RESONANCE SCATTERING OF PHOTONS BY PHOTONS VIA INTERMEDIATE BOUND STATES

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Formulas are derived for the cross section of resonance scattering of a photon by a photon via intermediate bound states of e^+e^- (positronium) and $\mu^+\mu^-$. At resonance the cross section reaches 10^{-20} cm^2.

THE scattering of light by light predicted by quantum electrodynamics [1] should take on resonance characteristics whenever the energy of the colliding photons in the center-of-mass system (c.m.s.) is close to the energy of any discrete state of interacting fields which has the same quantum numbers as the two-photon system.

Oraevskiĭ [2] has considered the resonance scattering of a photon by a photon via a π^0 meson in the intermediate state. An analogous description can be used for scattering via an η meson ($m = 548$ MeV). One must only allow for the fact that the partial width associated with the decay $\eta \to 2\gamma$ is not the same as the total energy width of the η meson.

The role of the discrete intermediate state can also be played by bound states of pairs of charged particles (e^+e^-), ($\mu^+\mu^-$), ($\pi^+\pi^-$), (K^+K^-), ($p\bar{p}$), and so on. In this paper we consider resonances caused by purely electrodynamic systems, (e^+e^-), ($\mu^+\mu^-$).

The matrix element which describes the scattering of a photon by a photon via positronium,

$$\gamma_1 + \gamma_2 \to (e^+e^-) \to \gamma_3 + \gamma_4,$$

near resonance can be represented in the form ($\hbar = c = 1$, $e^2/4\pi = \alpha = 1/137$)

$$M = M(1234) + M(2134) + M(1243) + M(2143), \tag{1}$$

where

$$M(1234) \equiv M(k_1, e_1; k_2, e_2; k_3, e_3; k_4, e_4)$$
$$= \frac{e^4}{4(\omega_1\omega_2\omega_3\omega_4)^{1/2}} \int d^4x_1 d^4x_2 d^4x_3 d^4x_4 \exp[i(k_1x_1$$
$$+ k_2x_2 - k_3x_3 - k_4x_4)]\, \overset{-+}{G}_{\alpha\sigma,\beta\tau}(x_3x_4, x_1x_2)$$
$$\times C_{\gamma\tau}C^{-1}_{\sigma\delta}[\hat{e}_1 S^c(x_1 - x_2)\hat{e}_2]_{\beta\gamma}[\hat{e}_4 S^c(x_4 - x_3)\hat{e}_3]_{\delta\alpha}, \tag{2}$$

k_i and e_i are the momentum and the polarization vector of the i-th photon, C is the charge-conjugation matrix, $\hat{e} = e_\mu\gamma_\mu$, $kx = \mathbf{k}\cdot\mathbf{x} - \omega t$,

$$S^c(x) = \int \frac{d^4p}{(2\pi)^4} \frac{i(i\hat{p} - m)}{p^2 + m^2 - i0} e^{ipx} \tag{3}$$

and $G^{-+}_{\alpha\sigma,\beta\tau}(x_3x_4, x_1x_2)$ is the two-particle Green's function which describes the propagation of an electron from the point x_1 to x_3 and a positron from x_2 to x_4 and takes into account the interaction between them.[1)]

We are to take into account in G^{-+} only the contribution of bound states (positronium). Furthermore, for simplicity we shall describe the relative motion of the particles in positronium in the nonrelativistic approximation, neglecting the fine structure. In this approximation we can write

$$G^{-+}_{\alpha\sigma,\beta\tau}(34,12) = \int d^3p \sum_{(n)} \Psi^{\alpha\sigma}_{\mathbf{p}(n)}(34)\, \Psi^{+\beta\tau}_{\mathbf{p}(n)}(12)\, \theta(t_{34} - t_{12}), \tag{4}$$

where $\Psi^{\alpha\sigma}_{\mathbf{p}(n)}$ are the normalized wave functions of positronium, which are of the form

$$\Psi^{\alpha\sigma}_{\mathbf{p}(n)}(34) = (2\pi)^{-3/2} \exp[i(\mathbf{p}\mathbf{x}_{34} - E_n t_{34})]\, u_{nlm}(\mathbf{x}_3 - \mathbf{x}_4)\chi^{\alpha\sigma}_{ss_z}, \tag{5}$$

where p is the momentum of the positronium atom as a whole,

$$\mathbf{x}_{34} = \tfrac{1}{2}(\mathbf{x}_3 + \mathbf{x}_4), \quad t_{34} = \tfrac{1}{2}(t_3 + t_4), \quad (n) \equiv nlmss_z,$$
$$E_n = (\mathbf{p}^2 + M_n^2)^{1/2}, \quad M_n = 2m - \alpha/2a_0n^2,$$
$$a_0 = 2/m\alpha, \quad n = 1, 2, 3, \ldots,$$

$u_{nlm}(\mathbf{x})$ is the normalized wave function of the relative motion, and χ^z_{ss} is the spin wave function.

