COUPLED MAGNETO-ELASTIC WAVES IN ANTIFERROMAGNETICS WITH A MAGNETIC STRUCTURE OF THE $MnCO_3$ TYPE

V. G. BAR'YAKHTAR, M. A. SAVCHENKO, V. V. GANN, and P. V. RYABKO

Physico-technical Institute, Academy of Sciences, Ukrainian S.S.R.

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Coupled magneto-elastic waves in antiferromagnets with weak ferromagnetism are considered. The experimental possibility of determining the exchange integrals on the basis of ferro-acoustic resonance in such substances is pointed out.

In the present paper we study coupled magneto-elastic vibrations in antiferromagnetics with weak ferromagnetism (crystals of the $MnCO_3$ type), the magnetic properties of which have been well explored at the present time.[1,2] These substances have a rhombohedral lattice with two magnetic ions per unit cell (Fig. 1). The space group of the crystal D_{3d}^6, in addition to translation and inversion, possesses two independent symmetry elements: C_3 and σ_{dt} [where t means the translation ($\frac{1}{2}\,\frac{1}{2}\,\frac{1}{2}$)]. The magnetic structure is even[3] relative to translation and inversion; therefore, in the construction of invariants, it suffices to consider the elements C_3 and σ_y, with the magnetic structure even relative to the (C_3^+) axis and odd relative to the glide plane (σ_y^-). Consequently, $\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2$ transforms as an axial vector, while $\mathbf{L} = \mathbf{M}_1 - \mathbf{M}_2$ undergoes an additional change in sign under the transformation σ_y^-, since the translation t enters into it.

We write down the Hamiltonian of $MnCO_3$ in the form

$$\mathcal{H} = \mathcal{H}_s + \mathcal{H}_p + \mathcal{H}_{sp};\ \mathcal{H}_{sp} = \mathcal{H}_{sp}^{(1)} + \mathcal{H}_{sp}^{(2)}. \tag{1}$$

The Hamiltonian (1) contains the magnetic energy $\mathcal{H}_s$, the elastic energy $\mathcal{H}_p$, the energy of magnetostriction $\mathcal{H}_{sp}^{(1)}$, and the energy of interaction, which is brought about by rotation of the trigonal axis $\mathcal{H}_{sp}^{(2)}$[4], while

$$\mathcal{H}_s = \int dV \left\{\frac{\alpha}{2}\left[\left(\frac{\partial \mathbf{M}_1}{\partial x_i}\right)^2 + \left(\frac{\partial \mathbf{M}_2}{\partial x_i}\right)^2\right] + \alpha' \frac{\partial \mathbf{M}_1}{\partial x_i}\frac{\partial \mathbf{M}_2}{\partial x_i} + \delta \mathbf{M}_1\mathbf{M}_2 - \frac{1}{2}\beta[(\mathbf{n}\mathbf{M}_1)^2 + (\mathbf{n}\mathbf{M}_2)^2] - \beta'(\mathbf{n}\mathbf{M}_1)(\mathbf{n}\mathbf{M}_2) - (\mathbf{M}_1 + \mathbf{M}_2, \mathbf{H}) - d(M_{1x}M_{2y} - M_{1y}M_{2x})\right\}, \tag{2}$$

$$\mathcal{H}_p = \frac{1}{2}\int dV \left\{\rho\dot{\mathbf{u}}^2 + \lambda_1(\operatorname{div}\mathbf{u})^2 + \lambda_2 u_{ik}^2 + \lambda_3 u_{zz}\operatorname{div}\mathbf{u} + \lambda_4(u_{zi})^2 + \lambda_5 u_{zz}^2 + \lambda_6[u_{zx}(u_{xx} - u_{yy}) - 2u_{zy}u_{xy}]\right\}, \tag{3}$$

