SCATTERING OF NEUTRONS BY QUANTIZED MAGNETIC FLUX LINES IN TYPE II SUPERCONDUCTORS

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We consider the diffraction of neutrons by quantized magnetic flux lines in type II superconductors. The formulae obtained enable us to determine the magnetic structure of a superconductor from neutron diffraction data.

IT is well known[1] that a magnetic field larger than H_{c1} penetrates into the interior of type II superconductors in the form of quantized magnetic flux lines which form a regular lattice in two dimensions, either square or triangular.[1,2] In the papers by De Gennes and Matricon[3] and F. Shapiro (private communication) it was suggested to study this magnetic structure using neutron diffraction. In the present paper we evaluate this effect, considering both a square and a triangular lattice.

The transition probability for neutron scattering is given by the equation

$$dw = \frac{2\pi}{\hbar} |M|^2 \frac{d^3p}{(2\pi\hbar)^3} V, \tag{1}$$

where the transition matrix element is evaluated using free neutron functions,

$$M = \mu_n \frac{1}{V} \int e^{i\mathbf{q}\mathbf{r}} (\boldsymbol{\sigma}\mathbf{H})\, dV, \tag{2}$$

where $\mathbf{q} = (\mathbf{p}' - \mathbf{p})/\hbar$, $\mu_n \boldsymbol{\sigma}$ is the neutron magnetic moment, and $H(\boldsymbol{\rho})$ is the magnetic field in the sample[1]

$$H(\boldsymbol{\rho}) = \frac{\Phi_0}{2\pi\delta^2} \sum_m K_0\left(\frac{\boldsymbol{\rho} - \mathbf{R}_m}{\delta}\right), \tag{3}$$

$\boldsymbol{\rho}$ is the radius vector in the xy-plane which is perpendicular to **H**, the $\mathbf{R}_m$ correspond to the coordinates of the flux line centers, δ is the penetration depth of the magnetic field, $\Phi_0 = ch/2e$ is the flux quantum of the magnetic induction, and K_0 a MacDonald function.

Substituting (3) into (2) and evaluating the matrix element we get

$$M = \pm \mu_n \frac{\delta_{p_z' p_z}}{S} \frac{\Phi_0}{1 + \delta^2 q^2} \sum_m e^{-i\mathbf{q}\mathbf{R}_m}, \tag{4}$$

where S is the cross-sectional area of the sample in the xy-plane. The sum in (4) is non-vanishing only when the vector $\mathbf{q} = \boldsymbol{\tau}$ which is a vector of the reciprocal lattice, so that

$$\left|\sum_m e^{-i\mathbf{q}\mathbf{R}_m}\right|^2 = \left|N \sum_\tau \delta_{\mathbf{q},\boldsymbol{\tau}}\right|^2 = N^2 \sum_\tau \delta_{\mathbf{q}\boldsymbol{\tau}}$$

where N is the number of centers.

The square of the matrix element is equal to

$$|M|^2 = \mu_n^2 \frac{\delta_{p_z' p_z}}{S^2} N^2 \Phi_0^2 \frac{(2\pi)^2}{S} {\sum_\tau}' \frac{\delta(\mathbf{q} - \boldsymbol{\tau})}{(1 + \delta^2\tau^2)^2}, \tag{5}$$

so that the probability for a transition per unit time will be:

$$dw = \frac{2\pi}{\hbar} \frac{(2\pi N \mu_n \Phi_0)^2}{S^3} {\sum_\tau}' \frac{\delta(\mathbf{q} - \boldsymbol{\tau})}{(1 + \delta^2\tau^2)^2} \frac{S}{(2\pi\hbar)^2} \times \int \delta(\varepsilon - \varepsilon')\, \delta(p_z - p_z')\, d^3p'$$

or, finally,

$$dw = \frac{2\pi m_n}{\hbar^3} \mu_n^2 \Phi_0^2 \left(\frac{N}{S}\right)^2 {\sum_\tau}' \frac{\delta(\mathbf{q} - \boldsymbol{\tau})}{(1 + \delta^2\tau^2)^2}\, d\varphi'. \tag{6}$$

A neutron diffraction study of the structure of a substance can be performed by the Debye or by the von Laue method. If we use a polycrystalline or a not very homogeneous sample, the orientation of the magnetic lattice will vary over the sample. We must then apply the Debye method. The differential extinction coefficient in the Debye method, i.e., the ratio of the number of particles scattered into $d\varphi$ per unit time and unit volume to the flux density is equal to $dh_D = dw/v$ so that

$$dh_D = \frac{2\pi m_n^2}{\hbar^3 p} (\mu_n B)^2 {\sum_\tau}' \frac{\delta(\mathbf{q} - \boldsymbol{\tau})}{(1 + \delta^2\tau^2)^2}\, d\varphi', \tag{7}$$

where $B = \Phi_0 N/S$ is the magnetic induction in the sample.