Since the wavelength of the relative motion of

1) The expression (2) corresponds to the sum of the diagrams obtained by insertion of parallel photon lines into the square diagram that describes the scattering of a photon by a photon in the basic approximation of perturbation theory.

the photons is of the order of m^{-1}, only distances $\lesssim m^{-1} \ll a_0$ are important in the integrals over the relative coordinates in (2). Therefore the main contribution to the scattering comes from the s states of positronium ($l = 0$), and we need only take the values of the corresponding wave functions at the origin:

$$u_{n00}(0) = \pi^{-1/2}(a_0 n)^{-3/2}. \qquad (6)$$

Confining ourselves to the use of s states only, we must take $s = s_z = 0$ in (5), since owing to conservation of charge parity

$$C_{\text{pos}} = (-1)^{l+s} = C_{2\gamma} = 1.$$

The spin wave function for $s = 0$ (parapositronium) is given by

$$\chi_{00} = \frac{i}{\sqrt{2}}\begin{pmatrix} \sigma_y & 0 \\ 0 & 0 \end{pmatrix}, \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad (7)$$

in a representation in which the charge-conjugation matrix is

$$C = \alpha_y = \begin{pmatrix} 0 & \sigma_y \\ \sigma_y & 0 \end{pmatrix}. \qquad (8)$$

After substituting (3)—(8) in (2) we can easily do the integration and the matrix operations. The result in the c.m.s. is

$$M(1234) = \frac{e^4(2\pi)^3\delta(k_1 + k_2 - k_3 - k_4)}{4im^4(na_0)^3}\,\frac{(\mathbf{n}[\mathbf{e}_1\mathbf{e}_2])\,(\mathbf{n}'[\mathbf{e}_3\mathbf{e}_4])}{2\omega - M_n + i\Gamma_n/2};$$

$$\mathbf{n} = \frac{\mathbf{k}_1}{\omega} = -\frac{\mathbf{k}_2}{\omega}, \quad \mathbf{n}' = \frac{\mathbf{k}_3}{\omega} = -\frac{\mathbf{k}_4}{\omega}, \qquad (9)^*$$

and ω is the frequency in the c.m.s.; it is indicated explicitly that M_n has the imaginary part $-i\Gamma_n/2$, where Γ_n is the total width of the positronium level with $l = s = 0$ and principal quantum number n.

It is easy to see from (9) and (1) that the total matrix element is given by

$$M = 4M(1234). \qquad (10)$$

On the basis of (10) and (9) we easily get the following expression for the cross section in the c.m.s. for scattering of photon by photon via the s states of positronium:

$$d\sigma_n = \frac{\gamma_n^2}{4m^2}\,\frac{(\mathbf{n}[\mathbf{e}_1\mathbf{e}_2])^2(\mathbf{n}'[\mathbf{e}_3\mathbf{e}_4])^2}{(2\omega - M_n)^2 + \Gamma_n^2/4}\,d\Omega, \qquad (11)$$

*$[\mathbf{e}_1\mathbf{e}_2] = \mathbf{e}_1 \times \mathbf{e}_2$.

where γ_n is the annihilation width of the level, given by

$$\gamma_n = \frac{4\alpha^2}{m^2(na_0)^3} = \frac{0.8\cdot 10^{10}\ \text{sec}^{-1}}{n^3}. \qquad (12)$$

Averaging over the initial and summing over the final polarizations gives

$$d\sigma_n = \frac{d\Omega}{4m^2}\,\frac{\gamma_n^2}{(2\omega - M_n)^2 + \Gamma_n^2/4}. \qquad (13)$$

Integrating over a hemisphere, we get

$$\sigma_n = \frac{\pi}{2m^2}\,\frac{\gamma_n^2}{(2\omega - M_n)^2 + \Gamma_n^2/4}. \qquad (14)$$

The total width is practically the same as the annihilation width, $\Gamma_n \approx \gamma_n$, since the radiative widths of the ns states of parapositronium are of the order of $10^8\,\text{sec}^{-1}/n^3$ (cf. e.g., [3], page 418). At resonances $2\omega = M_n$, so that

$$\sigma_n = 2\pi / m^2 = 1\cdot 10^{-20}\ \text{cm}^2. \qquad (15)$$

In spite of such a large cross section, observation of the resonance scattering is difficult on account of the small width of the resonance.

The resonance scattering via bound states of $\mu^+\mu^-$ is obviously described by the same formulas, with the electron mass m replaced by the μ-meson mass m_μ. Then, for example, we get instead of (15) the value

$$\sigma_n = 2\pi / m_\mu^2 = 2.5\cdot 10^{-25}\ \text{cm}^2. \qquad (16)$$

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[1] A. I. Akhiezer and V. B. Berestetskiĭ, Kvantovaya élektrodinamika (Quantum Electrodynamics), Second Edition, Fizmatgiz, 1959.

[2] V. N. Oraevskiĭ, JETP 39, 1049 (1960), Soviet Phys. JETP 12, 730 (1961).

[3] H. A. Bethe and E. E. Salpeter, Quantum Mechanics of One- and Two-electron Atoms, Academic Press, New York, 1957.

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