INTERACTION OF 19.8 BeV/c PROTONS WITH NUCLEONS AND EMULSION NUCLEI

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Interactions of protons with nucleons and emulsion nuclei have been investigated. 7960 events have been detected in a total primary track length of 2927 meters. This corresponds to a mean free path $\lambda = 36.8 \pm 0.4$ cm. 1035 inelastic p-N interactions have been selected. The distribution of p-p events with respect to number of prongs is in agreement with hydrogen bubble chamber data. The mean numbers of charged secondary particles from p-p and p-n interactions are respectively 4.3 ± 0.2 and 4.5 ± 0.2. Showers with asymmetric emission of charged particles in the c.m.s. are studied. The distribution of the asymmetry of individual interactions can be explained by assuming that the shower particles are deflected in a random fashion from symmetric emission. The dependence of the multiplicity on the type of target nucleus is analyzed. The experimental data are compared with the predictions of various theoretical conceptions of the interaction mechanism between nucleons and nuclei. It is shown that the best agreement is obtained with the cascade model calculations performed at the Joint Institute for Nuclear Research.

1. EXPERIMENTAL METHOD

THE present work is devoted to study of the interaction of 19.8 BeV/c protons with nucleons. For this purpose we used a stack of 12 cm × 20 cm × 600 μ Ilford G-5 emulsions irradiated in the CERN proton synchrotron. Scanning was carried out along the tracks of the primary particles in MBI-9 microscopes with a magnification of 900 ×. To select interactions in free and quasifree nucleons we used the following criteria:

1. Only those stars are chosen in which there is not more than one gray or black prong ($N_h \leq 1$).
2. There is no recoil nucleus track in the star.
3. In a star with an even number of prongs, no β-electron track must be present.
4. In the case of a gray or black prong (a presumed proton) the emission angle of the secondary particle with respect to the direction of the primary particle must not exceed a limiting angle.[1]
5. For a given angle θ less than the limiting angle, the range (energy) of a gray or black prong must exceed the minimum length (energy) possible in the generation of visible charged particles.[2] This criterion is a generalization of that of Sternheimer [1] to angles less than the limiting angle.
6. For each interaction the condition of Birger and Smorodin [3] must be satisfied, assuming constant transverse momentum of the secondary particles [4]:*

$$m_t = \bar{p}_{\perp\pi}\left[\sum_{i=1}^{n_s}(z_\pi^2+\operatorname{ctg}^2\theta_{\pi i})^{1/2}-\sum_{i=1}^{n_s}\operatorname{ctg}\theta_{\pi i}\right] + \bar{p}_{\perp p}\left[(z_p^2+\operatorname{ctg}^2\theta_p)^{1/2}-\operatorname{ctg}\theta_p\right] \leqslant M+E_0-p_0,$$

where $\theta_{\pi i}$ is the emission angle of the i-th shower particle (a presumed π meson), θ_p is the emission angle for a gray or black track (a presumed proton), $\bar{p}_{\perp\pi}$ and $\bar{p}_{\perp p}$ are the mean values of transverse momentum of the secondary particles identified as π mesons and protons;

$$z_\pi^2 = \frac{p_{\perp\pi}^2+1}{p_{\perp\pi}^2}, \qquad z_p^2 = \frac{p_{\perp p}^2+(M/m_\pi)^2}{p_{\perp p}^2}$$

Each of the enumerated criteria is necessary but not sufficient; however, together they guarantee a high probability of selecting interactions of a proton with free and quasifree nucleons.

To exclude elastic interactions we used the usual criteria: the relation between the angles θ_1 and θ_2, and the coplanarity condition ($\Phi \leq 1°$, where the angle 1° is twice the mean square error in the noncoplanarity angle Φ). In analysis of a gray or black prong, the association of an event with an elastic interaction was evaluated from the relation between the angle and energy of the par-

*ctg = cot.

