_\mu \nu_e)$. After integrating over the variables p_2 and p_5 ($Q = p_2 + p_5 = p_1 + p_4 - p_3 - p_6$, $p_7 = p_1$

$-Q$, $p_8 = -p_3 - Q$), we have

$$\sigma_{78}(\mu^- e^+ \bar{\nu}_\mu \nu_e) = \frac{G^4}{16\pi^8 E^2}\frac{\pi}{6}\int \frac{d\mathbf{p}_3 d\mathbf{p}_6}{E_3 E_6}\frac{1}{p_7^2 p_8^2}\{8Q^2(p_1Q)(p_3Q)$$

$$-8Q^2(p_1p_3)^2 - 4Q^2(p_1p_3)(p_1Q) + 4Q^2(p_1p_3)(p_3Q)$$

$$-2Q^4(p_1p_3) + 8(p_1p_3)(p_1Q)(p_3Q)\} \qquad (B.23)$$

[the expression in the curly brackets is obtained from the corresponding expression for $\sigma_{78}(\mu^- e^- \bar{\nu}_\mu \nu_e)$ by changing the sign of the whole expression and the sign of p_3]. We now do the integration over dp_3. For this we choose a system in which p_8 has only the fourth component: $E_8 \equiv -M$. In this system $Q^2 = M(M - 2E_3)$,

$$(p_1p_3) = E_1E_3(1-x), \quad p_3Q = E_3M,$$

$$(p_1Q) = E_1M - E_1E_3(1-x),$$

$$p_7^2 = (p_1 - Q)^2 = M^2 - 2M(E_1 + E_3) + 2E_1E_3(1-x), \qquad (B.24)$$

and the limits for the integration over $d\mathbf{p}_3$ are: $0 \le E_3 \le M/2$; $-1 \le x \le 1$ (x is the cosine of the angle between the vectors $\mathbf{p}_1$ and $\mathbf{p}_3$).

We put the resulting expression in covariant form by making the replacements

$$M^2 \to p_8^2, \quad E_1M \to -(p_1p_8), \quad E_1^2 \to (p_1p_8)^2/p_8^2.$$

The result is then

$$\sigma_{78}(\mu^- e^+ \bar{\nu}_\mu \nu_e) = -\frac{G^4}{16\pi^8 E^2}\frac{\pi^2}{3}\int \frac{d\mathbf{p}_6}{E_6}(p_4p_6)\left\{\frac{(p_1p_8)\,p_8^2}{2} + 2(p_1p_8)^2\right.$$

$$\left. -2(p_1p_8)^2\left[\frac{2(p_1p_8)}{p_8^2} + 1\right]\ln\frac{p_8^2 + 2(p_1p_8)}{2(p_1p_8)}\right\}. \qquad (B.25)$$

The integration over dp_6 can be done in an elementary way in the center-of-mass system of particles 1 and 4. The final result is

$$\sigma_{78}(\mu^- e^+ \bar{\nu}_\mu \nu_e) = \frac{2G^4E^8}{45}(\pi^2 - 10). \qquad (B.26)$$

The cross section for the process $e^- e^+ \to \mu^+ e^- \nu_\mu \bar{\nu}_e$ is equal to that for $e^- e^+ \to \mu^- e^+ \bar{\nu}_\mu \nu_e$, since these are charge-conjugate processes.

Process 10:

$$\underset{1\ 4}{e^+e^-} \to \underset{2\ 3\ 5\ 6}{\mu^+\mu^-\nu_\mu\bar{\nu}_\mu}$$

This process is described by the single matrix element M_7. It is easy to see that

$$\sigma(\mu^+\mu^-\nu_\mu\bar{\nu}_\mu) = \sigma_7(e^+\mu^-\nu_e\bar{\nu}_\mu) = G^4E^6/135\pi^5. \quad (B.27)$$

Process 11:

$$\underset{3\ 2}{e^+e^-} \to \underset{1\ 4\ 6\ 5}{\mu^+\mu^-\nu_e\bar{\nu}_e}$$