$$\begin{aligned}\mathcal{H}_{sp}^{(1)} = \int dV \{&[\gamma_1(\mathbf{M}_1^2 + \mathbf{M}_2^2) + \gamma_2\mathbf{M}_1\mathbf{M}_2 + \gamma_3(M_{1z}^2 + M_{2z}^2) \\ &+ \gamma_4 M_{1z}M_{2z} + \gamma_5(M_{1x}M_{2y} - M_{1y}M_{2x})]u_{ii} \\ &+ [\gamma_6(\mathbf{M}_1^2 + \mathbf{M}_2^2) + \gamma_7\mathbf{M}_1\mathbf{M}_2 + \gamma_8(M_{1z}^2 + M_{2z}^2) \\ &+ \gamma_9 M_{1z}M_{2z} + \gamma_{10}(M_{1x}M_{2y} - M_{1y}M_{2x})]u_{zz} \\ &+ [\gamma_{11}(M_{1i}M_{1k} + M_{2i}M_{2k}) + \gamma_{12}M_{1i}M_{2k}]u_{ik} \\ &+ [\gamma_{13}(M_{1z}M_{1i} + M_{2z}M_{2i}) + \gamma_{14}(M_{1z}M_{2i} + M_{1i}M_{2z})]u_{iz} \\ &+ \gamma_{15}[u_{ix}(M_{1i}M_{1y} - M_{2i}M_{2y}) - u_{iy}(M_{1i}M_{1x} - M_{2i}M_{2x})] \\ &+ \gamma_{16}[u_{zx}(M_{1z}M_{1y} - M_{2z}M_{2y}) - u_{zy}(M_{1z}M_{1x} - M_{2z}M_{2x})] \\ &+ \gamma_{17}[u_{zx}(M_{1z}M_{2y} - M_{2z}M_{1y}) - u_{zy}(M_{1z}M_{2x} - M_{2z}M_{1x})] \\ &+ \gamma_{18}[(u_{xx} - u_{yy})(M_{1z}M_{1x} + M_{2z}M_{2x}) \\ &- 2u_{xy}(M_{1z}M_{1y} + M_{2z}M_{2y})] \\ &+ \gamma_{19}[(u_{xx} - u_{yy})(M_{1z}M_{1y} - M_{2z}M_{2y}) \\ &+ 2u_{xy}(M_{1z}M_{1x} - M_{2z}M_{2x})] \\ &+ \gamma_{20}[u_{zx}(M_{1x}^2 - M_{1y}^2 + M_{2x}^2 - M_{2y}^2) \\ &- 2u_{zy}(M_{1x}M_{1y} + M_{2x}M_{2y})] \\ &+ \gamma_{21}[u_{zy}(M_{1x}^2 - M_{1y}^2 - M_{2x}^2 + M_{2y}^2) \\ &+ 2u_{zx}(M_{1x}M_{1y} - M_{2x}M_{2y})] \\ &+ \gamma_{22}[(u_{xx} - u_{yy})(M_{1z}M_{2x} + M_{2z}M_{1x}) \\ &- 2u_{xy}(M_{1z}M_{2y} + M_{2z}M_{1y})] \\ &+ \gamma_{23}[(u_{xx} - u_{yy})(M_{1z}M_{2y} - M_{2z}M_{1y}) \\ &+ 2u_{xy}(M_{1z}M_{2x} - M_{2z}M_{1x})] \\ &+ \gamma_{24}[u_{xz}(M_{1x}M_{2x} - M_{1y}M_{2y}) \\ &- u_{zy}(M_{1x}M_{2y} + M_{2x}M_{1y})]\},\end{aligned} \tag{4}$$

$$\mathcal{H}_{sp}^{(2)} = \int dV \{\beta(M_{1i}M_{1z} + M_2\, M_{2z}) + \beta'(M_{1i}M_{2z} + M_{2i}M_{1z}) + de_{ijk}M_{1j}M_{2k}\}\,\varepsilon_{iz}. \tag{5}$$

