ENERGY LOSS OF SLOW PROTONS IN METALS

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Formulas describing the energy loss of slow protons in a degenerate plasma are derived. Two plasma models are considered: that of an ideal degenerate gas and that of a Fermi liquid. The results are in satisfactory agreement with the experimental data.

1. A number of workers (see [1]) have studied the energy loss of slow ions (~ 10 keV) of light elements in thin metallic films. The experimental results are satisfactorily described by the linear relation:

$$-dE/dx = Av. \qquad (1)$$

As we will show below, this dependence can be obtained theoretically by assuming that the velocity of the incident particle is much less than the Fermi velocity of the electrons in the metal, $v \ll v_0$.

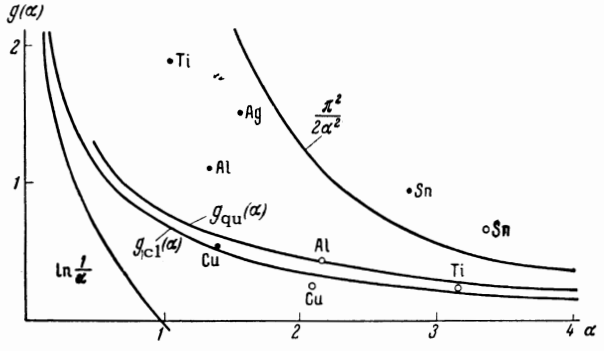
Slowing down of the particles can be produced by polarization losses in the electron plasma and also by collision with lattice atoms. However, it has been observed [1] that the coefficient A in formula (1) is the same for protons and deuterons, i.e., it does not depend on the mass of the moving particle. This fact indicates that collisions with lattice atoms apparently do not play an important role, and we can assume that the slowing down is produced only by losses to electrons.

The energy loss of a slow charged particle ($v \ll v_0$) in a degenerate electron gas was first discussed by Fermi and Teller [2] who obtained the formula

$$-\frac{dE}{dx} = v\frac{2e^4m^2}{3\pi\hbar^3}\ln\frac{\hbar v_0}{e^2}, \qquad (2)$$

where m is the electron mass and v_0 is the velocity at the Fermi surface. This formula was obtained in the Born approximation ($e^2/\hbar v_0 \ll 1$) and is inapplicable to real metals, where $e^2/\hbar v_0$ turns out to be greater than unity.

In the present paper we have attempted to refine formula (2) and have shown that a systematic treatment of the polarization of the medium in terms of a gas model leads to expressions which are in satisfactory agreement with experimental results [1] (see the figure). Since the applicability of the gas model itself becomes doubtful for $e^2/\hbar v_0 \gtrsim 1$, we have also attempted in this work to discuss the electrons in metals as a Fermi liquid. It turns out that this treatment leads qualitatively to the same expression for dE/dx as the gas model.



2. The results obtained below for $v \ll v_0$ can be written in the form

$$-\frac{dE}{dx} = v\frac{2}{3\pi}\frac{e^4m^2}{\hbar^3}g(\alpha), \qquad \alpha = \frac{e^2}{\hbar v_0}. \qquad (3)$$

For $\alpha \ll 1$ we have $g(\alpha) \cong \ln \alpha^{-1}$ and Eq. (3) becomes the Fermi-Teller formula, Eq. (2). In the opposite extreme case the function $g(\alpha)$ turns out to be

$$g(\alpha) \cong \pi^2/2\alpha^2, \qquad \alpha \gg 1, \qquad (4)$$

in which case

$$-\frac{dE}{dx} = v\frac{\pi}{3}\frac{p_0^2}{\hbar}, \qquad p_0 = mv_0. \qquad (5)$$

Expressing the Fermi limiting momentum p_0 in terms of the density, we can write formula (5) in the form

$$-dE/dx = v(\pi^7/3)^{1/3}\hbar n^{2/3} = 10\, v\hbar n^{2/3}. \qquad (6)$$

This simple formula is in satisfactory agreement with experiments (see the figure) and it is appropriate to give its qualitative derivation here.