This process is also described by the single matrix element M_7:

$$\sigma(\mu^+\mu^-\nu_e\bar{\nu}_e) = \frac{G^4}{16\pi^8E^2}\int \frac{d\mathbf{p}_1 d\mathbf{p}_4 d\mathbf{p}_5 d\mathbf{p}_6}{E_1E_4E_5E_6}$$

$$\times \delta^4(p_2 + p_3 - p_1 - p_4 - p_5 - p_6)$$

$$\times \frac{1}{p_7^4}(p_1p_5)(p_4p_6)[2(p_2p_7)(p_3p_7) - p_7^2(p_2p_3)]. \quad (B.28)$$

Introducing the variables $Q' = p_1 + p_5 = p_2 - p_7$, $R' = p_4 + p_6 = p_3 + p_7$ and transforming the integral into an integral over Q', we have

$$\sigma(\mu^+\mu^-\nu_e\bar{\nu}_e)$$

$$= \frac{G^4}{16\pi^8E^2}\pi^2\int d^4Q'\,\frac{Q'^2R'^2[2(p_2p_7)(p_3p_7) - p_7^2(p_2p_3)]}{p_7^4}$$

$$= \frac{G^4}{16\pi^8E^2}\frac{\pi^2}{2}\int d^4Q'\left\{-Q'^2R'^2 - \frac{Q'^2R'^2(Q'R')}{2p_7^2} - \frac{Q'^4R'^4}{p_7^4}\right\}. \qquad (B.29)$$

When we now use the integrals (B.5) and (B.6), we get finally

$$\sigma(\mu^+\mu^-\nu_e\bar{\nu}_e) = G^4E^6/180\pi^5. \qquad (B.30)$$

APPENDIX C

THE TWO-COMPONENT METHOD

At high energies, when the lepton masses can be neglected, it is especially convenient to use the two-component way of writing spinors[1,10,11]

$$\psi(x) = u(x) + v(x), \qquad u(x) = \frac{1}{2}(1 + \gamma_5)\psi(x),$$

$$v(x) = \frac{1}{2}(1 - \gamma_5)\psi(x). \qquad (C.1)$$

The two-component spinors $u(p)$ satisfy the equation

$$(E + \boldsymbol{\sigma}\mathbf{p})u(p) = 0 \qquad (C.2)$$

(where $\boldsymbol{\sigma}$ is the two-row Pauli matrices), and for $E > 0$ this is

$$(1 + \boldsymbol{\sigma}\mathbf{n})u(p) = 0, \quad \mathbf{n} = \mathbf{p}/|\mathbf{p}| \qquad (C.3)$$

and has the solution $u = \binom{0}{1}$, which describes the state of the lepton with spin projection $s_z = -1/2$ (the z axis is along the momentum).

If we introduce the matrix four-vector

$$\zeta_\mu = \{1, -\boldsymbol{\sigma}\}, \qquad (C.4)$$

Eq. (C.2) can be written in the form

$$p_\mu\zeta_\mu u = 0. \qquad (C.5)$$

Besides the four-vector ζ_μ it is convenient to use also the four-vector $\bar{\zeta}_\mu$ given by

$$\bar{\zeta}_\mu = \{1, \boldsymbol{\sigma}\}. \qquad (C.6)$$

We note some properties of the matrices ζ_μ and $\bar{\zeta}_\mu$ which follow immediately from the definitions (C.4) and (C.6):

