NONLINEAR GALVANOMAGNETIC EFFECTS, NEGATIVE RESISTANCE REGIONS OF CURRENT-VOLTAGE CHARACTERISTICS, AND RUNAWAY ELECTRONS

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The dependence of conductivity in semiconductors on external electric and magnetic fields is investigated. The conditions producing negative resistance regions on the current-voltage characteristic are considered. The relation between negative resistance and runaway electrons is discussed.

AT low temperatures a nonlinear relationship between electric current and the electric field exists within the lattices of semiconductors even at relatively low voltages. The associated phenomena have often been studied theoretically. It has been noted that regions of negative resistance (negative differential conductivity) appear on the current-voltage characteristics of semiconductors in association with certain mechanisms of carrier scattering by various impurities and phonons. A. Gurevich was the first to mention the occurrence of this effect in a plasma;[1] it has been studied in semiconductors by Bok,[2] Adawi,[3] and Kogan,[4] and in the presence of magnetic fields by Kazarinov and Skobov[5,6] and by Gurevich and Korenblit.[7]

In the present work the current-voltage characteristics of semiconductors in high electric and magnetic fields are studied. We shall analyze the influence of boundary conditions and of the general properties of mechanisms of electron scattering by a lattice and by impurities on the form of the current-voltage characteristics. We shall also investigate the conditions under which negative resistance regions appear. The possible spatial inhomogeneity of samples will not be considered, nor shall we investigate the conditions of current instability associated with the negative resistance.

One of two different procedures is usually employed in investigating nonlinear effects; one can either solve a kinetic equation, from which a distribution function is obtained,[5,6,8,9] or one can assume an equilibrium distribution at an effective temperature determined from energy balance which is derived from a kinetic equation.[4,7,10,11] The latter approach is to some extent equivalent to the so-called elementary theory which is widely utilized in studying the nonlinear propagation of electromagnetic waves in the ionosphere.[12,13] When calculating macroscopic quantities, such as conductivity, mean energy, the Hall field etc., both methods yield practically equivalent results. However, only the first method can be used when the electron distribution must be investigated directly. We note that when the carrier concentrations are large and carrier-carrier scattering is important, carrier-distribution equilibrium is established at some effective temperature and the first procedure coincides essentially with the second. The present work is based mainly on the effective temperature method as permitting simpler calculations, although the distribution function method will sometimes be employed.

1. EQUATION FOR THE EFFECTIVE TEMPERATURE OF AN ELECTRON GAS IN ELECTRIC FIELDS. GALVANOMAGNETIC EFFECTS

The electric current in a semiconductor within a magnetic field is related to the electric field by the equation

$$j_i = \sigma_{ik}(\mathbf{E}, \mathbf{H}) E_k, \tag{1.1}$$

where j_i is the i-th component of the electric current, σ_{ik} are conductivity tensor components, E_k is the k-th electric field component; i, k = x, y, z; summation is performed over identical subscripts. In a coordinate system (1, 2, 3) with the magnetic field along the 3 axis, the conductivity tensor components for a semiconductor with an isotropic quadratic law of carrier dispersion are given by

$$\sigma_{11} = \sigma_{22} = -\frac{8\sqrt{2}\pi e^2 m^{1/2}}{3}\int_0^\infty \frac{\tau(\varepsilon)\varepsilon^{3/2}}{1+(e\tau(\varepsilon)H/mc)^2}\frac{df(\varepsilon)}{d\varepsilon}d\varepsilon,$$

$$\sigma_{12} = -\sigma_{21} = -\frac{8\sqrt{2}\pi e^3 H}{3m^{1/2}c}\int_0^\infty \frac{\tau^2(\varepsilon)\varepsilon^{3/2}}{1+(e\tau(\varepsilon)H/mc)^2}\frac{df(\varepsilon)}{d\varepsilon}d\varepsilon,$$

$$\sigma_{33} = -\frac{8\sqrt{2}\pi e^2 m^{1/2}}{3}\int_0^\infty \tau(\varepsilon)\varepsilon^{3/2}\frac{df(\varepsilon)}{d\varepsilon}d\varepsilon,$$

$$\sigma_{13} = \sigma_{31} = \sigma_{23} = \sigma_{32} = 0. \quad (1.2)$$

Here e is the charge, m is the effective mass, ϵ is the electron energy, $\tau(\epsilon)$ is the momentum relaxation time, and $f(\epsilon)$ is the symmetric part of the carrier distribution function.