Here α, α', δ are exchange integrals, β and β' are the constants of anisotropy, d is the Dzyaloshinskiĭ constant, e_{ijk} is a unit antisymmetric tensor, $\mathbf{n}$ is the unit vector along the z axis,

$$\varepsilon_{ij} = \tfrac{1}{2}(\partial u_i/\partial x_j - \partial u_j/\partial x_i),$$
$$u_{ij} = \tfrac{1}{2}(\partial u_i/\partial x_j + \partial u_j/\partial x_i).$$

We assume that the field is directed along the y axis, the magnetic moments M_{10} and M_{20} lie in the basal plane, and φ_1 and φ_2 are the angles between M_{10}, M_{20} and the y axis (Fig. 2).

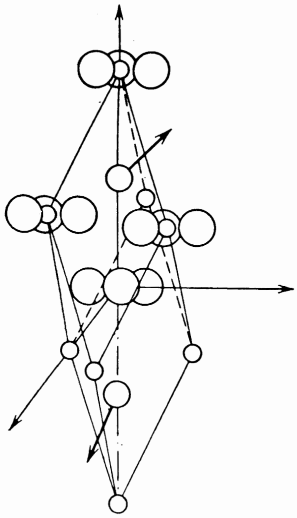

FIG. 1.

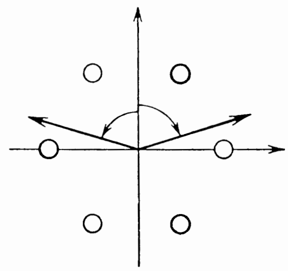

FIG. 2.

We write down the Hamiltonian (1) in terms of the operators of creation and absorption of phonons (b_{fs}, b^+_{fs}) and spin waves (a_{kj}, a^+_{kj}), using the Holstein-Primakoff transformation;[5]

$$M_{jx} = (M_0 - \mu a_j^+ a_j)\sin\varphi_j + i(\mu M_0/2)^{1/2}(a_j - a_j^+)\cos\varphi_j,$$

$$M_{jy} = (M_0 - \mu a_j^+ a_j)\cos\varphi_j - i(\mu M_0/2)^{1/2}(a_j - a_j^+)\sin\varphi_j,$$

$$M_{jz} = (\mu M_0/2)^{1/2}(a_j + a_j^+), \qquad (6)$$

and the Fourier expansion for the operators a_j and $\mathbf{u}(\mathbf{r})$:

$$a_j(\mathbf{r}) = \frac{1}{\sqrt{V}}\sum_{\mathbf{k}} a_{j\mathbf{k}} e^{i\mathbf{kr}},$$

$$\mathbf{u}(\mathbf{r}) = \frac{1}{\sqrt{2\rho V}}\sum_{\mathbf{f}s}\frac{\mathbf{e}_{\mathbf{f}s}}{\sqrt{\omega_{\mathbf{f}s}}}(b_{\mathbf{f}s}e^{i\mathbf{fr}} + b_{\mathbf{f}s}^+ e^{-i\mathbf{fr}}),$$

$$\omega_l^2 = f^2(\lambda_1 + \lambda_2)/\rho, \qquad \omega_t^2 = f^2\lambda_2/2\rho. \qquad (7)$$

Here j is the index of the sublattice, s is the index of polarization.