Table I

E, Bev	Source of data	Method	Number of Prongs					
			2	4	6	8	10	12
20	[5]	p. e.	47±8	33±7	12±4	6±3	1±1	1±1
20	Present work	p. e.	29±2	41±3	19±2	8±1	2±1	1±1
23.5	[6]	p. e.	38±8	33±8	19±6	8±4	2±2	—
24	[7]	h. b. c.	29±3	43±4	22±3	6±2	1±1	—
24	[8]	p. e.	24±5	30±5	30±5	8±3	6±2	2±1
27	[9]	p. e.	20±3	34±4	28±4	13±3	4±2	1±1

Note. p. e. = photographic emulsion method, h. b. c. = hydrogen bubble chamber method.

Table II

E, BeV	Source of data	Method	Number of Prongs					
			1	3	5	7	9	11
20	[5]	p. e.		81±16	7±4	9±5	3±3	—
20	Present work	p. e.		50±3	32±3	13±2	4±1	1±1
				42±3	27±2	11±1	4±1	1±1
			15±2					
24	[8]	p. e.		41±8	22±6	29±7	7±3	1±1
27	[9]	p. e.		35±6	40±6	16±4	5±2	4±2

Note. p. e. = photographic emulsion method.

Table III

Source of data	[11]	[12, 13]	[14]	Present work	[5]	[6]	[7]	[8]	[9]
Method	p. e.	p. e.	p. e.	p. e.	p. e.	p. e.	h. b. c.	p. e.	p. e.
E, BeV	9.0	9.0	9.0	20	20	23.5	24	24	27
$\langle n\rangle_{pn}$, exp.	3.6±0.6	3.2±0.1	3.9±0.1	4.5±0.2	3.7±0.7	—	—	5.1±0.7	5.1±0.5
$\langle n\rangle_{pp}$, exp.	3.2±0.1	3.2±0.3	3.4±0.1	4.3±0.2	3.5±0.4	4.1±0.6	4.2±0.1	4.9±0.5	5.0±0.4
$\langle n\rangle_{pp}$, stat. theory	3.68	3.68	3.68	4.56	4.56	4.82	4.84	4.84	4.96

Note. Values of $\langle n\rangle_{pn}$ are given without taking into account one-prong events; p. e. = photographic emulsion method; h. b. c. = hydrogen bubble chamber method.

ticle. All scatterings at an angle $\theta \leq 30'$ were considered to be diffraction scattering.[5]

2. PROTON-NUCLEON INTERACTIONS

Using the criteria described above, in the total scanned track length of 2927 m we identified 1035 inelastic proton-nucleon (p-N) collisions, of which 546 are p-p interactions and 489 are p-n interactions.

The number-of-prong distributions for p-p and p-n collisions are given in Tables I and II, respectively (in %). Data of other authors in the energy range 20-27 BeV are also given here for comparison. Table I shows that the number-of-prong distribution for p-p interactions obtained in the present work is in good agreement with that obtained by Dodd et al [7] in a hydrogen bubble chamber at 24 BeV. It should be noted that, in the work of Abraham and Kalbach [5] at 20 BeV, the peak in the prong distribution for inelastic p-p interactions occurs for two-pronged events, which disagrees not only with the results of the present work but also with the data of other authors listed in Table I for energies greater than 20 BeV. It can be seen from Table II that the number of three-pronged interactions found by Abraham and Kalbach is considerably greater than the corresponding values from the other investigations.

When the energy changes from 20 to 27 BeV, some redistribution is observed in the number of events, such that the contribution of multipronged events increases. This appears in the energy dependence of the average number of prongs per interaction for p-p and p-n collisions. The corresponding data are listed in Table III. Also listed for comparison are the values calculated from the statistical theory of Hagedorn.[10] Our data agree with the theoretical values within two standard deviations.

It is evident from Table III that the average number of charged particles found by Abraham and Kalbach [5] turns out to be less than the corresponding values listed by the other authors both for p-p and p-n interactions. It should be mentioned that the good agreement of the number-of-charged-particle distributions obtained in the present work and by Dodd et al [7] with a bubble chamber apparently confirms the correctness and rigor of the selection criteria used by us for p-N interactions.