If we start by assuming binary collisions, the rate of energy loss of an ion to electrons is given by the relation

$$-\frac{dE}{dx} = m\left|\int d\mathbf{p}'f(\mathbf{p}')\left|\mathbf{v}-\frac{\mathbf{p}'}{m}\right|\left(\mathbf{v}-\frac{\mathbf{p}'}{m}\right)\sigma_{tr}\right|, \qquad (7)$$

$$\sigma_{tr} = \int (1 - \cos\theta) \frac{d\sigma}{d\Omega} d\Omega, \tag{8}$$

where $f(p')$ is the electron distribution function $\int f(p')\,dp' = n$. Formula (2) is obtained from (7) if we take σ_{tr} to be the Coulomb cross section

$$\sigma = 4\pi \frac{e^4}{m^2} \frac{1}{|v - p'/m|^4} \ln \frac{1}{\theta_{min}}. \tag{9}$$

Here $\ln \theta_{min}^{-1}$ is the "Coulomb logarithm."

The Debye-Hückel radius for a degenerate plasma is equal to

$$d = \left(4\pi e^2 \frac{\partial n}{\partial \mu} \right)^{-1/2} = \left(\frac{mv_0^2/3}{4\pi n e^2} \right)^{1/2} \tag{10}$$

($\mu = mv_0^2/2$ is the chemical potential), and therefore Fermi and Teller assume

$$\theta_{min} = \rho_{min} / \rho_{max} = \lambda / d \sim \sqrt{e^2 / \hbar v_0} \ll 1, \tag{11}$$

which also leads to formula (2) if $v \ll v_0$. From this it is evident that the second extreme case $e^2/\hbar v_0 \gg 1$ corresponds formally to small values of d. Here the screened Coulomb potential for the interaction of electrons with an ion will have the form of a δ-function:

$$U_{eff} = \lim_{d \to 0} \frac{-e^2}{r} e^{-r/d} = -4\pi e^2 d^2 \delta(\mathbf{r}). \tag{12}$$

For this potential the elastic scattering cross section in the Born approximation, in contrast to Eq. (9), turns out to be

$$\frac{d\sigma}{d\Omega} = \frac{m^2}{4\pi^2\hbar^4} \left| \int U_{eff}(\mathbf{r})\, e^{-i\mathbf{q}\mathbf{r}} d\mathbf{r} \right|^2 = \frac{4m^2 e^4 d^4}{\hbar^4} = \frac{\pi^2 \lambda_0^2}{4},$$

$$\lambda_0 = \frac{\hbar}{p_0}. \tag{13}$$

For the potential (12) the Born approximation turns out to be applicable in spite of the condition $e^2/\hbar v_0 \gg 1$.

As a matter of fact, potential (12) can be considered as a spherically symmetric well of the form

$$U_{eff} = \begin{cases} -U_0 & r < d \\ 0 & r > d \end{cases} \qquad U_0 = 3\frac{e^2}{d}. \tag{14}$$

The scattering cross section of slow particles ($\lambda \gg d$) in such a potential is

$$d\sigma / d\Omega = |(\tan \varkappa d - \varkappa d) / \varkappa|^2, \qquad \varkappa = \sqrt{2mU_0/\hbar} \tag{15}$$

and for $d \to 0$ ($d = \sqrt{6me^2 d/\hbar} \ll 1$) we find

$$d\sigma / d\Omega = (\varkappa^3 d^3 / 3\varkappa)^2 = d^2 (\varkappa d)^4 / 9, \tag{16}$$

which agrees with the Born formula (13).

Since $d\sigma/d\Omega$ does not depend on angle, the transport cross section (8) will be equal to the total cross section:

$$\sigma_{tr} = 4\pi \frac{d\sigma}{d\Omega} = \frac{(p_0/\hbar)^4}{9\pi n^2} = \pi^3 \lambda_0^2. \tag{17}$$

Taking σ_{tr} out from under the integral sign in formula (7), for $v \ll v_0$ and for a Fermi distribution we obtain formula (5):

$$-\frac{dE}{dx} = m\sigma_{tr} \left| \int d\mathbf{p}' f(\mathbf{p}') \left| \mathbf{v} - \frac{\mathbf{p}'}{m} \right| \left(\mathbf{v} - \frac{\mathbf{p}'}{m} \right) \right|$$

$$= \frac{4}{3} \sigma_{tr} v \int d\mathbf{p}' f(\mathbf{p}') |\mathbf{p}'| = v \frac{\pi}{3} \frac{p_0^2}{\hbar}. \tag{18}$$

It must be noted that for $e^2/\hbar v_0 \gg 1$ the potential energy due to interaction of the electrons exceeds their kinetic energy ($e^2 n^{1/3} \gg mv_0^2$) and it is necessary to take into account particle correlations. We will show later, however, that taking into account correlations in terms of the Fermi liquid model does not change the result obtained [formula (18)].