The Hamiltonian (2) takes the form

$$\mathcal{H}_s = \sum_{\mathbf{k}}\{A(a^+_{1\mathbf{k}}a_{1\mathbf{k}} + a^+_{2\mathbf{k}}a_{2\mathbf{k}}) + B(a_{1\mathbf{k}}a_{2-\mathbf{k}} + a^+_{1\mathbf{k}}a^+_{2-\mathbf{k}})$$
$$+ C(a_{1\mathbf{k}}a^+_{2\mathbf{k}} + a^+_{1\mathbf{k}}a_{2\mathbf{k}}) + {}^1/_2 D(a_{1\mathbf{k}}a_{1-\mathbf{k}}$$
$$+ a_{2\mathbf{k}}a_{2-\mathbf{k}} + a^+_{1\mathbf{k}}a^+_{1-\mathbf{k}} + a^+_{2\mathbf{k}}a^+_{2-\mathbf{k}})\}, \qquad (8)$$

where

$$A = \mu M_0(\alpha k^2 + \delta - {}^1/_2\beta + 2d\eta\nu),$$

$$B = \mu M_0[(\alpha' k^2 + \delta)\nu^2 - {}^1/_2\beta' + d\eta\nu],$$

$$C = \mu M_0[(\alpha' k^2 + \delta)\eta^2 - {}^1/_2\beta' - d\eta\nu], \quad D = -{}^1/_2\beta\mu M_0. \qquad (9)$$

The values of the angles $\nu = \sin\varphi$ and $\eta = \cos\varphi$ are found from the equation of the ground state:

$$H_0\nu/M_0 - 2\delta\eta\nu - d(\eta^2 - \nu^2) = 0. \qquad (10)$$

The Hamiltonian of the system of phonons (3) has the form

$$\mathcal{H}_p = \sum_{\mathbf{k},s}\omega_{\mathbf{k}s}b^+_{\mathbf{k}s}b_{\mathbf{k}s}$$
$$+ \sum_{\mathbf{k},s,s'}\beta_{\mathbf{k}ss'}[b^+_{\mathbf{k}s}b_{\mathbf{k}s'} + b_{\mathbf{k}s}b^+_{\mathbf{k}s'} - b_{\mathbf{k}s}b_{-\mathbf{k}s'} - b^+_{\mathbf{k}s}b^+_{-\mathbf{k}s'}]. \qquad (11)$$

We shall not write out the tensor $\beta_{\mathbf{k}ss'}$ for an arbitrary direction of propagation of the sound [the corresponding expression can be obtained from Eqs. (3) and (7)].

If the sound wave is propagated along a threefold axis, then the tensor $\beta_{\mathbf{k}ss'}$ has the form

$$\beta_{\mathbf{k}ss'} = \frac{k^2}{16\rho(\omega_{\mathbf{k}s}\omega_{\mathbf{k}s'})^{1/2}}[e^z_{\mathbf{k}s}e^z_{\mathbf{k}s'}(4\lambda_3 + 3\lambda_4 + 4\lambda_5) + \lambda_4\delta_{ss'}],$$

whence it is easy to see that in this case two waves are transverse while the third is longitudinal. The velocity of the transverse waves is equal to

$$c_t^2 = (2\lambda_2 + \lambda_4)/4\rho,$$

while the velocity of longitudinal waves is

$$c_l^2 = (\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5)/\rho.$$

With the help of the canonical u, v transformation,

$$a_{j\mathbf{k}} = u_{j1}c_{1\mathbf{k}} + u_{j2}c_{2\mathbf{k}} + v_{j1}{}^*c^+{}_{1-\mathbf{k}} + v_{j2}{}^*c^+{}_{2-\mathbf{k}}, \qquad j = 1, 2, \qquad (13)$$

we reduce the Hamiltonian (8) to diagonal form:

$$\mathcal{H}_s = \sum_{\mathbf{k},j}\varepsilon_{j\mathbf{k}}c_{j\mathbf{k}}{}^+c_{j\mathbf{k}}, \qquad (14)$$

where ϵ_{jk} is the spectrum of spin waves:[6]

$$\varepsilon_1 = [(A + B + C + D)(A + C - B - D)]^{1/2},$$

$$\varepsilon_2 = [(A + B - C - D)(A + D - B - C)]^{1/2}. \qquad (15)$$

Similarly, one can reduce the Hamiltonian (11) to diagonal form

$$\mathcal{H}_p = \sum_{\mathbf{k},s}\tilde{\omega}_{\mathbf{k}s}d_{\mathbf{k}s}{}^+d_{\mathbf{k}s}, \qquad \omega_{ks} = c_s k, \qquad (16)$$

where d^+_{ks} and d_{ks} are the creation and annihilation operators of modified acoustic waves,

$$b_{\mathbf{k}s} = u_{ss}d_{\mathbf{k}s} + v_{ss}{}^*d_{-\mathbf{k}s}{}^+. \qquad (17)$$