The neutron wavelength is given, but the orientation of the reflecting planes is random. We rewrite the δ-function in (7) as follows:

$$\delta(\mathbf{q}-\boldsymbol{\tau})=\delta\left[\frac{p}{\hbar}-\left(\frac{p^2}{\hbar^2}+\tau^2-2\frac{p}{\hbar}\tau\cos\varphi\right)^{1/2}\right]$$
$$\times\frac{\hbar}{p}\,\delta(\vartheta-\vartheta_\tau)=\frac{\delta(\varphi-\varphi_0)}{\tau\sin\varphi_0}\,\frac{\hbar}{p}\,\delta(\vartheta-\vartheta_\tau), \qquad (8)$$

where φ is the angle between **p** and τ, ϑ the scattering angle, ϑ_T the Bragg angle, and $\varphi_0 = (\pi - \vartheta)/2$. Averaging over the angle φ we get the following expression for dh_D:

$$dh_D=\frac{m_n{}^2}{\hbar^3 p}(\mu_n B)^2\sum_\tau\frac{\hbar\delta(\vartheta-\vartheta_\tau)}{p\tau\cos(\vartheta_\tau/2)\,(\tau^2\delta^2+1)^2}\,d\vartheta. \qquad (9)$$

The intensity of the n-th maximum from the plane with Miller indices k, l is equal to

$$J_n^{(k,\,l)}=\frac{m_n^2}{\hbar^2p^2}(\mu_nB)^2\left(\frac{d_{kl}}{2\pi n}\right)$$
$$\times\left[1-\left(\frac{n\lambda}{2d_{kl}}\right)^2\right]^{-1/2}\left[1+\left(\frac{2\pi n\delta}{d_{kl}}\right)^2\right]^{-2}. \qquad (10)$$

The scattering angle ϑ_T is determined by the Bragg condition

$$2d_{kl}\sin(\vartheta_\tau/2)=n\lambda,$$

d_{kl} is the distance between the planes for the system of (kl) planes:

$$d_{kl}=a\,/\,(k^2+l^2)^{1/2} \qquad (11)$$

for a quadratic lattice, and

$$d_{kl}=a\sqrt{3}\,/\,2(k^2+l^2-kl)^{1/2} \qquad (11')$$

for a triangular lattice. Here a is the lattice period which is equal to $(\Phi_0/B)^{1/2}$ for a square and to $(2\Phi_0/\sqrt{3}B)^{1/2}$ for a triangular lattice. It follows from Eq. (10) that if $\lambda \ll \delta$ (this is usually satisfied since λ is a few angstrom), the intensity decreases for large n as n^{-5}.

If the experiment is performed on a very homogeneous superconductor, we can assume that the sample will be a single crystal as far as the magnetic structure is concerned[1)] and apply the von Laue method which enables us to determine the type of lattice. We shall assume that the neutrons have a Maxwellian distribution with a temperature T and that their average wavelength is appreciably smaller than the period of the structure.

The transition probability must in that case be averaged over the momentum distribution of the incident neutrons. To obtain the extinction coefficient dh we must refer the averaged transition probability per unit volume to the averaged flux. We have thus

$$dh_L=\int_0^\infty dw\exp\left(-\frac{p^2}{2mT}\right)dp\Bigg/\int_0^\infty v\exp\left(-\frac{p^2}{2mT}\right)dp. \qquad (12)$$

Substituting here Eq. (6) we get

$$dh_L=\frac{2\pi m_n}{\hbar^2T}(\mu_nB)^2\sum_\tau\frac{\delta(\vartheta-\vartheta_\tau)}{\tau(1+\tau^2\delta^2)^2\sin(\vartheta_\tau/2)}\cdot d\vartheta. \qquad (13)$$

The angle ϑ_T is in the von Laue method determined solely by the crystallographic planes and each maximum in the diagram corresponds to the sum of the reflections of all orders from the given family of planes. The angle ϑ_T depends in Eq. (13) only on the Miller indices. We must take the quantity τ to be equal to $2\pi n/d_{kl}$ [see (11) and (11′)] and sum over all n.

If $\delta/d_{kl} > 1$, the sum over n converges fast. In the opposite limiting case, $\delta/d_{kl} \ll 1$, we can get the asymptotic formula

$$dh_L=\frac{m_n(\mu_nB)^2}{\hbar^2T}\sum_{k,\,l}d_{kl}\ln\left[\frac{\gamma d_{kl}^2}{(2\pi\delta)^2\sqrt{e}}\right]\frac{1}{\sin(\vartheta_{kl}/2)}$$
$$\times\,\delta(\vartheta-\vartheta_{kl})\,d\vartheta. \qquad (14)$$

It is not difficult to determine all scattering planes. We shall give here the formulae to determine the Miller indices from the scattering angle for two directions of the neutron beam for each lattice.

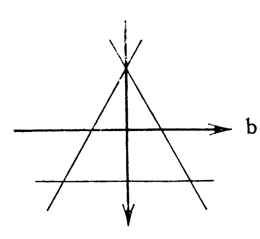

FIG. 1

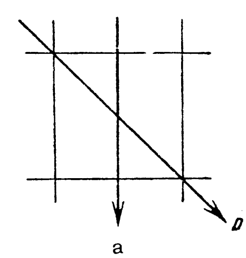

FIG. 2

1. In the case of the triangular lattice (Fig. 1)

$$\frac{l}{k}=\cos\left(\frac{\vartheta}{2}-\frac{\pi}{6}\right)\Bigg/\sin\frac{\vartheta}{2}, \qquad (a)$$

$$\frac{l}{k}=\cos\frac{\vartheta}{2}\Bigg/\cos\left(\frac{\pi}{3}-\frac{\vartheta}{2}\right). \qquad (b)$$

2. In the case of a square lattice (Fig. 2)

$$l\,/\,k=\cot(\vartheta\,/\,2), \qquad (a)$$
$$l\,/\,k=\cot(\vartheta\,/\,2+\pi\,/\,4). \qquad (b)$$

We can use these equations to draw beforehand the Laue diagrams to be expected.

1)If we take a single crystal with one of its principal axes along the field, we can assume that the crystallographic anisotropy will favor the stability of the magnetic structure.

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[1] A. A. Abrikosov, JETP **32**, 1442 (1957), Soviet Phys. JETP **5**, 1174 (1957).

[2] Kleiner, Roth, and Autler, Phys. Rev. **133**, A1226 (1964).

[3] P. G. De Gennes and J. Matricon, Revs. Modern Phys. **36**, 45 (1964).

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