3. If we consider the plasma as a continuous medium, the energy loss of a slow particle can be calculated from the formula (see [3])

$$-\frac{dE}{dx} = \frac{ie^2}{2\pi^2 v} \int \frac{(\mathbf{kv})\, d\mathbf{k}}{k^2 \varepsilon(k, \mathbf{kv})}$$

$$= \frac{2e^2}{\pi} \int_0^1 y\,dy \int_0^\infty \frac{k\,dk}{\varepsilon^2(k,0)} \operatorname{Im} \varepsilon(k, \mathbf{kv}). \tag{19}$$

Here $y = \cos(\mathbf{k}, \mathbf{v})$, and $\epsilon(k, \omega)$ is the longitudinal dielectric constant. It is assumed that the real part of ϵ is an even function of ω and the imaginary part is an odd function. Furthermore, since the particle is assumed to be quite slow, ($v \to 0$), in the denominator of ϵ we can place $\omega = \mathbf{k} \cdot \mathbf{v} = 0$.

The electrons in a metal can be approximately considered as a degenerate electron gas with a temperature equal to zero. If we use the classical kinetic equation, we obtain for $\epsilon(k, \omega)$ the well known expression

$$\varepsilon(k, \omega) = 1 + \frac{4\pi e^2}{k^2} \int \frac{d\mathbf{p}}{\omega - \mathbf{kp}/m} \left(\mathbf{k} \frac{\partial f}{\partial \mathbf{p}} \right). \tag{20}$$

Substituting in this the Fermi distribution, we find:

$$\varepsilon(\omega, k) = 1 + \frac{4\alpha}{\pi k^2 \lambda_0^2} \eta\left(\frac{\omega}{k v_0} \right);$$

$$\eta(s) = 1 + \frac{s}{2} \ln \left| \frac{1-s}{1+s} \right| + i\pi \frac{s}{2}, \qquad s = \frac{\omega}{k v_0} \tag{21}$$

where $\alpha = e^2/\hbar v_0$ and $s = \omega/kv_0$, and from this,

$$\varepsilon(k, 0) = 1 + \frac{4\alpha}{\pi k^2 \lambda_0^2}, \quad \operatorname{Im} \varepsilon(k, \mathbf{kv}) = \frac{2\alpha}{k^2 \lambda_0^2} \frac{v}{v_0} y, \tag{22}$$

where $y = \cos(\mathbf{k}, \mathbf{v})$ and v is the velocity of the particle whose energy loss we wish to determine.

Substituting (22) into (19), we obtain

$$-\frac{dE}{dx}=v\frac{2m^2e^4}{3\pi\hbar^3}g_{cl}(\alpha),$$

$$\text{where } g_{cl}(\alpha)=\int_0^{k_{max}}\frac{2dk}{k(1+4\alpha/\pi k^2\lambda_0^2)^2}. \qquad (23)$$

Here, in accordance with (19), it would be necessary to assume that $k_{max}=\infty$, but in this case the integral turns out to be divergent. The correct cutoff of this integral can be obtained only on the basis of a systematic quantum-mechanical treatment.

For this purpose it is necessary to use the quantum-mechanical kinetic equation, from which, in contrast to the classical formula (20), we obtain for $\epsilon(k,\omega)$ the well known expression (see for example [3])

$$\varepsilon(k,\omega)=1+\frac{4\pi e^2}{\hbar k^2}\int\frac{d\mathbf{p}}{\omega-\mathbf{kp}/m}\times\left[f\left(\mathbf{p}+\frac{\hbar\mathbf{k}}{2}\right)-f\left(\mathbf{p}-\frac{\hbar\mathbf{k}}{2}\right)\right], \qquad (24)$$

which becomes identical to Eq. (20) only in the limit $\hbar k \ll p_0$ (or $k\to 0$).