To clarify the dependence of the angular distribution on the multiplicity, all events were grouped according to number of prongs. For each such group the half-angles $\theta_{1/2}$ of emission in the laboratory system were evaluated for all particles. The values of $\theta_{1/2}$ for p-p and p-n interactions are listed in Tables IV and V, respectively. For p-p and p-n collisions a rapid increase of the half-angle with increasing number of prongs is observed in few-pronged interactions; beginning with five-pronged events, the value of $\theta_{1/2}$ remains practically constant. This behavior of the angular distribution is associated with shower particles, in view of the fact that the half-angles for gray and black prongs remain constant within the experimental errors.

It is of interest to compare the different kinematical methods of evaluating the Lorentz factor γ_c in the c.m.s. Values of γ_c were calculated from the half-angle for each group of particles with the same multiplicity, with the assumption $p_{\perp i} = \mathrm{const}(\gamma_c)$ and $\beta_c = \beta_i'(\gamma_c')$. It was assumed that the shower particles are π mesons and the gray and black tracks are protons. We used the mean values $\bar{p}_{\perp\pi} = (1.8 \pm 0.2)\mu_\pi c$ and $\bar{p}_{\perp p} = (2.3 \pm 0.4)\mu_\pi c$, obtained in measurement of the momentum of 110 secondary particles from p-n interactions. Values of γ_c for p-p and p-n collisions are listed in Tables IV and V, respectively. In the region of low multiplicity (up to n = 5) the value of γ_c exceeds the expected value $\gamma_c^0 = 3.33$. This is apparently due to the fact that for low multiplicity the contribution of particles other than π mesons is large and cannot be neglected, which is confirmed by a comparison

Table IV

Quantity	Number of Prongs					
	2	4	6	8	10	12
Photographic emulsion method (E = 20 BeV, $\gamma_c^0 = 3.33$)						
$\theta_{1/2}$	$7°34'^{+1°50'}_{-1°24'}$	$10°18'^{+0°48'}_{-0°36'}$	$12°27'^{+0°48'}_{-0°57'}$	$13°42'^{+1°14'}_{-1°06'}$	$16°12'^{+2°36'}_{-2°12'}$	$13°42'^{+2°00'}_{-2°42'}$
B	0,15±0.02	0.31±0.02	0.49±0,04	0.56±0.06	0.5±0.1	0.7±0.2
γ_c ($p_\perp$ = const)	$6,1^{+1.2}_{-1.3}$	$4.5^{+0.2}_{-0.3}$	$3,8^{+0.2}_{-0.3}$	$3,4^{+0,3}_{-0,3}$	$2,9^{+0.4}_{-0.4}$	$3,4^{+0.4}_{-0.8}$
γ_c' ($p_\perp$ = const)	$7,5^{+1.5}_{-1.7}$	$5.5^{+0.4}_{-0.3}$	$4.5^{+0.3}_{-0.4}$	$4.1^{+0.4}_{-0.4}$	$3.4^{+0.5}_{-0.6}$	$4.1^{+0.5}_{-0.1}$
γ_c'/γ_c^0 ($\beta_c = \beta_i'$)	$2.2^{+0.5}_{-0.5}$	$1.6^{+0.2}_{-0.1}$	$1,3^{+0.1}_{-0.1}$	$1,2^{+0.1}_{-0.1}$	$1,0^{+0.2}_{-0.2}$	$1.2^{+0.2}_{-0.1}$
Hydrogen bubble chamber method (E = 24 BeV, $\gamma_c^0 = 3.64$)						
γ_c'	—	5.7	5.2	4.4		
γ_c'/γ_c^0 ($\beta_c = \beta_i'$)	—	1.6	1.4	1.2		