In accordance with the effective temperature method we assume a Maxwellian distribution:

$$f(\varepsilon) = Ne^{-\varepsilon/\Theta} / (2\pi m\Theta)^{3/2} \quad (1.3)$$

where N is the electron concentration. From the kinetic equation we obtain the following equation for the effective temperature Θ:

$$\mathbf{jE} = \sigma_{ik}(\Theta)E_iE_k = \frac{2N}{\pi^{1/2}\Theta^{3/2}T}\left(1 - \frac{T}{\Theta}\right)\int_0^\infty A(\varepsilon)\,\varepsilon^{1/2}e^{-\varepsilon/\Theta}d\varepsilon. \quad (1.4)$$

$A(\epsilon)$ is the electron diffusion coefficient in energy space and is related to the energy relaxation time; T is the equilibrium temperature.

In a real semiconductor several types of both momentum scattering and energy scattering can occur simultaneously. Then $A(\epsilon)$ and $\tau(\epsilon)$ are represented by

$$A(\varepsilon) = \sum_i A_i(\varepsilon), \qquad \tau^{-1}(\varepsilon) = \sum_k \tau_k^{-1}(\varepsilon). \quad (1.5)$$

The summations are performed over all types of energy and momentum scattering; in general, $i \neq k$.

Our subsequent discussion requires that we determine the energy dependences $A(\epsilon)$ and $\tau(\epsilon)$. It can be shown that for any type of scattering we have

$$A(\varepsilon) = A_0(T)(\varepsilon / T)^r, \qquad \tau(\varepsilon) = \tau_0(T)(\varepsilon / T)^q; \quad (1.6)$$

r and q for different types of scattering are given in the accompanying table, where T_d is the Debye temperature. Electron scattering is assumed to occur everywhere except in polar semiconductors. The impurities will be assumed to have infinite masses and to change only the momentum direction of a scattered electron, while the electron energy is conserved; this has been denoted by dashes in the column for r in the cases of scattering by impurities. For scattering by lattice vibrations we shall assume that momentum transfer to the lattice will go to the same branch as for energy transfer. We note that several scattering mechanisms can, in general, participate simultaneously; scattering by the lattice is accompanied by energy transfer and scattering on impurities is accompanied by momentum transfer.

Objects causing scattering	r	q
Acoustic vibrations,	$3/2$	$-1/2$
Optical vibrations, $T < T_d$	1	0
Optical vibrations, $T > T_d$	$-1/2$	$1/2$
Piezo-acoustic vibrations	$1/2$	$1/2$
Polar semiconductors, scattering by optical vibrations, $T > T_d$	$-1/2$	$3/2$
Neutral impurities	–	0
Charged impurities	–	$3/2$
Dipolar impurities	–	$1/2$

For the sake of brevity each particular scattering mechanism will be denoted by its relative numerical position in the table; Roman numerals from I to V will be used for r, and Arabic numerals from 1 to 8 will be used for q. Thus when energy transfer occurs through scattering by acoustic phonons and momentum transfer occurs through scattering by ionized impurities, the scattering type will be designated as I7. When electron energy for $T > T_d$ in ionic crystals is transferred to optical and acoustic vibrations while momentum is transferred to neutral impurities, the scattering will be denoted by I, III6 etc.

We shall now derive the equation for determining the temperature. It is assumed that a potential difference along the z axis generates a given static electric field E_z. We must now determine the dependence of j_z on E_z. The other electric field and electric current components must be found as functions of E_z from additional conditions (which we shall call boundary conditions) corresponding to the experimental setup.

We shall discuss four cases.

A. Consider a semiconductor in the form of a rectangular parallelepiped with its longest sides parallel to the z axis (Fig. 1A). Current does not flow through the faces which are perpendicular to the x and y axes. The electric fields E_x and E_y are derived from the conditions

$$j_x = \sigma_{xx}E_x + \sigma_{xy}E_y + \sigma_{xz}E_z = 0,$$
$$j_y = \sigma_{yx}E_x + \sigma_{yy}E_y + \sigma_{yz}E_z = 0. \quad (1.7)$$

Representing the remaining components of σ_{ik} by σ_{11}, σ_{12}, and σ_{13}, this equation yields

$$\frac{E_x}{E_z} = \frac{\sigma_{12}\sigma_{13}\sin\alpha}{(\sigma_{11}^2 + \sigma_{12}^2)\cos^2\alpha + \sigma_{11}\sigma_{33}\sin^2\alpha},$$

$$\frac{E_y}{E_z} = \frac{[\sigma_{12}^2 - \sigma_{11}(\sigma_{33} - \sigma_{11})]\sin\alpha\cos\alpha}{(\sigma_{11}^2 + \sigma_{12}^2)\cos^2\alpha + \sigma_{11}\sigma_{33}\sin^2\alpha}. \quad (1.8)$$

Here α is the angle between the magnetic field and the z axis. It is easily seen that E_x is an odd function of the magnetic field, while E_y is an even function. E_x is called the Hall field, and E_y is the field of the longitudinal-transverse magnetoresistive effect.

