THEORY OF THERMOMAGNETIC PHENOMENA IN METALS IN A STRONG MAGNETIC FIELD

V. G. BAR'YAKHTAR and S. V. PELETMINSKIĬ

Submitted to JETP editor June 24, 1964

J. Exptl. Theoret. Phys. (U.S.S.R.) **48**, 187-203 (January, 1965)

Formulas are derived for a macroscopic current and heat flow in a body. The kinetic coefficients determining the galvanomagnetic as well as thermomagnetic effects due to scattering of electrons by a short-range potential in metals in a strong magnetic field are calculated. The cases of a degenerate and nondegenerate electron gas are considered for $\omega_H \gg T$ (ω_H is the electron Larmor frequency and T is the temperature). It is shown that the relative amplitude of the oscillations at "resonance," when the chemical potential is $\zeta = \omega_H(N + 1/2)$ (N is an integer), may become of the order of unity.

1. INTRODUCTION

THE investigation of galvanomagnetic phenomena in metals in a strong magnetic field has been the subject of a large number of papers. If the impurity concentration is not too small, then the main mechanism for the scattering of the electrons at low temperatures is scattering by impurities.

The present paper is devoted to a derivation of general formulas for the kinetic coefficients in the presence of an electric field **E** and gradients of the chemical potential and the temperature T. With the aid of these formulas we calculate the galvanomagnetic and thermomagnetic coefficients due to the scattering of electrons by a short-range potential. The electric conductivity for this case was calculated by Skobov[1], while the thermal emf for a nondegenerate electron gas was calculated by Ansel'm and Askerov[2]. We shall determine all the kinetic coefficients for both degenerate and nondegenerate electron gas.

We shall show that for sufficiently low temperatures $T \ll \omega_H$ (ω_H = Larmor frequency of the electrons), and for ζ close to $\omega_H(N + 1/2)$ (N = positive integer), the relative amplitude of the quantum oscillations can reach a magnitude on the order of unity.

2. STATIONARY DENSITY MATRIX

In order to determine the mean value of the microscopic current **j** and of the energy flux density **q**, we must know the stationary density matrix of the system w. To obtain the latter we start from the following equation[1]):

$$i\frac{\partial w}{\partial t} = [H - \zeta N, w], \tag{1}$$

where H—Hamiltonian of the system, N—particle-number operator, and ζ—chemical potential. The representation in which this equation is written is connected with the usual Schrodinger representation for the density matrix w_S by the relation $w = \exp(i\zeta Nt)\, w_S \exp(-i\zeta Nt)$.

In solving (1) we start from the assumption that at each instant of time there is established in each macroscopically small element of volume, in first approximation, a quasi-equilibrium distribution with temperature $T(\mathbf{r}) = \beta^{-1}(\mathbf{r})$ and chemical potential $\zeta(\mathbf{r})$. Accordingly, we represent the quantity $H - \zeta N$ in the form

$$H - \zeta N = \mathcal{H} + V;$$

$$\mathcal{H} = \beta^{-1}\int \beta(\mathbf{r})\varepsilon(\mathbf{r})\,d\mathbf{r} - \beta^{-1}\int \beta(\mathbf{r})\zeta(\mathbf{r})\rho(\mathbf{r})\,d\mathbf{r},$$

$$V = \beta^{-1}\int [\beta - \beta(\mathbf{r})]\varepsilon(\mathbf{r})\,d\mathbf{r} - \beta^{-1}\int [\beta\zeta - \beta(\mathbf{r})\zeta(\mathbf{r})]\rho(\mathbf{r})\,d\mathbf{r} + e\int \varphi(\mathbf{r})\rho(\mathbf{r})\,d\mathbf{r}, \tag{2}$$

where $\epsilon(\mathbf{r})$—energy density operator, $\rho(\mathbf{r})$—particle number density operator, and $\varphi(\mathbf{r})$—potential of the external electric field. Accurate to terms linear in the "interaction" V, the solution of Eq. (1) can be represented in the form

$$w(t) = w_0 + i\int_0^t dt' [w_0, V(t' - t)];$$

$$w_0 = \exp(-\beta\mathcal{H}) / \operatorname{Sp}\exp(-\beta\mathcal{H}),$$

$$V(t) = \exp(iHt)\, V \exp(-iHt). \tag{3}$$

The mean value of the operator $A(\mathbf{r})$ at the instant of time t is determined by the formula

[1])We use a system of units in which $\hbar = c = 1$.

$$\bar{A}(\mathbf{r}, t) = \mathrm{Sp}\, A(\mathbf{r}) w_0 - i \int_0^t dt' \langle [A(\mathbf{r}, t), V] \rangle, \qquad (4)$$

where $\langle \ldots \rangle$ denotes averaging with an equilibrium density matrix. The time t in these formulas should be much larger than the time τ for the establishment of the quasi-equilibrium distribution, and much shorter than the time of attenuation of the macroscopic fluxes, which is determined by the size of the inhomogeneities of T and ζ.

Noting that when $t > 0$ we have

$$\langle [A(t), V] \rangle = -i \int_0^\beta d\lambda \langle \dot{V}(\lambda) A(t) \rangle,$$

where

$$\dot{V}(\lambda) = i[H, V(\lambda)], \qquad V(\lambda) = e^{\lambda H} V e^{-\lambda H},$$

we can rewrite (4) in the form

$$A(\mathbf{r}, t) = \mathrm{Sp}\, A(\mathbf{r}) w_0 - \int_0^t dt' \int_0^\beta d\lambda \langle \dot{V}(\lambda) A(\mathbf{r}, t) \rangle.$$

Since [2)]

$$\dot{\rho}(\mathbf{r}, \lambda) \equiv i[H, \rho(\mathbf{r}, \lambda)] = -e^{-1} \operatorname{div} \mathbf{j}(\mathbf{r}, \lambda),$$
$$\dot{\varepsilon}(\mathbf{r}, \lambda) \equiv i[H, \varepsilon(\mathbf{r}, \lambda)] = -\operatorname{div} \mathbf{q}(\mathbf{r}, \lambda),$$

we get for $\dot{V}(\lambda)$

$$\dot{V}(\lambda) = -\beta^{-1} \int [\beta - \beta(\mathbf{r})] \operatorname{div} \mathbf{q}(\mathbf{r}, \lambda) d\mathbf{r} + \frac{1}{e} \beta^{-1} \int [\beta\zeta - \beta(\mathbf{r})\zeta(\mathbf{r})] \operatorname{div} \mathbf{j}(\mathbf{r}, \lambda) d\mathbf{r} - \int \varphi(\mathbf{r}) \operatorname{div} \mathbf{j}(\mathbf{r}, \lambda) d\mathbf{r}.$$

Integrating this expression by parts, we obtain

$$\dot{V}(\lambda) = -\beta^{-1} \int \left[\mathbf{q} - \frac{1}{e}\zeta \mathbf{j}\right] \nabla \beta d\mathbf{r} - \int \mathbf{j} \left[\mathbf{E} - \frac{1}{e} \nabla \zeta\right] d\mathbf{r}. \quad (5)$$

Using (4) and (5) we get

$$\mathbf{j} = \mathrm{Sp}\, \mathbf{j}\, w_0 + \hat{S}^{(1)}(\mathbf{E} - \nabla\zeta / e) + \hat{S}^{(2)} \beta^{-1} \nabla \beta,$$

$$\mathbf{q} - \zeta \mathbf{j} / e = \mathrm{Sp}\, (\mathbf{q} - \zeta \mathbf{j} / e) w_0 + \hat{S}^{(3)}(\mathbf{E} - \nabla\zeta / e) + \hat{S}^{(4)} \beta^{-1} \nabla \beta,$$

where

$$S_{ki}^{(1)} = \frac{1}{V} \int_0^\infty dt \int_0^\beta d\lambda \langle j_i(\lambda) j_k(t) \rangle,$$

$$S_{ki}^{(2)} = \frac{1}{V} \int_0^\infty dt \int_0^\beta d\lambda \langle \tilde{q}_i(\lambda) j_k(t) \rangle,$$

$$S_{ki}^{(3)} = \frac{1}{V} \int_0^\infty dt \int_0^\beta d\lambda \langle j_i(\lambda) \tilde{q}_k(t) \rangle,$$

$$S_{ki}^{(4)} = \frac{1}{V} \int_0^\infty dt \int_0^\beta d\lambda \langle \tilde{q}_i(\lambda) \tilde{q}_k(t) \rangle; \qquad \tilde{\mathbf{q}} = \mathbf{q} - \frac{\zeta}{e} \mathbf{j},$$

$$\mathbf{q}(t) = \int \mathbf{q}(\mathbf{r}, t) d\mathbf{r}, \qquad \mathbf{j}(t) = \int \mathbf{j}(\mathbf{r}, t) d\mathbf{r}. \qquad (7)$$

In the derivation of (7) we have replaced the upper limit of integration with respect to t by infinity, since the known value of the product of different components of the fluxes differ from zero only if $t \lesssim \tau$.