In the case under consideration, the difference $u_{ss} - v_{ss}$ is equal to

$$u_{ss} - v_{ss} = (\omega_{ks}/\tilde{\omega}_{ks})^{1/2}. \qquad (18)$$

By use of Eqs. (4), (5), (13), (17), and (18), one can find the spin-phonon interaction Hamiltonian, $\mathcal{H}_{sp}$ in this case:

$$\mathcal{H}_{sp} = \sum_{j,\mathbf{k},s}(\gamma_{j\mathbf{k}s}c_{j\mathbf{k}} - \gamma^*_{j\mathbf{k}s}c^+_{j-\mathbf{k}})(d_{-\mathbf{k}s} - d^+_{\mathbf{k}s}), \qquad (19)$$

where

$$\gamma_{1kl} = \gamma_{2kt_y} = \gamma_{1kt_x} = 0,$$

$$\gamma_{1kt_y} = \frac{1}{2}\left(\frac{M_0^2}{\rho c_t^2}\right)^{1/2}(\mu M_0\tilde{\omega}_t)^{1/2}\left[i\left(\frac{A+C-B-D}{2\varepsilon_1}\right)^{1/2}A_{yz} - \left(\frac{A+B+C+D}{2\varepsilon_1}\right)^{1/2}B_{yz}\right],$$

$$\gamma_{2kt_x} = -\frac{1}{2}\left(\frac{M_0^2}{\rho c_t^2}\right)^{1/2}(\mu M_0\tilde{\omega}_t)^{1/2}\left[\left(\frac{A+D-B-C}{2\varepsilon_2}\right)^{1/2}B_{xz} + i\left(\frac{A+B-C-D}{2\varepsilon_2}\right)^{1/2}A_{xz}\right],$$

$$\gamma_{2kl} = \left(\frac{M_0^2}{\rho c_l^2}\right)^{1/2}(\mu M_0\tilde{\omega}_l)^{1/2}(S+B_{\|})\left(\frac{A+D-B-C}{2\varepsilon_2}\right)^{1/2},$$

$$A_{yz} = (\gamma_{17}-\gamma_{15}-\gamma_{16})\nu + (\gamma_{12}+\gamma_{14}+2\gamma_{11}+\gamma_{13})\eta + (\beta+\beta')\eta - \nu d,$$

$$B_{yz} = 2\gamma_{20}(\nu^2-\eta^2) - \gamma_{24} + 4\gamma_{21}\eta\nu,$$

$$A_{xz} = (2\gamma_{11}+\gamma_{13}-\gamma_{12}-\gamma_{14})\nu + (\gamma_{15}+\gamma_{16}+\gamma_{17})\eta - (\beta'-\beta)\nu - \eta d,$$

$$B_{xz} = 2\gamma_{21}(\eta^2-\nu^2) + 4\gamma_{20}\eta\nu, \quad S = -2\gamma_2\eta\nu + \gamma_5(\eta^2-\nu^2),$$

$$B_{\|} = -2\gamma_7\eta\nu + \gamma_{10}(\eta^2-\nu^2).$$

We note that in this case only the transverse sound wave with polarization along the y axis interacts with the low-frequency branch of spin waves ϵ_1. The other sound waves interact only with the high-frequency spin waves.