The imaginary part $\mathrm{Im}\,\epsilon(k,\omega)$ arises from a half-circuit of the pole $\omega-\mathbf{k}\cdot\mathbf{p}/m=0$, which in the limiting case $\omega\to 0$ being discussed lies at $p_{\parallel}=m\omega/k\to 0$. For this point to fall on one of the Fermi spheres $f(p\pm\hbar k/2)$, it is necessary that

$$\hbar k/2\leqslant p_0 \quad\text{or}\quad k\leqslant k_{max}=2/\lambda_0. \qquad (25)$$

Obviously this upper limit must also be substituted in (23), which leads to the expression

$$g_{cl}(\alpha)=2\int_0^1\frac{d\xi}{\xi(1+\alpha/\pi\xi^2)^2}=\ln\frac{\alpha+\pi}{\alpha}+\frac{\alpha}{\alpha+\pi}-1,$$

$$\alpha=\frac{k\lambda_0}{2}. \qquad (26)$$

For $\alpha\ll 1$ we have

$$g_{cl}(\alpha)=\ln\frac{\pi/e}{\alpha}+O(\alpha), \qquad (27)$$

which corresponds to the Fermi-Teller formula, Eq. (2), and for $\alpha\gg 1$

$$g_{cl}(\alpha)=\pi^2/2\alpha^2+O(1/\alpha^3), \qquad (28)$$

which agrees with (4). A plot of the function $g_{cl}(\alpha)$ (2) is given in the figure.

4. The results obtained above can be called semiclassical, since on the one hand they use the quantum-mechanical Fermi distribution and on the other hand they use for ϵ the classical formula (20).

These results can be somewhat refined if we obtain $\epsilon(k,\omega)$ from the quantum-mechanical formula (24). Substituting in (24) a Fermi distribution, we find (see also [3]):

$$\varepsilon(k,\omega)=1+\frac{2\alpha}{\pi k^2\lambda_0^2}\left\{1+\frac{1}{2k\lambda_0}\left[\left(1-\left(\frac{\omega+\hbar k^2/2m}{kv_0}\right)^2\right)\times\ln\frac{\omega+\hbar k^2/2m+kv_0}{\omega+\hbar k^2/2m-kv_0}-\left(1-\left(\frac{\omega-\hbar k^2/2m}{kv_0}\right)^2\right)\times\ln\frac{\omega-\hbar k^2/2m+kv_0}{\omega-\hbar k^2/2m-kv_0}\right]\right\}. \qquad (29)$$

From this expression we can obtain

$$\varepsilon(k,0)=1+\alpha\varphi(\xi),$$

$$\varphi(\xi)=\frac{1}{4\pi\xi^3}\left[2\xi+(1-\xi^2)\ln\frac{1+\xi}{1-\xi}\right]. \qquad (30)$$

Here $\alpha=e^2/\hbar v_0$ and $\xi=k\lambda_0/2$. For the imaginary part of (29) we have, when $\omega\to 0$:

$$\mathrm{Im}\,\varepsilon(k,\omega)=\begin{cases}-\alpha\omega/2\xi^2kv_0 & 0<\xi<1\\ 0 & \xi>1\end{cases}. \qquad (31)$$

Substituting these expressions into (19), we find for the rate of energy loss

$$-\frac{dE}{dx}=v\frac{2}{3\pi}\frac{e^4m^2}{\hbar^3}g_{qu}(\alpha),$$

$$g_{qu}(\alpha)=2\int_0^1\frac{d\xi}{\xi[1+\alpha\varphi(\xi)]^2}. \qquad (32)$$

A similar problem has been discussed by Smirnov [4]. However, in view of the inaccuracies in Smirnov's computation of the polarization operator, the loss formula given in his paper, which is claimed to be an improvement over the Fermi-Teller formula, is incorrect.

The function $g_{qu}(\alpha)$, obtained by numerical integration, is shown in the figure. For $\alpha\ll 1$ the main contributions comes from small values of ξ ($\xi\ll 1$), for which we have from (30) $\varphi(\xi)=1/\pi\xi^2$. In this case $\epsilon=1+\alpha\varphi=1+\alpha/\pi\xi^2$, and we again arrive at the semiclassical formula (26). In the other extreme case $\alpha\gg 1$, neglecting unity in $\epsilon(k,0)$ we obtain by numerical integration:

$$g_{qu}(\alpha)=C/\alpha^2,\quad C=2\int_0^1\frac{d\xi}{\xi\varphi^2(\xi)}=1.06\pi^2, \qquad (33)$$

which is somewhat different from the limiting form of the function $g_{cl}(\alpha)$ [see Eq. (28)].