We note that the quantity $\mathbf{q} - \zeta \mathbf{j}/e$ is equal to

$$\mathbf{q} - \zeta \mathbf{j} / e = \mathbf{q}' - \zeta' \mathbf{j} / e,$$

where $\mathbf{q}' = \mathbf{q} - \varphi \mathbf{j}$—energy flux density in the presence of an electric field, and $\zeta'/e = \varphi + \zeta/e$—electrochemical potential.

From the invariance of the equations of motion against time reversal, follows the existence of a time-independent operator U, satisfying the equation

$$\psi^*(-t, \mathbf{r}; -\mathbf{H}) = U \psi(t, \mathbf{r}, \mathbf{H}) U^{-1}, \qquad (8)$$

where $\psi(t, \mathbf{r}, \mathbf{H})$—operation of electron annihilation at the point $\mathbf{r}$ at the instant of time t ($\mathbf{H}$—external magnetic field). Using the definition of the operators of the macroscopic current $\mathbf{j}(\mathbf{r}, t)$ and the energy flux $\mathbf{q}(\mathbf{r}, t)$ [see (11) below and (8)], we obtain

$$U \mathbf{j}(\mathbf{r}, t; \mathbf{H}) U^{-1} = -\mathbf{j}(\mathbf{r}, -t; -\mathbf{H}),$$
$$U \mathbf{q}(\mathbf{r}, t; \mathbf{H}) U^{-1} = -\mathbf{q}(\mathbf{r}, -t; -\mathbf{H}),$$
$$U w_0(\mathbf{H}) U^{-1} = w_0(-\mathbf{H}). \qquad (9)$$

It follows therefore

$$\mathrm{Sp}\, w_0(\mathbf{H}) j_i(\mathbf{H}) = -\mathrm{Sp}\, w_0(-\mathbf{H}) j_i(-\mathbf{H}),$$
$$\mathrm{Sp}\, w_0(\mathbf{H}) q_i(\mathbf{H}) = -\mathrm{Sp}\, w_0(-\mathbf{H}) q_i(-\mathbf{H}). \qquad (9')$$

Thus, the first term in (6) vanishes in the absence of a magnetic field. On the other hand, if $\mathbf{H} \neq 0$, then the quantities $\mathrm{Sp}\, w_0 \mathbf{j}$ and $\mathrm{Sp}\, w_0 \mathbf{q}$ are generally speaking different from zero.

It also follows from (9) that

$$\langle j_i(\lambda) j_k(t) \rangle_{\mathbf{H}} = \langle j_k(\lambda) j_i(t) \rangle_{-\mathbf{H}},$$
$$\langle \tilde{q}_i(\lambda) \tilde{q}_k(t) \rangle_{\mathbf{H}} = \langle \tilde{q}_k(\lambda) \tilde{q}_i(t) \rangle_{-\mathbf{H}},$$
$$\langle j_i(\lambda) \tilde{q}_k(t) \rangle_{\mathbf{H}} = \langle \tilde{q}_k(\lambda) j_i(t) \rangle_{-\mathbf{H}}.$$

---

2) As can be seen from these formulas, it is necessary to take j and q to mean the operators of the current and the heat flux in the absence of an electric field.

Using these formulas, and also formulas (7), we obtain

$$S_{ki}^{(1)}(\mathbf{H}) = S_{ik}^{(1)}(-\mathbf{H}), \quad S_{ki}^{(2)}(\mathbf{H}) = S_{ik}^{(3)}(-\mathbf{H}),$$
$$S_{ki}^{(4)}(\mathbf{H}) = S_{ik}^{(4)}(-\mathbf{H}). \tag{10}$$

These relations express the Onsager principle of symmetry of the kinetic coefficients.

We shall henceforth take into account only the interaction of the conduction electrons with the impurities. In this case the expressions for $\mathbf{j}$ and $\mathbf{q}$ take the form

$$j_i = \frac{ie}{2m}\left(\frac{\partial\psi^+}{\partial x_i} + ieA_i\psi^+\right)\psi + \text{Herm. conj.},$$
$$q_i = -\frac{1}{2m}\left(\frac{\partial\psi^+}{\partial x_i} + ieA_i\psi^+ \quad \psi + \text{Herm. conj.},\right. \tag{11}$$

and the operator $\psi(\mathbf{r}, t)$ is equal to

$$\psi(\mathbf{r}, t) = \sum_{\varkappa} a_\varkappa \psi_\varkappa(\mathbf{r}) e^{-i\varepsilon_\varkappa t}, \tag{12}$$

where ϵ_κ and $\psi_\kappa(\mathbf{r})$—energy and wave function of the electron in the periodic field of the lattice in the presence of impurities and an external magnetic field, and a_κ—operator of annihilation of the electron in the state κ.

If we use the relation

$$\int_0^\infty dt \int_0^\beta d\lambda \langle A(\lambda)B(t) + B(\lambda)A(t)\rangle = \beta \int_{-\infty}^\infty dt \langle A(0)B(t)\rangle,$$

then we can greatly simplify the following combinations of tensors S:

$$S_{ik}^{(1)} + S_{ki}^{(1)} = \frac{\beta}{V}\int_{-\infty}^\infty dt \langle j_k(0) j_i(t)\rangle,$$
$$S_{ki}^{(2)} + S_{ik}^{(3)} = \frac{\beta}{V}\int_{-\infty}^\infty dt \langle j_k(0) \tilde{q}_i(t)\rangle,$$
$$S_{ik}^{(4)} + S_{ki}^{(4)} = \frac{\beta}{V}\int_{-\infty}^\infty dt \langle \tilde{q}_k(0) \tilde{q}_i(t)\rangle. \tag{13}$$

Using (11), (12), we can represent the current density $\mathbf{j}$, and the energy flux density $\mathbf{q}$ in the form

$$\mathbf{j} = \sum_{\varkappa\varkappa'} \mathbf{j}_{\varkappa\varkappa'} f_{\varkappa\varkappa'}, \qquad \mathbf{q} = \sum_{\varkappa\varkappa'} \mathbf{q}_{\varkappa\varkappa'} f_{\varkappa\varkappa'}, \tag{14}$$

where

$$\mathbf{j}_{\varkappa\varkappa'}(\mathbf{r}, t) = \frac{ie}{2m}(\nabla\psi_\varkappa^* + ie\mathbf{A}\psi_\varkappa^*)\psi_{\varkappa'} - \frac{ie}{2m}\psi_\varkappa^*(\nabla\psi_{\varkappa'} - ie\mathbf{A}\psi_{\varkappa'}),$$
$$\mathbf{q}_{\varkappa\varkappa'}(\mathbf{r}, t) = \frac{ie}{2m}(\nabla\psi_\varkappa^* + ie\mathbf{A}\psi_\varkappa^*)\varepsilon_{\varkappa'}\psi_{\varkappa'}$$
$$- \frac{i}{2m}\varepsilon_\varkappa\psi_\varkappa^*(\nabla\psi_{\varkappa'} - ie\mathbf{A}\psi_{\varkappa'})$$

and $f_{\kappa\kappa'}$—stationary electron density matrix, defined by the relation

$$f_{\varkappa\varkappa'} = \operatorname{Sp} w_0 a_\varkappa^+ a_{\varkappa'} + \mathbf{A}_{\varkappa\varkappa'}(\mathbf{E} - \nabla\zeta / e) + B_{\varkappa\varkappa'}\beta^{-1}\nabla\beta;$$
$$\mathbf{A}_{\varkappa\varkappa'} = \int_0^\infty dt \int_0 d\lambda \langle \mathbf{j}(\lambda) a_\varkappa^+(t) a_{\varkappa'}(t)\rangle,$$
$$\mathbf{B}_{\varkappa\varkappa'} = \int_0^\infty dt \int_0^\beta d\lambda \langle \tilde{\mathbf{q}}(\lambda) a_\varkappa^+(t) a_{\varkappa'}(t)\rangle. \tag{15}$$