$$\zeta_\mu\zeta_\mu = \bar{\zeta}_\mu\bar{\zeta}_\mu = -2,$$
$$\zeta_\mu\bar{\zeta}_\nu + \zeta_\nu\bar{\zeta}_\mu = \bar{\zeta}_\mu\zeta_\nu + \bar{\zeta}_\nu\zeta_\mu = 2\delta_{\mu\nu},$$
$$\zeta_\mu\bar{\zeta}_\nu\zeta_\mu = -2\zeta_\nu, \quad \bar{\zeta}_\mu\zeta_\lambda\bar{\zeta}_\mu = -2\bar{\zeta}_\lambda,$$
$$\bar{\zeta}_\mu\zeta_\lambda\bar{\zeta}_\rho\zeta_\mu = \zeta_\mu\bar{\zeta}_\lambda\zeta_\rho\bar{\zeta}_\mu = 4\delta_{\lambda\rho},$$
$$\bar{\zeta}_\mu\zeta_\nu\bar{\zeta}_\lambda = \chi_{\mu\nu\lambda\sigma}\bar{\zeta}_\sigma, \quad \zeta_\mu\bar{\zeta}_\nu\zeta_\lambda = \chi^*_{\mu\lambda\nu\sigma}\zeta_\sigma. \qquad (C.7)$$

In concrete calculations it is convenient to use a number of identities which hold for the tensor $\chi_{\mu\lambda\nu\sigma}$:

$$\chi_{\mu\lambda\nu\sigma} = \chi^*_{\lambda\mu\nu\sigma}, \quad \chi_{\mu\lambda\nu\sigma} = \chi^*_{\mu\lambda\sigma\nu}, \quad \chi_{\mu\lambda\nu\sigma} = \chi_{\nu\sigma\mu\lambda},$$
$$\chi_{\mu\lambda\nu\tau}\chi_{\tau\rho\sigma\varkappa} = \chi^*_{\nu\sigma\lambda\tau}\chi_{\mu\rho\tau\varkappa} = \chi_{\lambda\rho\sigma\tau}\chi_{\mu\tau\nu\varkappa},$$
$$\chi_{\mu\lambda\nu\tau}\chi_{\tau\nu\sigma\varkappa} = 4\delta_{\lambda\sigma}\delta_{\mu\varkappa}, \quad \chi_{\mu\lambda\nu\tau}\chi_{\tau\rho\mu\varkappa} = 4\delta_{\nu\lambda}\delta_{\rho\varkappa},$$
$$\chi_{\mu\lambda\nu\tau}\chi_{\tau\rho\lambda\varkappa} = 4\chi_{\mu\rho\nu\varkappa}, \quad \chi_{\mu\lambda\nu\tau}\chi_{\tau\rho\nu\varkappa} = -2\chi_{\mu\rho\lambda\varkappa}. \qquad (C.8)$$

These identities can also be obtained easily if we write out the products of sets of matrices ζ_μ and $\bar{\zeta}_\mu$ in different ways by using the last of the formulas (C.7).

The equation (C.5) has a corresponding Green's function G(p),

$$p_\mu\zeta_\mu G(p) = 1, \qquad G(p) = (p_\mu\zeta_\mu)^{-1} = p^{-2}\bar{\zeta}_\mu p_\mu. \quad (C.9)$$

In terms of two-component spinors the matrix element of the weak-interaction Hamiltonian

$$\frac{G}{\sqrt{2}}(\bar{\psi}_1\gamma_\mu(1+\gamma_5)\psi_2)(\bar{\psi}_3\gamma_\mu(1+\gamma_5)\psi_4)$$

can be written in the form

$$4\frac{G}{\sqrt{2}}(u_1{}^*\zeta_\mu u_2)(u_3{}^*\zeta_\mu u_4). \qquad (C.10)$$

We shall take the spinors u normalized with the condition $u^+u = 2E$. Then in the calculation of transition probabilities the normalization constants will be chosen in the usual way, i.e., $N_i = (2E_i)^{-1}$ for each of the particles in the initial and final states.