Table V

Quantity	Number of Prongs				
	3	5	7	9	11
Photographic emulsion method (E = 20 BeV, $\gamma_c^0 = 3.33$					
$\theta_{1/2}$	$6°15'^{+0°51'}_{-0°39'}$	$11°18'^{+0°48'}_{-1°12'}$	$12°42'^{+1°35'}_{-0°57'}$	$15°00'^{+1°24'}_{-1°48'}$	$18°24'^{+3°56'}_{-3°24'}$
B	0.19±0.02	0.40±0.03	0,72±0.08	0.52±0.08	0.7±0.2
γ_c ($p_\perp$ = const)	$7.4^{+0.9}_{-0.8}$	$4.1^{+0.3}_{-0.5}$	$3.7^{+0.4}_{-0.3}$	$3.1^{+0.3}_{-0.4}$	$2.6^{+0.4}_{-0.5}$
γ_c' ($p_\perp$ = const)	$9.1^{+1.1}_{-1.1}$	$5.0^{+0.3}_{-0.6}$	$4.4^{+0.5}_{-0.4}$	$3.7^{+0.3}_{-0.5}$	$3.0^{+0.6}_{-0.7}$
γ_c'/γ_c^0 ($\beta_c = \beta_i'$)	$2.7^{+0.3}_{-0.3}$	$1.5^{+0.1}_{-0.2}$	$1.3^{+0.2}_{-0.1}$	$1.1^{+0.1}_{-0.1}$	$0.9^{+0.2}_{-0.2}$

with the 24-BeV hydrogen bubble chamber data of Meyer et al [15] (see Table IV). Beginning with $n \geq 5$, the value of γ_c obtained assuming $p_\perp = \text{const}$ agrees satisfactorily with the value expected for this energy, while the assumption $\beta_c = \beta_i'$ leads to a systematic exaggeration. It should be noted that the ratio of $\gamma_c'(\beta_c = \beta_i')$ to the true value γ_c^0 is in good agreement with the ratio found from hydrogen bubble chamber data under the same assumptions. This again confirms that in the p-p interactions selected by us the contribution of proton-nuclear interactions is negligibly small.

Figure 1 shows the distribution of γ_c for individual interactions with $n \geq 5$, calculated on the assumption of constant transverse momentum of the secondary particles. The peak of the distribution is in the vicinity of the expected value (denoted by the dashed line). The width of the distribution is characterized by the fact that 71% of the events have values of γ_c between 2 and 5. Showers are observed in which γ_c exceeds the expected value by 3-4 times. This is due to the asymmetric scattering of the shower particles in the c.m.s.

To determine the nature of the secondary-particle angular distribution in the c.m.s., we used a method of angle scaling based on the assumption of constant transverse momentum.[4] As a qualitative characteristic of the degree of isotropy in the c.m.s., we used the quantity $B = 2N_1/3N_2$, where N_1 is the number of particles incident in the interval $-0.6 \leq \cos\theta' \leq 0.6$; N_2 is the number of particles incident in the remaining angles (for an isotropic distribution $B \equiv 1$). Values of B for p-p and p-n interactions are listed in Tables IV and V, respectively. A tendency is observed for an increase in the degree of isotropy with increased multiplicity.

3. ANALYSIS OF THE ASYMMETRY OF EMISSION OF SECONDARY CHARGED PARTICLES IN THE C.M.S.

The question of the existence of showers with asymmetrical emission of secondary particles, both at energies $E \geq 10^{11}$ eV and at energies which can be reached in accelerators, has been discussed in a number of papers.[16-18] Since we have noted earlier [19] that asymmetrical emission of charged particles can be due to random deflections from symmetry, we have specially studied this question. In the individual events we found the asymmetry coefficient

$$\alpha = (n_+ - n_-) / (n_+ + n_-),$$

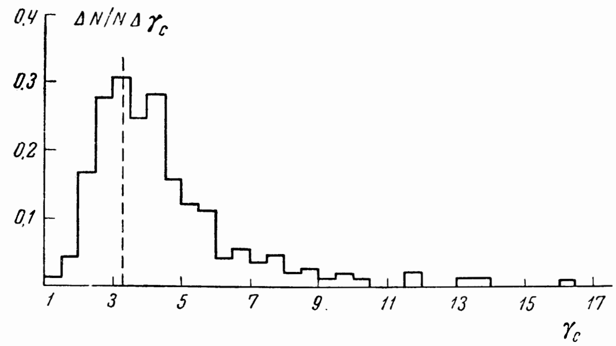

FIG. 1. Distribution of γ_c for individual interactions with $n \geq 5$, calculated on the assumption of constant transverse momentum.