3. KINETIC COEFFICIENT

In order to calculate the parts of the fluxes $\mathbf{j}$ and $\mathbf{q}$ which are not connected with the scattering of the electrons by the impurities, we must know the single-particle density matrix $f_{\kappa\kappa'}$ in the absence of impurities. When calculating $f_{\kappa\kappa'}$ we shall assume that the conduction electrons have a quadratic dispersion law. The particle-number density $\rho(\mathbf{r})$ and the energy density $\epsilon(\mathbf{r})$, which enter into expression (2) for $\mathscr{H}$, can be represented in the form

$$\rho(\mathbf{r}) = \sum_{\varkappa\varkappa'} \psi_\varkappa^*(\mathbf{r})\psi_{\varkappa'}(\mathbf{r}) a_\varkappa^+ a_{\varkappa'}, \qquad \varepsilon(\mathbf{r}) = \sum_{\varkappa\varkappa'} \Lambda_{\varkappa\varkappa'}(\mathbf{r}) a_\varkappa^+ a_{\varkappa'},$$
$$\Lambda_{\varkappa\varkappa'}(\mathbf{r}) = \frac{1}{2m}(\nabla\psi_\varkappa^* + ie\mathbf{A}\psi_\varkappa^*)(\nabla\psi_{\varkappa'} - ie\mathbf{A}\psi_{\varkappa'}).$$

The electron wave functions ψ_κ, as is well known, take the form

$$\psi_\varkappa = \frac{(eH)^{1/4}}{(L_y L_z)^{1/2}} e^{i(p_y y + p_z z)} \frac{e^{-\eta^2/2}}{(2^n n! \sqrt{\pi})^{1/2}} H_n(\eta);$$
$$\eta = (x - \xi)\sqrt{eH}, \qquad \xi = \frac{p_y}{eH},$$

where n—oscillation quantum number, p_y and p_z—components of the generalized momentum along the y and z axes, $H_n(\eta)$—Hermite polynomials, and L_y, L_z—dimensions of the normalization volume along the y and z axes [the gauge of the vector potential of the magnetic field is chosen in the form A = (0, Hx, 0)].

In the quasi-classical approximation (large quantum numbers n) the wave functions ψ_κ differ from zero only when $|x - \xi| \lesssim r_H$ (r_H—radius of the electron Larmor orbit). Therefore, if the temperature T and the chemical potential ζ change little over distances of the order of r_H, then the "Hamiltonian" $\mathscr{H}$ can be represented, accurate to terms linear in the gradients of β and ζ, in the form

$$\mathscr{H} = \beta^{-1}\sum_\varkappa a_\varkappa^+ a_\varkappa(\varepsilon_\varkappa - \zeta(\xi))\beta(\xi)$$
$$+ \sum_{\varkappa\varkappa'} a_\varkappa^+ a_{\varkappa'}\left\{\frac{\partial\beta(\xi)}{\partial\xi}\int (x - \xi)[\Lambda_{\varkappa\varkappa'}(\mathbf{r}) - \zeta(\xi)\psi_\varkappa^*\psi_{\varkappa'}]\,d\mathbf{r}\right.$$

$$- \beta(\xi) \frac{\partial \zeta(\xi)}{\partial \xi} \int (x-\xi) \psi_\varkappa^*(\mathbf{r}) \psi_{\varkappa'}(\mathbf{r}) d\mathbf{r} \Big\} \beta^{-1}.$$

The integrals entering in this expression can be easily calculated. We then obtain the following formula for $\mathscr{H}$:

$$\mathscr{H} = \mathscr{H}_0 + U; \qquad \mathscr{H}_0 = \beta^{-1} \sum_\varkappa a_\varkappa^+ a_\varkappa (\varepsilon_\varkappa - \zeta(\xi)) \beta(\xi),$$

$$U = \beta^{-1} \sum_\varkappa \frac{1}{2\sqrt{eH}} \Big\{ \frac{\partial \beta}{\partial \xi} \Big[(2(n+1))^{1/2} \Big(\varepsilon_\varkappa - \zeta(\xi) + \frac{1}{2}\omega_H \Big) a_\varkappa^+ a_{\varkappa+1}$$

$$+ \sqrt{2n} \Big(\varepsilon_\varkappa - \zeta(\xi) - \frac{1}{2}\omega_H \Big) a_\varkappa^+ a_{\varkappa-1} \Big]$$

$$- \beta \frac{\partial \zeta}{\partial \xi} [(2(n+1))^{1/2} a_\varkappa^+ a_{\varkappa+1} + \sqrt{2n}\, a_\varkappa^+ a_{\varkappa-1}] \Big\}.$$

Since U is proportional to $\nabla\beta$ and $\nabla\zeta$, we can represent w_0, accurate to terms linear in the gradients, in the form

$$w_0 = \exp(-\beta\mathscr{H}_0) \Big(1 - \int_0^\beta d\lambda U(\lambda) \Big) \Big/ \operatorname{Sp} \exp(-\beta\mathscr{H}_0). \quad (16)$$

Using (15) and (16) we readily get

$$\operatorname{Sp} w_0 a_\varkappa^+ a_{\varkappa'} = f_\varkappa^0 \delta_{\varkappa\varkappa'} - \frac{\sqrt{2n'}}{2\sqrt{eH}} \frac{1}{\omega_H} (f_{\varkappa'-1}^0 - f_{\varkappa'}^0)$$

$$\times \Big\{ \Big[\varepsilon_{\varkappa'} - \zeta(\xi) - \frac{1}{2}\omega_H \Big] \frac{1}{\beta} \frac{\partial \beta}{\partial \xi} - \frac{\partial \zeta}{\partial \xi} \Big\} \delta_{\varkappa, \varkappa'-1}$$

$$- \frac{\sqrt{2n}}{2\sqrt{eH}} \frac{1}{\omega_H} (f_{\varkappa-1}^0 - f_\varkappa^0)$$

$$\times \Big\{ \Big[\varepsilon_\varkappa - \zeta(\xi) - \frac{1}{2}\omega_H \Big] \frac{1}{\beta} \frac{\partial \beta}{\partial \xi} - \frac{\partial \zeta}{\partial \xi} \Big\} \delta_{\varkappa, \varkappa'+1},$$

$$f_\varkappa^0 = (\exp\{\beta(\xi)(\varepsilon_\varkappa - \zeta(\xi))\} + 1]^{-1}. \quad (17)$$

The quantities $A_{\kappa\kappa'}$ and $B_{\kappa\kappa'}$, which determine the single-particle density matrix $f_{\kappa\kappa'}$, can be calculated by neglecting scattering of the electrons by the impurities. We present here only the final results:

$$A_{\varkappa\varkappa'}^0 = \frac{\sqrt{2n}}{2\omega_H \sqrt{eH}} (f_{\varkappa-1}^0 - f_\varkappa^0) \delta_{\varkappa, \varkappa'+1}$$

$$- \frac{(2(n+1))^{1/2}}{2\omega_H \sqrt{eH}} (f_{\varkappa'}^0 - f_{\varkappa'-1}^0) \delta_{\varkappa, \varkappa'-1},$$

$$B_{\varkappa\varkappa'}^0 = \frac{1}{2} (2\zeta(\xi) - \varepsilon_\varkappa - \varepsilon_{\varkappa'}) A_{\varkappa\varkappa'}^0 \quad (18)$$

(we denote by $A_{\kappa\kappa'}^0$ and $B_{\kappa\kappa'}^0$ the quantities $(A_{\kappa\kappa'})_x$ and $(B_{\kappa\kappa'})_x$ with the scattering of electrons by the impurities neglected).

Using (15), (17), and (18), we obtain the following expression for the single-particle density matrix, neglecting collision of the electrons with the impurities,

$$f_{\varkappa\varkappa'}^0 = f_\varkappa^0 \delta_{\varkappa\varkappa'} - \frac{eE}{2\omega_H \sqrt{eH}} \{ \sqrt{2n} (f_\varkappa^0 - f_{\varkappa-1}^0) \delta_{\varkappa, \varkappa'+1}$$

$$+ \sqrt{2n'} (f_{\varkappa'}^0 - f_{\varkappa'-1}^0) \delta_{\varkappa', \varkappa+1} \}. \quad (19)$$

We note that this expression contains only the electric field E. The terms in (17) proportional to $\partial\beta/\partial\xi$ and $\partial\zeta/\partial\xi$ are cancelled out by analogous terms in $f_{\kappa\kappa'}$, determined by $A_{\kappa\kappa'}^0$ and $B_{\kappa\kappa'}^0$.