By taking (14), (16), and (19) into account, one can reduce the Hamiltonian (1) to diagonal form, transforming to the creation and annihilation operators of coupled magneto-elastic waves, and finding the dispersion law of these waves:[7]

$$\begin{aligned}(\omega^2-\varepsilon_1^2)(\omega^2-\varepsilon_2^2)(\omega^2-\tilde{\omega}_l^2)(\omega^2-\tilde{\omega}_t^2) &- 4|\gamma_{1t_y}|^2\varepsilon_1\tilde{\omega}_t(\omega^2-\varepsilon_2^2)(\omega^2-\tilde{\omega}_l^2)\\ &- 4|\gamma_{2\,x}|^2\varepsilon_2\tilde{\omega}_t(\omega^2-\varepsilon_1^2)(\omega^2-\tilde{\omega}_l^2)\\ &- 4|\gamma_{2l}|^2\varepsilon_2\tilde{\omega}_l(\omega^2-\varepsilon_1^2)(\omega^2-\tilde{\omega}_t^2) = 0. \qquad (20)\end{aligned}$$

If the sound wave is propagated along the x axis, then

$$\beta_{kss'} = \frac{k^2}{16\rho(\omega_{ks}\omega_{ks'})^{1/2}}[\lambda_4 e^z_{ks}e^z_{ks'} + 2\lambda_6 e^z_{ks}e^x_{ks'}]. \qquad (21)$$

It is then seen that the transverse wave with the polarization vector along the y axis does not interact with the other two acoustic waves. One can show that only this transverse wave interacts with the low-frequency spin waves, while the dispersion equation which describes this interaction has the form

$$(\omega^2-\varepsilon_1^2)(\omega^2-\tilde{\omega}_t^2) - 4|\gamma_{1t_y}|^2\varepsilon_1\tilde{\omega}_t = 0; \qquad (22)$$

$$\gamma'_{1t_y} = \frac{1}{2}\left(\frac{M_0^2}{\rho c_t^2}\right)^{1/2}(\mu M_0\tilde{\omega}_t)^{1/2} \times\left[i\left(\frac{A+C-B-D}{2\varepsilon_1}\right)^{1/2}A_{xy} - \left(\frac{A+B+C+D}{2\varepsilon_1}\right)^{1/2}B_{xy}\right], \qquad (23)$$

$$A_{xy} = 2[(\gamma_{19}-\gamma_{23})\nu - (\gamma_{18}+\gamma_{22})\eta],$$

$$B_{xy} = \gamma_{12} - 4\gamma_{15}\eta\nu + 2\gamma_{11}(\eta^2-\nu^2).$$

As is seen from the dispersion equations (20) and (22), the coupling between the sound and spin waves is especially large at resonance, when the frequency of the spin wave is identical with the frequency of the sound wave. Far from resonance, the dispersion equations describe sound and spin waves that are virtually non-interacting.

As is well known, the ferro-acoustic resonance can be observed from the increase in sound absorption, by the change in the velocity and by the rotation of the plane of polarization of the sound. By knowing the frequency of the ferro-acoustic resonance, one can experimentally determine the exchange integrals in the spin wave spectrum for $MnCO_3$.

The energy spectrum (15) can be represented in the form

$$\varepsilon_1^2 = \Theta_{\|}^2(ak_{\|})^2 + \Theta_{\perp}^2(ak_{\perp})^2 + \mu^2 H_0(H_0+H_d), \qquad (24)$$

$$\varepsilon_2^2 = \Theta_{\|}^2(ak_{\|})^2 + \Theta_{\perp}^2(ak_{\perp})^2 + \mu^2 H_\delta H_A; \qquad (25)$$

$$\Theta_{\|} = a^{-1}\mu M_0[2\delta(\alpha_{\|}-\alpha_{\|}')]^{1/2},$$

$$\Theta_{\perp} = a^{-1}\mu M_0[2\delta(\alpha_{\perp}-\alpha_{\perp}')]^{1/2},$$

$$H_\delta = 2\delta M_0, \quad H_A = M_0(\beta'-\beta), \quad H_d = dM_0.$$

In view of the fact that the second branch of the spin waves ϵ_2 always has a large energy gap, the observation of ferro-acoustic resonance with this branch is difficult. The gap in the spectrum is brought about by the external magnetic field; therefore, in weak magnetic fields H_0, one can observe resonance with the sound wave.