where n_+ and n_- are the number of particles emitted forward and backward in the c.m.s., respectively. Then we plotted a distribution in $|\alpha|$ for p-p, p-n, and p-N interactions, beginning with $n \geq 5$ (Fig. 2a, b, c). The experimental distributions in $|\alpha|$ were compared with a histogram corresponding to the assumption of random deflection of particles from a symmetrical distribution. The probability of observing a certain value of α was determined from the binomial distribution

$$P_n(\alpha) = \frac{n!}{n_+! n_-!} p^{n_+} q^{n_-}$$

assuming independent and symmetrical emission of particles in the c.m.s. ($p = q = \frac{1}{2}$). This probability was then summed over intervals $\Delta\alpha = 0.25$, taking into account the distribution in number of prongs of $P(\alpha)$. Table VI lists the experimental and expected numbers of events with $0 \leq |\alpha| \leq 0.5$ and $0.5 < |\alpha| \leq 1$ for p-p, p-n, and p-N interactions.

From Fig. 2 and Table VI it is evident that within the framework of the assumptions made, which do not depend on a specific model of meson production, the occurrence of showers with asymmetrical emission of charged particles is consistent with the assumption of random deflection from symmetrical emission. In this scheme we have not taken into account the conservation of energy and momentum of the generated particles, since the mechanism of their production is un-

Table VI

Values of α		Interactions		
		pp	pn	pN
$0 \leqslant \lvert\alpha\rvert \leqslant 0.5$	Experiment Theory	128±11 128.8	142±12 149	270±16 277
$0.5 < \lvert\alpha\rvert \leqslant 1$	Experiment Theory	33±6 32.2	67±8 60	100±10 93

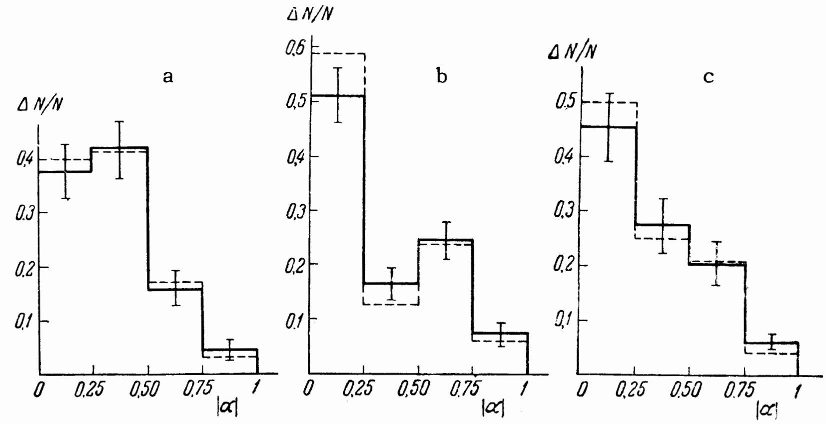

FIG. 2. Distributions of $|\alpha|$: a) p-p interactions, b) p-n interactions, c) p-N interactions; the solid lines are the experimental distributions, and the dashed lines are theoretical.

known. However, to illustrate the applicability of the method used and to determine the effect of energy and momentum conservation within the framework of one particular model (the statistical theory), we obtained distributions in α from 201 random stars at an energy of 9 BeV [20] (Fig. 3). It is evident from this figure that the assumption of constant transverse momentum is in good agreement with the observed distributions obtained both from the table of random stars and from the binomial law. It must be noted that the law of conservation of energy and momentum limits the frequency of appearance of asymmetrical showers within the framework of the model considered; however, the difference from the binomial distribution arising here did not exceed the statistical errors. Comparison of the distributions by the method of Kolmogorov [21] shows that, at the 85% confidence level, the observed discrepancies can be ascribed to random fluctuations.