The expression given above for the distribution function $f_{\kappa\kappa'}^0$ does not coincide with the well known expression for the stationary diagonal distribution function of the electrons $f_{\lambda\lambda'}^0$ in a strong magnetic field, obtained by Titeica[3]. The reason is that in the Titeica distribution function the states λ represent the states of an electron in the presence of both magnetic and constant electric fields. In order to go over from the distribution function $f_{\kappa\kappa'}^0$ to the distribution function $f_{\lambda\lambda'}^0$ it is necessary to make the following transformation:

$$f_{\lambda\lambda'}^0 = \sum_{\varkappa\varkappa'} f_{\varkappa\varkappa'}^0 \Big\{ \int \varphi_\lambda^* \psi_\varkappa d\mathbf{r} \Big\} \Big\{ \int \varphi_{\lambda'}^* \psi_{\varkappa'} d\mathbf{r} \Big\},$$

where φ_λ—wave functions of the electrons in the presence of an electric field:

$$\varphi_\lambda = \frac{(eH)^{1/4}}{(L_y L_z)^{1/2}} e^{i(p_y y + p_z z)} \frac{e^{-\eta^2/2}}{(2^n n! \sqrt{\pi})^{1/2}} H_n(\eta),$$

$$\eta = \sqrt{eH} \Big(x - \frac{p_y}{eH} - \frac{E}{H\omega_H} \Big).$$

Noting that, accurate to terms linear in the electric field E,

$$\int \varphi_{\lambda'}^* \psi_\varkappa d\mathbf{r} = \delta_{\varkappa\lambda'} - \frac{E\sqrt{eH}}{2\omega_H H} \{ \sqrt{2n'}\, \delta_{\varkappa, \lambda'-1} - \sqrt{2n}\, \delta_{\varkappa, \lambda'+1} \},$$

we obtain for $f_{\lambda\lambda'}^0$, the following expression: $f_{\lambda\lambda'}^0 = f_\lambda^0 \delta_{\lambda\lambda'}$; it coincides with the distribution obtained by Titeica.

Let us calculate now the collisionless parts of the electric current $\mathbf{j}$ and of the energy flux $\mathbf{q}$. Since in the quasi-classical approximation $\mathbf{j}_{\kappa\kappa'}$ and $\mathbf{q}_{\kappa\kappa'}$ differ from zero only when $|x - \xi| \lesssim r_H$, in the approximation considered here we can expand $f_{\kappa\kappa'}$ in powers of $\xi - x$ near the point x:

$$f_{\varkappa\varkappa'} = f_{\varkappa\varkappa'}|_{\xi=x} + (\xi - x) \frac{\partial f_{\varkappa\varkappa'}}{\partial \xi}\Big|_{\xi=x} + \ldots,$$

after which we can readily sum over p_y in (14).

We present here an expression for the circular components of the current density j^+ in the energy flux density q^+:

$$j^+ = j_x + ij_y = -\frac{i\sqrt{eH}}{m}\frac{e}{V}\sum_\varkappa \sqrt{2n}\, f_{\varkappa-1,\varkappa}\Big|_{\xi=x}$$

$$+\frac{ie}{m}\frac{\partial}{\partial x}\frac{1}{V}\sum_\varkappa\Big(n+\frac{1}{2}\Big)f_{\varkappa\varkappa'}\Big|_{\xi=x},$$

$$\tilde q^+ = \tilde q_x + i\tilde q_y = -\frac{i\sqrt{eH}}{mV}\sum_\varkappa \sqrt{2n}\Big(\varepsilon_\varkappa - \zeta - \frac{1}{2}\omega_H\Big) f_{\varkappa-1,\varkappa}\Big|_{\xi=x}$$

$$+\frac{i}{m}\frac{\partial}{\partial x}\frac{1}{V}\sum_\varkappa(\varepsilon_\varkappa-\zeta)\Big(n+\frac{1}{2}\Big)f_{\varkappa\varkappa'}\Big|_{\xi=x}. \qquad (20)$$

Substituting here the expression for $f^0_{\kappa\kappa'}$, we obtain

$$j^+ = \frac{i}{H}\frac{\partial}{\partial x}\frac{1}{V}\sum_\varkappa \varepsilon_\perp f_\varkappa^0 - i\frac{eE}{H}\frac{1}{V}\sum_\varkappa f_\varkappa^0,$$

$$\tilde q^+ = \frac{i}{eH}\frac{1}{V}\sum_\varkappa(\varepsilon_\varkappa-\zeta)\varepsilon_\perp\frac{\partial f_\varkappa^0}{\partial x} - i\frac{E}{H}\frac{1}{V}\sum_\varkappa(\varepsilon_\varkappa+\varepsilon_\perp-\zeta)f_\varkappa^0;$$

$$\varepsilon_\perp = \omega_H\Big(n+\frac{1}{2}\Big). \qquad (21)$$

These formulas determine only the off-diagonal components of the kinetic coefficients. Account of the interaction with the impurities can lead only to small corrections relative to the parameter $1/\omega_H\tau$ (τ—time of free path of the electron at $H = 0$).

We now proceed to calculate the diagonal components of the kinetic coefficients. These components differ from zero only if account is taken of the scattering of the electrons by the impurities. Since, in accordance with formulas (9′), Sp $w_0 j$ and Sp $w_0 q$ are odd functions of the magnetic field H, they can contribute only to the off-diagonal components of the kinetic coefficients. Therefore the diagonal components of the kinetic coefficients are determined by the quantities $S^{(i)}$.

To obtain the latter we note that the operators $\mathbf{j}$ and $\mathbf{q}$ can be represented in the form

$$j_i = e\frac{d}{dt}\hat x_i, \quad q_i = \frac{1}{2}\frac{d}{dt}\widehat{\{x_i, H\}};$$

$$x_i \equiv \int dr\psi^+ x_i\psi, \quad \widehat{\{x_i, H\}} \equiv \int dr\psi^+\{x_i, H\}\psi.$$

According to Kubo et al[4], we break up the quantities x_i into two terms: $x_i = X_i + \xi_i$, where

$$X = -\frac{i}{eH}\frac{\partial}{\partial y}, \quad Y = y + \frac{i}{eH}\frac{\partial}{\partial x}$$

are the coordinates of the center of the electron Larmor orbit (they commute with the Hamiltonian H_0 of the electron in the absence of impurities), and ξ_i are the coordinates of the electron relative to the center of the Larmor orbit. In view of the bounded nature of the coordinate ξ, the correlation between the quantities q_i and $\xi_k(t)$ or j_i and $\xi_k(t)$ disappears as $t \to \pm\infty$, and consequently[3)]

$$\langle q_i\xi_k(t)\rangle_{t\to\pm\infty} \approx \langle q_i\rangle\langle\xi_k\rangle = 0,$$

$$\langle j_i\xi_k(t)\rangle_{t\to\pm\infty} = \langle j_i\rangle\langle\xi_k\rangle = 0.$$

Therefore formulas (13) take the form

$$S_{xx}^{(1)} = \frac{e^2\beta}{2V}\int_{-\infty}^{\infty} dt\,\langle\dot X(0)\dot X(t)\rangle,$$

$$S_{xx}^{(2)} + S_{xx}^{(3)} = \frac{e\beta}{2V}\int_{-\infty}^{\infty} dt\,\langle\dot X\{\widehat{H-\zeta, X(t)}\}^{\cdot}\rangle,$$

$$S_{xx}^{(4)} = \frac{\beta}{8V}\int_{-\infty}^{\infty} dt\,\langle\{\widehat{H-\zeta, X(0)}\}^{\cdot}\{\widehat{H-\zeta, X(t)}\}^{\cdot}\rangle, \qquad (22)$$

where

$$\hat X = \int dr\psi^+ X\psi, \quad \widehat{\{H, X\}} = \int dr\psi^+\{H, X\}\psi;$$

We denote by $\{\ldots\}$ the derivative with respect to time.

Since the kinetic coefficients are symmetrical and the diagonal components of the tensors $S^{(i)}$ are even with respect to the magnetic field H, we have

$$S_{xx}^{(3)} = S_{xx}^{(2)}.$$

For an isotropic body $S^{(i)}_{xx} = S^{(i)}_{yy}$.