It is useful to note that, from formula (30) for $\epsilon(k,0)$, it follows that the effective potential for interaction of an electron with an ion is equal to

$$U_{int}^{qu}(\mathbf{r}) = -e^2 \int \frac{e^{i\mathbf{kr}}\, d\mathbf{k}}{2\pi^2 k^2 \varepsilon(k, 0)}$$

$$= -\frac{e^2}{2\pi^2} \int \frac{e^{i\mathbf{kr}}\, d\mathbf{k}}{k^2 [1 + \alpha\varphi(k\lambda_0/2)]}. \quad (34)$$

If we write the potential in the form

$$U(r) = -e^2 r^{-1} e^{-r/d} + v_{qu}(r), \quad (35)$$

where d is the Debye radius (10), the quantum-mechanical correction to the screened Coulomb potential will be

$$v_{qu}(r) = \frac{e^2}{2\pi^2} \int \frac{d\mathbf{k}}{k^2} e^{i\mathbf{kr}} \left\{ \left[1 + \frac{\alpha}{\pi} \left(\frac{2}{k\lambda_0} \right)^2 \right]^{-1} - \left[1 + \alpha\varphi\left(\frac{k\lambda_0}{2} \right) \right]^{-1} \right\}. \quad (36)$$

For the case $\alpha \gg 1$ expression (34) does not reduce to a δ-function [cf. (12)] and the scattering cross section turns out to be angle-dependent:

$$\left(\frac{d\sigma}{d\Omega} \right)_{Born} = 16\pi^4 \frac{m^2}{\hbar^4} |U_q|^2 = \frac{4m^2 e^4}{\hbar^4 q^4 [1 + \alpha\varphi(q\lambda_0/2)]^2}, \quad (37)$$

where $q = 2k \sin(\theta/2)$ and $k \cong p_0/\hbar$ (since $v \ll v_0$)

For $\alpha \gg 1$ the transport cross section, in contrast to (17), is equal to

$$\sigma_{tr} = \int (1 - \cos\theta) \frac{d\sigma}{d\Omega} d\Omega = 4\pi\lambda_0^2 \int_0^1 \frac{d\xi}{\xi\varphi^2(\xi)}, \quad (38)$$

which, in accordance with (18), also reduces to formula (33).

For real metals $\alpha = e^2/\hbar v_0 \sim 1$ and therefore it is necessary to take into account particle correlations. Let us attempt to do this in terms of the Fermi liquid theory developed by Landau.[5] In this theory the liquid is described by the ordinary non-quantum-mechanical kinetic equation

$$\frac{\partial f}{\partial t} + \frac{\partial \varepsilon}{\partial \mathbf{p}} \frac{\partial f}{\partial \mathbf{r}} - \frac{\partial \varepsilon}{\partial \mathbf{r}} \frac{\partial f}{\partial \mathbf{p}} = 0. \quad (39)$$

However, the particle energy $\epsilon(\mathbf{p}, \mathbf{r})$ depends functionally on the distribution f:

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}_0 + \delta\varepsilon, \quad \delta\varepsilon = \int \Phi(\mathbf{p}, \mathbf{p}') \delta f(\mathbf{p}') d\mathbf{p}' \quad (40)$$

(δf is the correction to the equilibrium distribution function).

The function Φ takes into account particle correlations. Equation (39) assumes that $\hbar k \ll p_0$, and this procedure can be considered as a generalization of the classical (non-quantum-mechanical) gas model discussed above in Sec. 3. Here $\Phi(\mathbf{p}, \mathbf{p}')$ depends only on the angle between the vectors p and p', and can be represented in the form

$$\Phi(\mathbf{p}; \mathbf{p}') = \sum_n \Phi_n P_n[\cos(\mathbf{p}, \mathbf{p}')], \quad (41)$$

where P_n are the Legendre polynomials. In practice Φ must be determined from one or two terms of the expansion, and for simplicity we assume that

$$\Phi = \Phi_0 + \Phi_1 \cos(\mathbf{p}, \mathbf{p}'), \quad (42)$$

assuming the remaining coefficients are equal to zero.