Let us consider some properties of the matrix element

$$u^*(p_2)\zeta_\mu u(p_1) \equiv f_\mu(p_2, p_1), \qquad (C.11)$$

which appears in the four-fermion interaction. We shall suppose first that $u(p_1)$ and $u(p_2)$ describe lepton (not antilepton) states. It is obvious that

$$p_{1\mu}f_\mu(p_2, p_1) = p_{2\mu}f_\mu(p_2, p_1) = 0, \qquad (C.12)$$
$$f_\mu(p_2, p_1) = f_\mu{}^*(p_1, p_2), \quad f_\mu(p, p) = 2p_\mu. \qquad (C.13)$$

If the vectors p_1 and p_2 are such that $(p_1 + p_2)^2 > 0$, then in the Lorentz system in which $p_1 \equiv \{p_0, p_x, p_y, p_z\} = E_1\{1, 0, 0, 1\}$, $p_2 = E_2\{1, 0, 0, -1\}$ the function $f_\mu(p_2, p_1)$ is of the form

$$f_\mu(p_2, p_1) = 2\sqrt{E_1E_2}\{0, 1, -i, 0\}. \qquad (C.14)$$

In order to find $f_\mu(p_2, p_1)$ in an arbitrary reference system, instead of using the formulas for the transformation of spinors we can write the properties (C.14) (sic) in the form of a system of invariant equations and find the f_μ that solves this system of equations. Besides (C.12) and (C.13) this system contains the equations

$$f_\mu{}^2 = 0, \quad (\mathrm{Re} f_\mu)^2 = (\mathrm{Im} f_\mu)^2 = -2(p_1p_2),$$
$$\mathrm{Re} f_\mu \cdot \mathrm{Im} f_\mu = 0,$$
$$\varepsilon_{\mu\nu\lambda\sigma}\mathrm{Re} f_\mu \cdot \mathrm{Im} f_\nu \cdot p_{1\lambda}p_{2\sigma} = 2(p_1p_2)^2. \qquad (C.15)$$

For the special case in which the momentum p_1 is directed along the z axis and the momentum p_2 ($|p_2| = |p_1|$) makes the angle θ with this axis we find from (C.12), (C.13), (C.15) the result

$$f_\mu(p_2, p_1) = 2E_1\left\{\cos\frac{\theta}{2}, \quad \sin\frac{\theta}{2}, \quad -i\sin\frac{\theta}{2}, \quad \cos\frac{\theta}{2}\right\}. \qquad (C.16)$$

By means of (C.16) we can, for example, obtain at once the expression for the matrix element for scattering of neutrinos by electrons:

$$M_{\nu+e} = 4\frac{G}{\sqrt{2}}(u_e{}^*(p_2)\zeta_\mu u_e(p_1))(u_\nu{}^*(p_4)\zeta_\mu u_\nu(p_3))$$
$$= 4\frac{G}{\sqrt{2}}f_\mu(p_2, p_1)f_\mu(p_4, p_3) = -32\frac{1}{\sqrt{2}}GE^2, \qquad (C.17)$$

if we recall that in the center-of-mass system of the electron and neutrino the initial momentum of the neutrino is $p_3 = -p_1$ and its final momentum is $p_4 = -p_2$, so that $f_\mu(p_4, p_3)$ is obtained from $f_\mu(p_2, p_1)$ by the substitution $\theta \to \pi - \theta$.

The case of antileptons differs from that of leptons only in that the creation operators correspond to the spinors u in the interaction Hamiltonian and the spinors u* contain the annihilation operators [the sign of p drops out of Eq. (C.5)]; that is, for antileptons the order in which the momenta are written in $f_\mu(p_1, p_2)$ is opposite to that for leptons. For example for the matrix element for scattering $\bar{\nu} + e \to \bar{\nu} + e$ we have by (C.13):

$$M_{\bar{\nu}+e} = \frac{4G}{\sqrt{2}}f_\mu(p_2, p_1)f_\mu(p_3, p_4) = -16\frac{1}{\sqrt{2}}GE^2(1+\cos\theta). \qquad (C.18)$$

From Eqs. (C.13) and (C.15) it is not hard to derive the practically important formula

$$f_\alpha(p_2, p_1) f_\beta^*(p_2, p_1) = 2\chi_{\alpha\beta\gamma\delta} p_{2\gamma} p_{1\delta}, \qquad (C.19)$$

by means of which calculations of the squares of matrix elements can be done conveniently.

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