Noting that

$$\dot{\hat X} = i[\hat H, \hat X] = i\sum_{\varkappa\varkappa'} a_\varkappa^+ a_{\varkappa'}[H, X]_{\varkappa,\varkappa'},$$

$$\widehat{\{H, X\}}^{\cdot} = i\sum_{\varkappa\varkappa'} a_\varkappa^+ a_{\varkappa'}\{[H, X], H\}_{\varkappa\varkappa'},$$

where H—Hamiltonian of the electron in the magnetic field in the presence of impurities, and the summation is carried out over the complete system of eigenfunctions of the Hamiltonian H with eigenvalues ϵ_κ, we obtain

$$S_{xx}^{(1)} = -\frac{\pi\beta e^2}{V}\sum_{\varkappa\varkappa'}\delta(\varepsilon_\varkappa-\varepsilon_{\varkappa'})f_\varkappa(1-f_{\varkappa'})[H, X]_{\varkappa\varkappa'}[H, X]_{\varkappa'\varkappa},$$

$$S_{xx}^{(3)} = -\frac{\pi\beta e}{4V}\sum_{\varkappa\varkappa'}\delta(\varepsilon_\varkappa-\varepsilon_{\varkappa'})f_\varkappa(1-f_{\varkappa'})$$
$$\times(\varepsilon_\varkappa+\varepsilon_{\varkappa'}-2\zeta)[H, X]_{\varkappa\varkappa'}[H, X]_{\varkappa'\varkappa},$$

$$S_{xx}^{(4)} = -\frac{\pi\beta}{4V}\sum_{\varkappa\varkappa'}\delta(\varepsilon_\varkappa-\varepsilon_{\varkappa'})f_\varkappa(1-f_{\varkappa'})$$
$$\times(\varepsilon_\varkappa+\varepsilon_{\varkappa'}-2\zeta)^2[H, X]_{\varkappa\varkappa'}[H, X]_{\varkappa'\varkappa}.$$

3)This is correct if $\omega_H\tau \gg 1$.

Introducing the function

$$G(E) = \mathrm{Sp}\{\delta(E-H)[H, X]\delta(E-H)[H, X]\}, \quad (23)$$

we represent (22) in the form

$$S_{xx}^{(1)} = \frac{\pi e^2}{V}\int_{-\infty}^{\infty}\frac{\partial f}{\partial E}G(E)\,dE,$$

$$S_{xx}^{(2)} = \frac{\pi e}{V}\int_{-\infty}^{\infty}(E-\zeta)\frac{\partial f}{\partial E}G(E)\,dE,$$

$$S_{xx}^{(4)} = \frac{\pi}{V}\int_{-\infty}^{\infty}(E-\zeta)^2\frac{\partial f}{\partial E}G(E)\,dE. \quad (24)$$

Thus, calculation of the diagonal elements of the kinetic coefficients reduces to a determination of the function G(E).

Formula (23) can be transformed to a form in which the Hamiltonian H_0 of the electron will be under the δ-function sign in the absence of impurities. To this end we introduce the operator T(E), satisfying the following integral equation:

$$T(E) = H_I + H_I(E + i\varepsilon - H_0)^{-1}T(E), \quad (25)$$

where H_I—Hamiltonian of interaction between the electrons and the impurities and $\epsilon \to +0$. The operator T(E) is connected with the scattering matrix S by the relation

$$S_{ab} = \delta(a-b) - 2\pi i\delta(E_a - E_b)T_{ba}(E_a). \quad (26)$$

From the definition of the operator T(E) we can obtain the formula

$$\delta(E-H) = \delta(E-H_0)$$
$$+\frac{i}{2\pi}\{(E+i\varepsilon-H_0)^{-1}T(E)(E+i\varepsilon-H_0)^{-1}$$
$$-(E-i\varepsilon-H_0)^{-1}T^{+}(E)(E-i\varepsilon-H_0)^{-1}\}.$$

Using further this formula, Eq. (25), and also the fact that X commutes with H_0, we can transform the function G(E) into

$$G(E) = \frac{1}{\pi}\,\mathrm{Sp}\{2\pi\delta(E-H_0)XT(E)\delta(E-H_0)XT^{+}(E)$$
$$-i(T(E)-T^{+}(E))X\delta(E-H_0)X\}.$$

From (26) and from the unitarity of S it follows that

$$T_{ba}(E_a) - T_{ba}^{+}(E_0) = -2\pi i[T(E_a)\delta(E_a-H_0)T^{+}(E_a)]_{ba},$$
$$E_a = E_b.$$

Therefore G(E) is equal to

$$G(E) = \mathrm{Sp}\{\delta(E-H_0)[X, T(E)]$$
$$\times\delta(E-H_0)[X, T^{+}(E)]\}. \quad (27)$$

Calculating the trace in the system of eigenfunctions of the Hamiltonian H_0, and introducing in accordance with (26) the probability of scattering of the electrons by the impurities

$$w_{\varkappa\varkappa'} = 2\pi\delta(\varepsilon_\varkappa - \varepsilon_{\varkappa'})\,|T_{\varkappa\varkappa'}(\varepsilon_\varkappa)|^2, \quad (28)$$

we get

$$G(E) = -\frac{1}{2\pi}\sum_{\varkappa\varkappa'}\delta(E-\varepsilon_\varkappa)(X_\varkappa - X_{\varkappa'})^2 w_{\varkappa\varkappa'},$$

$$X_\varkappa \equiv X_{\varkappa\varkappa'}. \quad (29)$$

If the concentration of the impurities is sufficiently small, then the amplitude for the scattering of the electrons by the impurities can be represented in the form of a sum of the amplitudes for the scattering by individual impurities:

$$T_{\varkappa\varkappa'} = \sum_j t_{\varkappa\varkappa'}(\mathbf{r}_j),$$

where $t_{KK'}(\mathbf{r}_j)$—amplitude for the scattering of the electron by the j-th impurity. Expanding $t_{KK'}(\mathbf{r}_j)$ in a Fourier series

$$t_{\varkappa\varkappa'}(\mathbf{r}_j) = \frac{1}{V}\sum_{\mathbf{k}} t_{\varkappa\varkappa'}(\mathbf{k})\,e^{i\mathbf{k}\mathbf{r}_j}$$

and averaging $|T_{KK'}|^2$ over the random distribution of the impurities, we obtain

$$\overline{|T_{\varkappa\varkappa'}|^2} = \frac{n_i}{V}\sum_k |t_{\varkappa\varkappa'}(k)|^2,$$

where n_i—concentration of the impurities and V—normalization volume.

The amplitude for the scattering of the electron in a strong magnetic field is [5]

$$t_{\varkappa\varkappa'}(\mathbf{r}_j) = \frac{2\pi a}{m}\frac{\psi_\varkappa^*(\mathbf{r}_j)\psi_{\varkappa'}(\mathbf{r}_j)}{1+iaK(\varepsilon_\varkappa)}$$

$$K(E) = K'(E) + iK''(E),$$

$$K'(E) = \left(\frac{eH}{2}\right)^{1/2}\sum_{n=0}^{N-1}\left(\frac{E}{\omega_H} - n - \frac{1}{2}\right)^{-1/2},$$

$$K''(E) = \left(\frac{eH}{2}\right)^{1/2}\frac{1}{\sqrt{\eta}} \quad (30)$$

where $2\pi a/m$ is the amplitude for the scattering of the electrons by the impurity with zero energy in the absence of a magnetic field, while η is determined by the equation $N - \eta = E/\omega_H - 1/2$, where N is a positive integer and $0 \le \eta < 1$.