To find the exchange integrals from the frequencies of ferro-acoustic resonance, we represent the dispersion equations in the form:[8]

$$(k^2-k_t^2)(k^2-k_\mu^2-ik_\mu\varkappa) - k^2\zeta = 0, \qquad (26)$$

where for $\mathbf{k} \parallel \mathbf{n}$

$$k_\mu^2 = [\omega^2 - \mu^2 H_0(H_0+H_d)]/\Theta_{\|}^2a^2, \quad k_\mu\varkappa = 2\delta\gamma\omega/\Theta_{\|}^2a^2,$$

$$\zeta = 4|\gamma_{1ty}|^2\varepsilon_1/\omega_t\Theta_{\|}^2a^2,$$

and for $\mathbf{k} \perp \mathbf{n}$

$$k_\mu^2 = [\omega^2 - \mu^2 H_0(H_0+H_d)]/\Theta_{\perp}^2a^2, \quad k_\mu\varkappa = 2\delta\gamma\omega/\Theta_{\perp}^2a^2,$$

$$\zeta = 4|\gamma_1'{}_{ty}|^2\varepsilon_1/\omega_t\Theta_{\perp}^2a^2,$$

γ is the relaxation constant of the magnetic moment.

As is seen from the solution of the dispersion equation (26)

$$k_{1,2}^2 = \frac{1}{2}\{k_t^2 + k_\mu^2 + \zeta + ik_\mu\varkappa \pm [(k_t^2 - k_\mu^2 - \zeta - ik_\mu\varkappa)^2 - 4k_t^2\zeta]^{1/2}\}, \quad (27)$$

the resonance sets in when

$$k_t^2 = k_\mu^2 + \zeta; \quad (28)$$

in this case, the width of the line of ferro-acoustic resonance is equal to

$$\Delta k / k = |(k_\mu\varkappa)^2 - 4k_t^2\zeta|^{1/2} / k_t^2. \quad (29)$$

Condition (28) is fulfilled when the frequency of sound $\omega > 10^9$.

For resolution of the lines of ferro-acoustic resonance relative to the lines of ferromagnetic resonance, it is necessary that $\Delta k/k \ll 1$. This condition is satisfied only for frequencies $\omega \gtrsim 10^{11}$, or at much lower frequencies in the special case of coincidence of the value of the coupling parameter ζ with the damping in the system κ ($k_\mu\kappa = 2k_t\sqrt{\zeta}$).

From (28), we get the coupling of the frequency of ferro-acoustic resonance ω_r with the frequency of ferromagnetic resonance $\omega_0 = [\mu^2 H_0 (H_0 + H_d)]^{1/2}$:

$$\omega_{r\parallel}^2 - \omega_0^2 = (\Theta_\parallel^2 a^2 / c_t^2)(\omega_{r\parallel}^2 - c_t^2\zeta), \quad (30)$$

$$\omega_{r\perp}^2 - \omega_0^2 = (\Theta_\perp^2 a^2 / c_t^2)(\omega_{r\perp}^2 - c_t^2\zeta). \quad (31)$$

By experimentally determining the dependence of the frequencies of ferro-acoustic and ferromagnetic resonance on the value of the external field H_0, one can find the Curie temperatures $\Theta_\parallel$ and $\Theta_\perp$.

Similarly, one can consider the interaction of the low frequency branch of the spin waves ϵ_1 with the longitudinal sound waves. In this case, it is necessary to direct the wave vector **k** at an angle with respect to the x, y, or z axes.

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