Under these assumptions the dielectric constant of a Fermi liquid turns out to be (see [6])

$$\varepsilon(k, \omega) = 1 + \frac{4\alpha}{\pi k^2 \lambda_0^2} \frac{\eta(s)}{(1 + A_1/3)(1 + A_0\eta(s)) + A_1 s^2 \eta(s)}, \quad (43)$$

where $s = \omega/kv_0$, and the function $\eta(s)$ is given by

$$\eta(s) = 1 + \frac{s}{2} \ln \left| \frac{1 - s}{1 + s} \right| + i\pi \frac{s}{2} \quad (44)$$

[cf. (43), (44) with (21)]. In formula (43) we have used the designation

$$A_{0,1} = \Phi_{0,1} p_0^2 / \pi^2 \hbar^3 v_0. \quad (45)$$

It turns out that $p_0 = mv_0(1 + A_1/3)$, so that an effective mass $m^* = m(1 + A_1/3)$ can be assigned to the electrons. For $A_0 = A_1 = 0$, formula (43) becomes the semiclassical gas formula (21) for $\epsilon(k, \omega)$. From (43) we find

$$\varepsilon(k, \omega) = 1 + \frac{4\alpha^*}{\pi k^2 \lambda_0^2}, \quad \operatorname{Im} \varepsilon(k, \omega) = \frac{2\alpha^*}{k^2 \lambda_0^2} \frac{\omega}{kv_0} \frac{1}{1 + A_0},$$

$$\alpha^* = \frac{\alpha}{(1 + A_0)(1 + A_1/3)}. \quad (46)$$

Substituting these expressions into (19), we find for the rate of energy loss

$$-\frac{dE}{dx} = v \frac{2e^2\alpha^*}{3\pi\lambda_0^2 v_0 (1 + A_0)} \int_0^{k_{max}} \frac{2dk}{k(1 + 4\alpha^*/\pi k^2 \lambda_0^2)^2}. \quad (47)$$

Assuming that the integral is cut off, as in formulas (23) and (32), at $k_{max} = 2/\lambda_0$ [see (25)], Eq. (47) can be written in the form

$$-\frac{dE}{dx} = v \frac{2e^4 m^2}{3\pi\hbar^3} \frac{1 + A_1/3}{(1 + A_0)^2} g_{cl}(\alpha^*), \quad (48)$$

where m is now the free mass of the electron (not the effective mass), and $g_{cl}(\alpha^*)$ is the classical function (26) with α^* as the argument.

For $\alpha^* \ll 1$ we obtain the Fermi-Teller formula, Eq. (2), but with a different factor in front of the logarithm, and for $\alpha^* \gg 1$ [see (28)] we have

Metal	Cu	Ag	Al	Sn	Ti
g_{exp}	0.56	1,53	1.13	0.94	1.89
m^*/m	1.5	1.0	1.6	1.2	3
$n \cdot 10^{-22}$	8.5	5.8	5.4	1	21
α_{free}	1.39	1.56	1.33	2.81	1.04
α_{eff}	2.08	1.56	2.12	3.37	3.14

$$-\frac{dE}{dx} = v\frac{2e^4m^2}{3\pi\hbar^3}\frac{\pi^2(1+A_1/3)^3}{2\alpha^2} = v\frac{\pi}{3}\frac{m^*}{m}\frac{p_0^2}{\hbar}, \quad (49)$$

which differs from formula (5) in the factor $m^*/m = 1 + A_1/3$; the Fermi limiting momentum p_0 is expressed in terms of the density as in an ideal gas: $p_0 = (3\pi^2\hbar^3 n)^{1/3}$. We can see that the formulas obtained from the Fermi liquid model agree qualitatively with those of the gas model.

5. Let us compare the formulas which we have obtained with the experimental results of Gott and Tel'kovskiĭ.[1] The table lists the experimentally observed values of the quantity

$$g_{exp} = \frac{dE/dx}{2ve^4m^2/3\pi\hbar^3}, \quad (50)$$

and also the values of effective mass and density as given by Kittel.[7]

The figure shows curves for $g_{cl}(\alpha)$, $g_{qu}(\alpha)$, and also the two extreme curves: the Fermi-Teller curve, corresponding to $\alpha \ll 1$, and the curve $\pi^2/2\alpha^2$, corresponding to $\alpha \gg 1$. The experimental results are shown in the same figure, the solid circles corresponding to the case when the mass m is assumed equal to the free electron mass, and the hollow circles to the case when m is taken as the effective mass m*. The values of α corresponding to these cases are listed in the table.

Comparison of the experiments with the formulas obtained from the Fermi liquid model (which as we have shown agree qualitatively with the gas model) is difficult since the correlation constant A_0 is unknown for metals.

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