Formula (30) for $t_{KK'}$ is valid if the radius of action of the scattering potential r_0 is much shorter than the wavelength of the electron $\lambda = 1/\sqrt{m\epsilon}$. In calculating the kinetic coefficients in metals, as can be seen from (24), the most important role is played by electrons with energy $\epsilon \sim \zeta$. Therefore formula (30) can be used if $r_0 \ll 1/p_0 \sim d$, where d—lattice constant. Using (28)—(30), we obtain for G(E) the expression

$$G(E) = -\frac{Vn_i}{\pi^2}a^2\left(\frac{2}{eH}\right)^{1/2}\frac{K'(E)I(E)}{(1+aK''(E))^2+(aK'(E))^2};$$

$$I(E) = \sum_{n=0}^{N-1}\left(n+\frac{1}{2}\right)\Big/\left(\frac{E}{\omega_H}-n-\frac{1}{2}\right)^{1/2}. \quad (31)$$

According to (6) and (21), the electric current $\mathbf{j}$ and the heat flux $\mathbf{q}$ can be represented in the form

$$\mathbf{j} = \sigma\mathbf{E} - \sigma'\nabla\frac{\zeta}{e} - \alpha\nabla T,$$

$$\mathbf{q} - \frac{\zeta}{e}\mathbf{j} = \beta\mathbf{E} - \beta'\nabla\frac{\zeta}{e} - \gamma\nabla T, \quad (32)$$

where

$$\sigma_{ii} = \sigma_{ii}' = S_{ii}^{(1)}, \quad \sigma_{12} = -\sigma_{21} = \frac{e}{H}n_e,$$

$$\sigma_{12}' = -\sigma_{21}' = \frac{e}{H}\frac{\partial}{\partial\zeta}n_e\overline{\varepsilon}_\perp,$$

$$\alpha_{ii} = T^{-1}S_{ii}^{(2)}, \quad \alpha_{12} = -\alpha_{21} = \frac{1}{H}\frac{\partial}{\partial T}n_e\overline{\varepsilon}_\perp,$$

$$\beta_{ii} = \beta_{ii}' = S_{ii}^{(2)}, \quad \beta_{12} = -\beta_{21} = \frac{n_e}{H}(\overline{\varepsilon} + \overline{\varepsilon}_\perp - \zeta),$$

$$\beta_{12}' = -\beta_{21}' = \frac{T}{H}\frac{\partial}{\partial T}n_e\overline{\varepsilon}_\perp,$$

$$\gamma_{ii} = T^{-1}S_{ii}^{(4)}, \quad \gamma_{12} = -\gamma_{21} = -\frac{1}{eH}\frac{\partial}{\partial T}[n_e(\overline{\varepsilon\varepsilon_\perp} - \zeta\overline{\varepsilon}_\perp)],$$

$$n_e = \frac{1}{V}\sum_\varkappa f_\varkappa, \quad n_e\overline{F} = \frac{1}{V}\sum_\varkappa F_\varkappa f_\varkappa. \quad (33)$$

4. MACROSCOPIC CURRENT

Formulas (32) and (33) determine the microscopic electric current $\mathbf{j}$ and energy flux $\mathbf{q}$ in the presence of an electric field $\mathbf{E}$ and of gradients of a temperature T and the chemical potential ζ. Since $\sigma_{12} \neq \sigma'_{12}$ and $\beta_{12} \neq \beta'_{12}$, we see from (32) and (33) that $\nabla\zeta$ and E enter independently into the expressions for $\mathbf{j}$ and $\mathbf{q}$.

We now determine the macroscopic current $\mathbf{J}$ which enters in the Maxwell equation*

$$\mathbf{J} = \mathbf{j} - \operatorname{rot}\mathbf{M},$$

where M—density of the magnetic moment of the body. We shall show that the quantities $\nabla\zeta$ and $\mathbf{E}$ enter in $\mathbf{J}$ only in the form of the combination $\mathbf{E} - \nabla\zeta/e = -\nabla\zeta'/e$.

The density of the magnetic moment is determined by the formula

$$M_z(\mathbf{r}) = -\frac{1}{V}\frac{\partial}{\partial H}\Omega = \frac{1}{VH}\sum_\varkappa(2\varepsilon_\parallel - \varepsilon_\perp)f_\varkappa\Big|_{\xi=x}$$

$$= \frac{n_e}{H}(2\overline{\varepsilon}_\parallel - \overline{\varepsilon}_\perp), \qquad \varepsilon_\parallel = \frac{1}{2m}p_z^2. \quad (34)$$

Hence

$$\operatorname{rot}_y\mathbf{M} = -\frac{\partial M_z}{\partial x} = \frac{1}{H}\frac{\partial}{\partial x}n_e(\overline{\varepsilon}_\perp - 2\overline{\varepsilon}_\parallel).$$

Using (21) for J_y, we get

$$J_y = -e\frac{E}{H}n_e + \frac{2}{H}\frac{\partial}{\partial x}n_e\overline{\varepsilon}_\parallel.$$

Noting that $2H^{-1}\partial(n_e\overline{\epsilon}_\parallel)/\partial\zeta = n_e/H$, we get

$$J_y = \tilde{\sigma}_{yx}\left(E - \frac{1}{e}\frac{\partial\zeta}{\partial x}\right) - \tilde{\alpha}_{yx}\frac{\partial T}{\partial x}, \quad (35)$$

$$\tilde{\sigma}_{yx} = \sigma_{yx} = \frac{en_e}{H},$$

$$\tilde{\alpha}_{yx} = -\frac{2}{H}\frac{\partial}{\partial T}n_e\overline{\varepsilon}_\parallel = -\frac{n_e}{HT}(2\overline{\varepsilon}_\parallel + \overline{\varepsilon} - \zeta). \quad (36)$$

The formula for M can be intuitively interpreted in the following fashion: the quantity $2\epsilon_\parallel - \epsilon_\perp$ is the effective current of the electron in the state κ, determining the Landau diamagnetism. Therefore $\mathbf{J}$ is the total electric current left after subtracting the current causing the diamagnetic moment M_z.

We now determine the energy flux $\mathbf{Q}$, which represents the energy flux after subtracting the flux carried by the diamagnetic currents:

$$\mathbf{Q} = \mathbf{q}' - \operatorname{rot}\mathbf{L},$$

where

$$\mathbf{L} = (0, 0, L_z),$$

$$L_z = \frac{1}{eHV}\sum_\varkappa(\varepsilon_\varkappa + e\varphi)(2\varepsilon_\parallel - \varepsilon_\perp)f_\varkappa. \quad (37)$$

It is easy to see that the quantity $Q_y - \zeta' J_y/e$ can be represented in the form

$$Q_y - \frac{\zeta'}{e}J_y = \beta_{yx}\left(E - \frac{\partial}{\partial x}\frac{\zeta}{e}\right) - \gamma_{yx}\frac{\partial}{\partial x}T, \quad (38)$$

where

$$\tilde{\beta}_{yx} = -\frac{n_e}{H}(2\overline{\varepsilon}_\parallel + \overline{\varepsilon} - \zeta) = T\tilde{\alpha}_{yx},$$

$$\tilde{\gamma}_{yx} = -\frac{2}{eHT^2}\frac{1}{V}\sum_\varkappa\varepsilon_\parallel(\varepsilon - \zeta)^2 f_\varkappa(1 - f_\varkappa)$$

$$= -\frac{1}{eHT}n_e\overline{(\varepsilon - \zeta)(\varepsilon - \zeta + 4\varepsilon_\parallel)}. \quad (39)$$

*rot = curl.

Thus, the fluxes $\mathbf{J}$ and $\mathbf{Q} - \zeta'\mathbf{J}/e$ satisfy both the Onsager principle and the Einstein relations. (Violation of the Einstein relations in the fluxes $\mathbf{j}$ and $\mathbf{q}$ is connected with the presence of the diamagnetic currents[6].) The coefficients $\tilde{\alpha}_{yx}$ and $\tilde{\beta}_{yx}$ coincide with the expressions obtained in the paper of Obraztsov[7].

5. KINETIC COEFFICIENTS FOR A NONDEGENERATE ELECTRON GAS

Let us consider first of all the case when the electron gas is nondegenerate (semiconductors at not too low a temperature, $T \gg \zeta$). Substituting in (33) the Boltzmann distribution function for f_K^0, and neglecting f_K^0 compared with unity, we obtain the following expressions for the off-diagonal components of the tensors α, β, γ, and σ:

$$\sigma_{12} = \frac{en_e}{H}, \quad \sigma_{12}' = \frac{e^2 n_e}{2mT}\operatorname{cth}\alpha,$$

$$\alpha_{12} = T^{-1}\beta_{12}' = \frac{en_e}{2mT}\left\{\frac{1}{2}\operatorname{cth}\alpha + \alpha\frac{1+\operatorname{ch}^2\alpha}{\operatorname{sh}^2\alpha}\right\},$$

$$\beta_{12} = \frac{en_e}{m}\left\{\operatorname{cth}\alpha + \frac{1}{4\alpha}\right\},$$

$$\gamma_{12} = \frac{n_e}{2m}\left\{\frac{3}{4}\operatorname{cth}\alpha + \alpha\frac{1+\operatorname{ch}^2\alpha}{\operatorname{sh}^2\alpha} + \alpha^2\frac{5+\operatorname{ch}^2\alpha}{\operatorname{sh}^2\alpha}\operatorname{cth}\alpha\right\},$$

$$\tilde{\alpha}_{12} = \frac{en_e}{2mT}\left\{\operatorname{cth}\alpha + \frac{3}{2\alpha}\right\},$$

$$\tilde{\gamma}_{12} = \frac{n_e T}{eH}\left\{\frac{15}{4} + 3\alpha\operatorname{cth}\alpha + 2\alpha^2\frac{1+\operatorname{ch}^2\alpha}{\operatorname{sh}^2\alpha}\right\}, \qquad (40)^*$$

where $\alpha = \omega_H/2T$.