Fridlander and Spirchez,[18] working at an energy close to that used by us, used a similar method and obtained an opposite result. The conclusion reached by these authors of the existence of asymmetrical showers not due to fluctuations is, in our view, unconvincing. In the first place it is based essentially on discussion of interactions with low multiplicity (n_s = 3 and 4) which, as we have already noted, have a systematic asymmetry of charged particle emission in the direction of motion of the center of mass, which is methodological in origin. In the second place, only 116 interactions were used, and these were selected on the basis of the questionable criterion $N_h < 4$, according to which the contribution of nucleon-nuclear events is large.

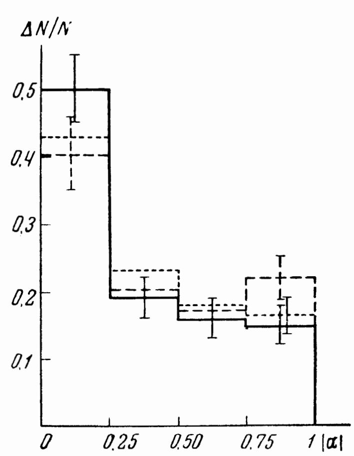

FIG. 3. Distribution of $|\alpha|$ for 201 random stars at 9 BeV. The solid line represents the experimental distribution, and the dashed line a binomial distribution; the dotted line is the distribution obtained assuming $p_\perp$ = const.

4. INTERACTION OF PROTONS WITH EMULSION NUCLEI

The total number of interactions observed is 7960, including 7890 inelastic collisions. This corresponds to a total mean free path $\lambda_{tot} = 36.8 \pm 0.4$ cm and a mean free path for inelastic interactions $\lambda_{inel} = 37.1 \pm 0.5$ cm.

The value of λ_{tot} found by us is compared in Table VII with values of mean free path obtained by different authors, also with good statistics.[14,6,9,12] It is evident from Table VII that the values listed are practically independent of energy and are in good agreement with the value $\lambda = 35.7$ cm [15] expected for emulsion, which demonstrates the high scanning efficiency.

The mean value of $\langle N_h \rangle = 7.42 \pm 0.08$ found by us is in good agreement with the other values

Table VII

Energy, BeV	Source of data	λ, cm
9.0	[12]	36.9±0.9
9.0	[14]	35.9±0.4
19.8	Present work	36.8±0.4
23.5	[6]	36.6±1.0
27.0	[9]	37.2±0.4

Table VIII

	$N_h \leqslant 1$		$1<N_h\leqslant 6$	$6<N_h\leqslant 16$	$N_h>16$	Average nucleus
	$p-N$	p-nuclear				
$\langle n_s\rangle$	3.9±0.1	4.1±0.2	4.86±0.09	6.6±0.1	8.1±0.3	5.89±0.06
		4.74±0.08		7.0±0.1		
$\langle N_h\rangle$	0.24±0.02	0.69±0.04	3.92±0.07	10.8±0.2	20.8±0.7	8.5±0.1
%	13.1	6.5	35.2	34.1	11.1	

listed for nearby energies.[15] The mean number of shower particles in an inelastic interaction, $\langle n_s\rangle = 5.63 \pm 0.06$, agrees satisfactorily with the empirical rule found by Meyer et al,[15] $\langle n_s\rangle = 0.65\,(E_{kin},\ \text{BeV})^{0.7}$. Since the energy dependence is stronger in this case than follows from the hydrodynamical and statistical theories ($n_s \sim E^{1/4}$), we can apparently assume that up to 20 BeV the generation of shower particles plays a role at the expense of cascade interactions.

Table VIII lists values of $\langle n_s\rangle$ and $\langle N_h\rangle$ for a group of interactions with different N_h. The table shows that with increasing N_h a rapid increase is observed in the average number of shower particles from a value 3.9 ± 0.1 for p-N collisions to a value 8.1 ± 0.3 for a group of stars with $N_h > 16$. In the last column of Table VIII are listed values of $\langle n_s\rangle_{\text{av.nuc.}}$ and $\langle N_h\rangle_{\text{av.nuc.}}$, which represent the average characteristics of proton-nuclear interactions.

