ON THE POSSIBILITY OF HARD VAVILOV-CERENKOV RADIATION

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Submitted to JETP editor July 3, 1964

J. Exptl. Theoret. Phys. (U.S.S.R.) 48, 272-274 (January, 1965)

It is shown that hard photons can occur in a two-quantum Vavilov-Cerenkov effect. The emission of hard photons can be either spontaneous or induced, i.e. due to soft photons. The spectrum and the total intensity of the induced emission are determined.

THE emission of hard photons can not occur in a one-quantum Vavilov-Cerenkov (V.-C.) effect. However, from the law of conservation of energy and momentum it is clear that a hard photon with 4-momentum $(\mathbf{k}, ik)$ and with index of refraction equal to unity can be emitted in a two-quantum V.-C. effect, accompanied by a soft photon $(\mathbf{q}, i\omega)$ $[|\mathbf{q}| = \omega n(\omega)]$, with index of refraction $n(\omega) > 1$, (we using a system of units in which $\hbar = c = 1$), provided that the radiating charged particle is ultrarelativistic. In fact, with $\epsilon_1 \gg m \gg \omega$, and at small ϑ, the energy of the hard photon is given by

$$k = 2\omega\gamma(\omega, \theta)\left[\left(\frac{m}{\varepsilon_1}\right)^2 + 2\frac{\omega}{\varepsilon_1}\gamma(\omega, \theta) + \vartheta^2\right]^{-1}, \qquad (1)$$

where ϵ_1 and m are the initial energy and mass of the particle, ϑ and θ are the angles of emission of the hard and soft photons relative to the initial momentum of the particle, and $\gamma(\omega, \theta) = n(\omega)\cos\theta - 1$. From (1) it is clear that the photons k are mainly hard when emitted inside the narrow cone

$$\vartheta \leqslant [(m/\varepsilon_1)^2 + 2(\omega/\varepsilon_1)\gamma(\omega, \theta)]^{1/2}.$$

The maximum value $k = k_m(\omega, \Theta)$ is attained at $\vartheta = 0$:

$$k_m(\omega, \theta) = \frac{\varepsilon_1^2}{\varepsilon_0(\omega, \theta) + \varepsilon_1}, \quad \varepsilon_0(\omega, \theta) = \frac{m^2}{2\omega\gamma(\omega, \theta)}. \qquad (2)$$

In the case of an electron, $\epsilon_0 \sim 10^{10}$ to 10^{11} ev for soft optical photons and $\epsilon_0 \sim 10^{15}$ to 10^{16} ev for photons in the centimeter wavelength band (with $\Theta = 0$). From (2) it is clear that when $\epsilon_1 \gtrsim \epsilon_0$ the maximum energy of the hard photons approaches ϵ_1. The radiation condition $k > 0$ is satisfied when $\cos\Theta > 1/n(\omega)$ and the angle Θ is inside the Cerenkov cone.

Calculations have shown that spontaneous emission of hard quanta in a two-quantum V.-C. effect has low intensity and is heavily masked by bremsstrahlung. We will not consider it here but remark only that it apparently can be made observable by the use of coincidence techniques.

The intensity of hard V.-C. radiation can be markedly enhanced by passing soft radiation of sufficiently high energy density through the medium, since the probability $d\omega_{\mathbf{k},\mathbf{q}^0}$ of induced two-quantum emission (which has been calculated by Tsytovich [1]) is proportional to the number N_0 of soft photons per unit volume. We will consider soft photons which are transverse, unpolarized, and all having the same momentum $\mathbf{q}_0(q_0, \Theta_0)$. In this case, according to (1), the angle of emission of the hard photons is uniquely related to their energy. For the spectrum of the energy lost by the particle to Cerenkov emission of hard quanta we obtain after some calculation:

$$\rho(k)\,dk = k\,dw_{\mathbf{k},\mathbf{q}_0}$$
$$= \frac{\pi e^4 N_0}{2\varepsilon_1^2}\frac{V_0}{n_0\omega_0}k\left\{a_0 + (1 + a_0)\left[1 + \frac{k^2}{\varepsilon_1(\varepsilon_1 - k)}\right] + \left[1 - 2\frac{\varepsilon_0 k}{\varepsilon_1(\varepsilon_1 - k)}\right]^2\right\}, \qquad (3)$$

where

$$e^2 = 1/137, \quad n_0 = n(\omega_0), \quad \omega_0 = q_0/n_0, \quad V_0 = (d\omega/dq)_{\omega=\omega_0},$$
$$\varepsilon_0 = \varepsilon_0(\omega_0, \theta_0), \quad a_0 = {}^1/_2(n_0^2 - 1)\gamma_0^{-2}\sin^2\theta_0.$$

In contrast to the bremsstrahlung spectrum, the spectrum of (3) has a mainly triangular shape with a maximum at the upper limit $k_m = k_m(\omega_0, \Theta_0)$. We present also the expression for the total intensity of the induced losses to hard V.-C. radiation for the case $\epsilon_1 \ll \epsilon_0$:

$$-\frac{d\varepsilon}{dx} = \int_0^{k_m}\rho(k)\,dk = \frac{4\pi}{3}\frac{e^4 V_0}{n_0 m^4}\left(1 + \frac{3}{2}a_0\right)\gamma_0^2 N_0\omega_0\varepsilon_1^2. \qquad (4)$$

With $\omega_0 = 1\ \text{cm}^{-1}$, $\Theta_0 = 0$, $n_0 = V_0^{-1} = 10$, $N_0 = 10^{20}\ \text{cm}^{-3}$, and $\epsilon_1 = 4 \times 10^{10}$ eV we have $\epsilon_0 = 0.73 \times 10^{15}$ eV, $k_m = 2.2 \times 10^6$ eV and $-d\epsilon/dx$

= 3.3 eV/cm. This amounts to about 10^{-4} of the bremsstrahlung losses in the soft part of the spectrum below k_m (in light elements).

However, V.-C. radiation is due to long-range interactions, in contrast to bremsstrahlung which is associated with near collisions. This lets one hope that the effect considered can be observed experimentally by injecting the particles into the medium through a tunnel with a diameter that is small compared with the wavelength of the soft photon (in which case there will be no bremsstrahlung, whereas the V.-C. radiation will be only slightly altered). We note also that $-d\epsilon/dx$ increases proportionally to N_0, ω_0, and ϵ_1^2. With ω_0 from the optical spectrum and $N_0 \sim 10^{20}$ cm^{-3} one can attain values comparable with bremsstrahlung losses (under the condition, which is difficult to fulfill, that the medium must withstand a sufficiently high density of optical radiation).

If the angle Θ_0 exceeds the limits of the Cerenkov cone, $\cos \Theta_0 < 1/n$, then instead of induced emission of hard quanta paired with soft quanta, there will occur radiation of hard quanta produced from soft quanta (which is effectively Compton scattering from the electrons moving in the medium). Formulae (3) and (4) retain the same form also in the case of the absorption of transverse quanta, but in formulae (1) and (2) it is necessary to change ω to $-\omega$, because $n(-\omega) = n(\omega)$. The question of the radiation of hard quanta following the absorption of soft longitudinal quanta has been considered by Ryazanov [2] and by Gaĭlitis and Tsytovich [3]. Radiation following absorption can occur even with $n_0 \leq 1$. The case $n_0 = 1$ has been considered in a number of works [4,5], and the results obtained there agree with ours.

The author sincerely thanks I. N. Toptygin for a series of discussions and for assistance, A. A. Rumyantsev, O. V. Konstantinov and V. I. Perel' for discussions, and Vit. V. Batygin for help with the calculations.

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Translated by A. E. Miller
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