In calculating the diagonal components of the tensors σ, α, β, and γ the problem reduces to obtaining an integral in the form

$$J_l = \frac{\pi}{V}\int_{\omega_H/2}^{\infty} E^l G(E)\frac{\partial f}{\partial E}dE, \qquad (41)$$

since the chemical potential ζ is much smaller than the average energy of the electrons T. The diagonal components of the tensors σ, α, β, and γ are expressed in terms of J_l by the formulas

$$\sigma_{11} = e^2 J_0, \quad \alpha_{11} = -eJ_1/T,$$
$$\beta_{11} = -eJ_1, \quad \gamma_{11} = J_2/T. \qquad (42)$$

Let us calculate the value of J_l in the quantum case, when $\omega_H \gg T$. In this case in the quantities $K'(E)$ and $I(E)$, which determine the function $G(E)$, we can confine ourselves to only one term with $n = 0$:

$$K'(E) = \left(\frac{eH}{2}\right)^{1/2}\left(\frac{E}{\omega_H} - \frac{1}{2}\right)^{-1/2},$$

$$I(E) = \frac{1}{2}\left(\frac{E}{\omega_H} - \frac{1}{2}\right)^{-1/2}. \qquad (43)$$

Substituting in (41) the function $G(E)$ and using (43), we get

$$J_l = \left(\frac{2\pi}{mT}\right)^{1/2}\frac{a^2 n_i n_e T^{l-1}}{m}\int_0^{\infty}\frac{(x+\omega_H/2T)^l e^{-x}}{x + a^2 eH\omega_H/2T}dx.$$

Assuming that $a^2 eH \ll 1$, we get

$$J_l = \left(\frac{2\pi}{mT}\right)^{1/2}\frac{a^2 n_i n_e}{mT}\left(\frac{\omega_H}{2}\right)^l \operatorname{Ei}(u), \qquad (44)$$

where

$$\operatorname{Ei}(u) = \int_0^{\infty}\frac{e^{-x}dx}{x+u}, \qquad u = \frac{ma^2\omega_H^2}{2T}.$$

Formulas (42) and (44) determine the diagonal components of the kinetic coefficients.

6. KINETIC COEFFICIENTS FOR DEGENERATE ELECTRON GAS

In the case of a degenerate electron gas ($T \ll \zeta$), it is necessary to use in the calculation of the kinetic coefficients a Fermi distribution function

$$f^0 = [\exp\{(\varepsilon - \zeta)/T\} + 1]^{-1}.$$

The calculation of the off-diagonal components of the kinetic coefficients is made in standard fashion and leads to the following results:

$$\sigma_{12} = \frac{e}{H}\frac{(2m\zeta)^{3/2}}{6\pi^2} - \frac{e}{H}\frac{(m\omega_H)^{3/2}}{4\pi^3}\sum_{r=1}(-1)^r\frac{1}{r^{3/2}}\Psi(\alpha_r) \times\cos\left(\frac{2\pi r\zeta}{\omega_H} + \frac{\pi}{4}\right),$$

$$\sigma_{12}' = \frac{e}{H}\frac{(2m\zeta)^{3/2}}{6\pi^2} + \frac{e^2\zeta}{2\pi^2}\left(\frac{m}{\omega_H}\right)^{1/2}\sum_{r=1}^{\infty}(-1)^r\frac{1}{r^{3/2}}\Psi(\alpha_r) \times\cos\left(\frac{2\pi r\zeta}{\omega_H} - \frac{\pi}{4}\right),$$

$$\alpha_{12} = T^{-1}\beta_{12}' = \frac{eT}{6\omega_H}(2m\zeta)^{1/2} - \frac{e\zeta}{2\pi}\left(\frac{m}{\omega_H}\right)^{1/2} \times\sum_{r=1}(-1)^r\frac{1}{r^{1/2}}\Psi'(\alpha_r)\cos\left(\frac{2\pi r\zeta}{\omega_H} - \frac{\pi}{4}\right),$$

$$\beta_{12} = \frac{eT^2}{6\omega_H}(2m\zeta)^{1/2} - \frac{\zeta}{H}\frac{(m\omega_H)^{3/2}}{4\pi^3}\sum_{r=1}^{\infty}(-1)^r\frac{1}{r^{3/2}}\Psi(\alpha_r) \times\cos\left(\frac{2\pi r\zeta}{\omega_H} + \frac{\pi}{4}\right),$$

*cth = coth, sh = sinh, ch = cosh.

$$\gamma_{12}=\frac{T\zeta(2m\zeta)^{1/2}}{9\omega_H}-\frac{\zeta T}{2}\left(\frac{m}{\omega_H}\right)^{1/2}\sum_{r=1}^{\infty}(-1)^r\frac{1}{r^{1/2}}\Psi''(\alpha_r)$$
$$\times\cos\left(\frac{2\pi r\zeta}{\omega_H}-\frac{\pi}{4}\right),$$

$$\tilde{\alpha}_{12}=\frac{eT}{6\omega_H}(2m\zeta)^{1/2}-\frac{e\sqrt{m\omega_H}}{(2\pi)^2}\sum_{r=1}^{\infty}\frac{(-1)^r}{r^{3/2}}\Psi'(\alpha_r)$$
$$\times\cos\left(\frac{2\pi r\zeta}{\omega_H}-\frac{\pi}{4}\right),$$

$$\tilde{\gamma}_{12}=\frac{T\zeta(2m\zeta)^{1/2}}{9\omega_H}+T\frac{\sqrt{m\omega_H}}{4\pi}\sum_{r=1}^{\infty}\frac{(-1)^r}{r^{3/2}}\Psi''(\alpha_r)$$
$$\times\cos\left(\frac{2\pi r\zeta}{\omega_H}+\frac{\pi}{4}\right), \tag{45}$$

where $\Psi(x) = x/\sinh x$ and $\alpha_r = 2\pi^2 rT/\omega_H$. We have assumed here that the condition $\omega_H \ll \zeta$ is satisfied.

To calculate the diagonal components of the kinetic coefficients it is necessary to know $K'(E)$ and $I(A)$. For $\omega_H \ll \zeta$, an important role is played in formulas (30) and (31) by a large number of terms, so that to calculate $K'(E)$ and $I(E)$ it is convenient to use the Poisson summation formula:

$$\sum_{n=0}^{\infty}\varphi(n)=\frac{1}{2}\varphi(0)+\sum_{r=-\infty}^{\infty}\int_0^{\infty}dx\varphi(x)e^{2\pi irx}.$$