In agreement with the work of Lohrmann and Teucher,[22] the admixture of interactions in light nuclei can be neglected in groups of events with $N_h > 6$. On the other hand Meyer et al [15] showed that in the group of events with $0 < N_h \leq 6$ there is a considerable admixture of interactions with heavy nuclei, which does not, however, affect in an important way the average multiplicity of showers of this group. Consequently, the value $\langle n_s\rangle_{l.\text{nuc.}} = 4.74 \pm 0.07$ listed in Table VIII for the group of light nuclei apparently is not greatly different from the true value, whereas the value $\langle n_s\rangle_{\text{h.nuc.}}$ for heavy nuclei is exaggerated. Proceeding from the composition of the emulsion and using the experimentally found values of $\langle n_s\rangle_{l.\text{nuc.}}$ and $\langle n_s\rangle_{\text{av.nuc.}}$, we can obtain the value $\langle n_s\rangle_{\text{h.nuc.}} = 6.18 \pm 0.08$, which is somewhat less than the value 7.0 ± 0.1 listed in Table VIII.

It is clear from Table VIII that events of the proton-nucleon type amount to 13.1% of the total number of inelastic interactions, which is consistent with the values obtained for nuclear emulsion according to the data of several authors.[7,23] In the group of showers with $N_h \leq 1$ the average multiplicities in proton-nucleon and proton-nuclear events agree within the limits of error, whereas $\langle N_h\rangle$ for proton-nuclear events is three times larger than $\langle N_h\rangle_{p-N}$.

Table IX lists the ratios of $\langle n_s\rangle_{l.\text{nuc.}}$, $\langle n_s\rangle_{\text{av.nuc.}}$, and $\langle n_s\rangle_{\text{h.nuc.}}$ to the mean number of shower particles for proton-nucleon events $\langle n_s\rangle_{p-N}$. The experimental values are compared with the corresponding values obtained using the "nucleon-tube" model [24] and the nuclear cascade model.[25] The calculations of Barashenkov et al[25] were made at an energy of 9 BeV. As can be seen from the table, for the different groups of nuclei the ratio $\langle n_s\rangle_{p-Z}/\langle n_s\rangle_{p-N}$ (the Z indicates a nucleus) increases with increasing atomic weight both for the experimental values and for the models considered. Consequently, the dependence of the multiplicity on the atomic number is relatively insensitive to the choice of model for the interaction (tube, cascade), but the best agreement in absolute value of the ratios $\langle n_s\rangle_{p-Z}/\langle n_s\rangle_{p-N}$ with experiment is observed for the calculations of Barashenkov et al.[25] This agreement is improved if we take for $\langle n_s\rangle_{\text{h.nuc.}}$ the value 6.18 ± 0.08 listed above. In this case the experimental value is $\langle n_s\rangle_{\text{h.nuc.}}/\langle n_s\rangle_{p-N} = 1.58 \pm 0.06$. The small difference which exists can evidently be explained by two factors: first, by the fact that the calculation was made for an energy of 9 BeV; second, by the fact that interactions of protons with quasifree nucleons were not included in the groups of interactions with light and heavy nuclei (exclusion of these interactions from the number of proton-nucleon events leads to a considerable reduction of the values of $\langle n_s\rangle_{l.\text{nuc.}}$, $\langle n_s\rangle_{\text{av.nuc.}}$, and $\langle n_s\rangle_{\text{h.nuc.}}$, without affecting the value of $\langle n_s\rangle_{p-N}$).

Table IX

	Tube model [24]	Cascade model at 9 BeV [25]	Experiment
$\langle n_s\rangle_{\text{l. nuc.}}/\langle n_s\rangle_{p-N}$	1.62	1.09	1.21±0.05
$\langle n_s\rangle_{\text{av.nuc.}}/\langle n_s\rangle_{p-N}$	2.00	1.39	1.50±0.05
$\langle n_s\rangle_{\text{h.nuc.}}/\langle n_s\rangle_{p-N}$	2.3	1.55	1.77±0.08

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Note added in proof (November 3, 1964). Recently Barashenkov et al[26] have published results of calculations of the average multiplicity at 25 BeV, using the cascade model. These results are in excellent agreement with the experimental data listed in Table VIII.

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