Using this formula, it is easy to obtain the following expressions for $K'(E)$ and $I(E)$:

$$K'(E)=(2m\omega_H)^{1/2}N^{1/2}\left\{1+N^{-1/2}\sum_{r=1}^{\infty}\frac{1}{\sqrt{2r}}\cos\left(2\pi r\eta-\frac{\pi}{4}\right)\right\},$$

$$I(E)=\frac{4}{3}N^{3/2}\left\{1+\frac{3}{2}N^{-1/2}\sum_{r=1}^{\infty}\frac{1}{\sqrt{2r}}\cos\left(2\pi r\eta-\frac{\pi}{4}\right)\right\},$$
$$N\gg1, \tag{46}$$

where N and η are determined by the formula $E = \omega_H(N + \tfrac{1}{2}) - \omega_H\eta$ (N—positive integer and $0 \le \eta < 1$), and

$$K''(E) = (eH/2\eta)^{1/2}.$$

For small values of η, the series in (46) behave like $\eta^{-1/2}$. Therefore, when calculating the diagonal components of the kinetic coefficient (if we use the expansion in the scattering amplitude a), a logarithmic divergence occurs when we multiply in formula (31) the series for $K'(E)$ by the series for $I(E)$, although this product does contain an additional small parameter $N^{-1/2}$. This means that in the corresponding terms for $G(E)$ it is necessary to take into account the difference between the denominator in formula (31) and unity, in which the function $K'(E)$ can be replaced by its asymptotic value for small η: $K'(E) = (m\omega_H/2\eta)^{1/2}$. Thus, using (46), we get

$$\sigma_{11}=\frac{8}{3\pi}n_ie^2a^2\left(\frac{\zeta}{\omega_H}\right)^2\left\{1+\frac{5}{2}\left(\frac{\omega_H}{\zeta}\right)^{1/2}\sum_{r=1}\frac{(-1)^r}{\sqrt{2r}}\Psi(\alpha_r)\right.$$
$$\left.\times\cos\left(\frac{2\pi r\zeta}{\omega_H}-\frac{\pi}{4}\right)+\Delta_\sigma\right\},$$

$$\alpha_{11}=T^{-1}\beta_{11}=\frac{16\pi}{9}n_iea^2\frac{T\zeta}{\omega_H^2}\left\{1-\frac{5}{2}\frac{\zeta}{T}\left(\frac{\omega_H}{T}\right)^{1/2}\right.$$
$$\left.\times\sum_{r=1}^{\infty}\frac{(-1)^r}{\sqrt{2r}}\Psi'(\alpha_r)\cos\left(\frac{2\pi r\zeta}{\omega_H}+\frac{\pi}{4}\right)+\Delta_\alpha\right\},$$

$$\gamma_{11}=\frac{8\pi}{9}n_ia^2\left(\frac{\zeta}{\omega_H}\right)^2T\left\{1-\frac{5}{2}\left(\frac{\omega_H}{\zeta}\right)^{1/2}\sum_{r=1}^{\infty}\frac{(-1)^r}{\sqrt{2r}}\Psi''(\alpha_r)\right.$$
$$\left.\times\cos\left(\frac{2\pi r\zeta}{\omega_H}-\frac{\pi}{4}\right)+\Delta_\gamma\right\},$$

$$\Delta_\sigma=-\frac{3}{4}\frac{\omega_H}{\zeta}\int_0^{\infty}\frac{\partial f(E)}{\partial E}\frac{dE}{(\sqrt{\eta}+a\sqrt{eH/2})^2+eHa^2/2},$$

$$\Delta_\alpha=\frac{9}{8\pi^2}\frac{\omega_H\zeta}{T^2}\int_0^{\infty}\frac{\partial f(E)}{\partial E}\frac{dE}{(\sqrt{\eta}+a\sqrt{eH/2})^2+eHa^2/2}\frac{E}{\zeta}$$
$$\times\left(1-\frac{E}{\zeta}\right),$$

$$\Delta_\gamma=-\frac{9}{4\pi^2}\frac{\omega_H\zeta}{T^2}\int_0^{\infty}\frac{\partial f(E)}{\partial E}\left(1-\frac{E}{\zeta}\right)^2$$
$$\times\frac{dE}{(\sqrt{\eta}+a\sqrt{eH/2})^2+eHa^2/2}. \tag{47}$$

To calculate the integrals contained in (47), we expand the function

$$\{(\sqrt{\eta}+a\sqrt{eH/2})^2+eHa^2/2\}^{-1}$$

which is periodic in E with period ω_H, in a Fourier series in the variable E:

$$\left\{\left(\sqrt{\eta}+a\sqrt{\frac{1}{2}eH}\right)^2+\frac{1}{2}eHa^2\right\}^{-1}$$
$$=\sum_{r=-\infty}^{\infty}(-1)^r\exp\left(2\pi ir\frac{E}{\omega_H}\right)F_r(z);$$

$$F_r(z)=\int_0^1\frac{e^{-2\pi ir\eta}}{(z+\sqrt{\eta})^2+z^2}d\eta,\qquad z=a\sqrt{eH/2}.$$

Substituting this expansion in (47) and integrating with respect to E, we get

$$\Delta_\sigma = \frac{3}{4}\frac{\omega_H}{\zeta}F_0(z) + \frac{3}{4}\frac{\omega_H}{\zeta}\mathrm{Re}\sum_{r=1}^{\infty}(-1)^r\Psi(\alpha_r)F_r(z) \times \exp\left(2\pi i r\frac{\zeta}{\omega_H}\right);$$

$$\Delta_\alpha = \frac{3}{8}\frac{\omega_H}{\zeta}F_0(z) - \frac{9}{8\pi^2}\frac{\omega_H}{T}\mathrm{Re}\sum_{r=1}^{\infty}(-1)^r i\Psi'(\alpha_r)F_r(z) \times \exp\left(2\pi i r\frac{\zeta}{\omega_H}\right),$$

$$\Delta_\gamma = -\frac{3}{4}\frac{\omega_H}{\zeta}F_0(z) - \frac{9}{4\pi^2}\frac{\omega_H}{\zeta}\mathrm{Re}\sum_{r=1}^{\infty}(-1)^r\Psi''(\alpha_r)F_r(z) \times \exp\left(2\pi i r\frac{\zeta}{\omega_H}\right). \quad (48)$$

The asymptotic form of $F_r(z)$ with $z \ll 1$ is $F_r(z) \approx -2\ln z^{-2}$.

If $T \ll \omega_H$ and $\zeta = \omega_H(N + 1/2) + \delta\omega_H$ ($0 \le \delta < 1$) is sufficiently close to $\omega_H(N + 1/2)$, then formulas (47) become meaningless, since the series in r begin to diverge. Physically, this signifies that the amplitude of the quantum oscillations, for sufficiently low temperatures

$$T \ll |\zeta - (N + 1/2)\omega_H| \ll \omega_H$$

becomes very large. The kinetic coefficients in this case are of the form

$$\sigma_{11} = -\frac{\pi e^2}{V}G(\zeta), \quad \alpha_{11} = -\frac{\pi^3 e}{3V}G'(\zeta),$$

$$\gamma_{11} = -\frac{\pi^3}{3V}TG(\zeta),$$

$$G(\zeta) = -\frac{V}{\pi^2}\omega_H n_i a^2\left(\sum_{n=0}^{N}\frac{2n+1}{(\zeta-\varepsilon_\perp)^{1/2}}\right)\left(\sum_{n'=0}^{N}\frac{1}{(\zeta-\varepsilon_\perp)^{1/2}}\right) \times\left\{\left[1 + a\left(\frac{eH}{2\varepsilon}\right)^{1/2}\right]^2 + \frac{a^2}{2m}(eH)^2\left(\sum_{n=0}^{N}\frac{1}{(\zeta-\varepsilon_\perp)^{1/2}}\right)^2\right\}^{-1},$$

$$\varepsilon_\perp = \omega_H\left(n + \frac{1}{2}\right). \quad (49)$$

Confining ourselves to only the last term n = N in the series (30) and (31) when $|\zeta - \omega_H(N + 1/2)| \ll \omega_H$ we get

$$G(\zeta) \approx -\frac{2}{\pi^2}Vn_i\frac{\zeta}{m\omega_H^2}.$$

We have assumed here that $\delta < \omega_H(p_0 a)^2/\zeta$. It is clear therefore that the relative amplitude of the oscillations when $T \ll \omega_H$ and $\delta \lesssim \omega_H/\zeta$ can reach a value on the order of unity.

The authors are grateful to A. I. Akhiezer, L. É. Gurevich, and A. L. Efros for a discussion of the work.

[1] V. Skobov, JETP **38**, 1304 (1960), Soviet Phys. JETP **11**, 941 (1960).

[2] A. Ansel'm and B. Askerov, FTT **3**, 3668 (1961), Soviet Phys. Solid State **3**, 2665 (1962).

[3] S. Titeica, Ann. Physik **22**, 128 (1953).

[4] Kubo, Hasegawa, and Hashitsume, J. Phys. Soc. Japan **14**, 56 (1959).

[5] V. Skobov, JETP **37**, 1467 (1959), Soviet Phys. JETP **10**, 1039 (1960).

[6] V. Silin and P. Zyryanov, JETP **46**, 537 (1964), Soviet Phys. JETP **19**, 366 (1964).

[7] Yu. Obraztsov, FTT **6**, 414 (1964), Soviet Phys. Solid State **6**, 331 (1964).

Translated by J. G. Adashko
30