The scalar product in the left-hand side of (1.4) will now be represented by E_z. From the boundary conditions we have

$$\mathbf{jE} = j_z E_z = \sigma_{zk} E_k E_z. \quad (1.9)$$

Substituting E_x and E_y from (1.8) into (1.9), we obtain

$$\mathbf{jE} = \sigma(\Theta) E_z^2, \quad (1.10)$$

where

$$\sigma(\Theta) = \sigma_A(\Theta),$$

$$\sigma_A(\Theta) = \frac{\sigma_{33}(\sigma_{12}^2 + \sigma_{11}^2)}{(\sigma_{11}^2 + \sigma_{12}^2)\cos^2\alpha + \sigma_{11}\sigma_{33}\sin^2\alpha}. \quad (1.11)$$

B. Here the boundary conditions are $j_x = 0$ and $E_y = 0$, which can be realized by a semiconductor in the form of a plate having its faces perpendicular to the x axis (Fig. 1B). The corresponding formulas are

$$\frac{E_x}{E_z} = \frac{\sigma_{12}}{\sigma_{11}} \sin\alpha; \quad (1.12)$$

jE is obtained from (1.10), where $\sigma(\Theta)$ is represented by

$$\sigma(\Theta) = \sigma_B(\Theta) = \frac{(\sigma_{11}^2 + \sigma_{12}^2)\sin^2\alpha + \sigma_{11}\sigma_{33}\cos^2\alpha}{\sigma_{11}}. \quad (1.13)$$

For $\alpha = \pi/2$ and $\alpha = 0$ we will have $\sigma_B(\Theta) = \sigma_A(\Theta)$.

C. Here the boundary conditions $E_x = 0$ and $j_y = 0$ represent a semiconducting plate with its faces perpendicular to the y axis (Fig. 1C). By analogy with the foregoing, we obtain

$$\frac{E_y}{E_z} = \frac{(\sigma_{11} - \sigma_{33})\sin\alpha\cos\alpha}{\sigma_{11}\cos^2\alpha + \sigma_{33}\sin^2\alpha}, \quad j_C = \sigma_C(\Theta) E_z, \quad (1.14)$$

$$\sigma_C(\Theta) = \frac{\sigma_{11}\sigma_{33}}{\sigma_{11}\cos^2\alpha + \sigma_{33}\sin^2\alpha}. \quad (1.15)$$

Here and subsequently the subscript z of j has been omitted.

D. The conditions $E_x = 0$ and $E_y = 0$ are realized for an unbounded sample; we then have

$$\sigma_D(\Theta) = \sigma_{11}\sin^2\alpha + \sigma_{33}\cos^2\alpha, \quad j_D = \sigma_D(\Theta) E_z. \quad (1.16)$$

For $\alpha = \pi/2$ this case has been considered in[5,6].

Thus, on the basis of (1.5) and (1.6), as well as (1.11), (1.13), (1.15), and (1.16), in all four cases the temperature equation becomes

$$\sigma_p(\Theta) E_z^2 = \frac{2N}{\sqrt{\pi}T}\left(1 - \frac{T}{\Theta}\right)\sum_i \Gamma(3/2 + r_i) A_{0i} \left(\frac{\Theta}{T}\right)^{r_i}, \quad (1.17)$$

where p = A, B, C, D.

Equation (1.17) in conjunction with the formula relating the current to the field,

$$j_p(\Theta) = \sigma_p(\Theta) E_z \quad (1.18)$$

determine in parametric form the current-voltage characteristic of a sample when written as

$$E_z^2 = \frac{2N}{\sqrt{\pi}T\sigma_p(u)}\left(1 - \frac{1}{u}\right)\sum_i \Gamma(3/2 + r_i) A_{0i} u^{r_i},$$

$$j^2(u) = \frac{2N\sigma_p(u)}{\sqrt{\pi}T}\left(1 - \frac{1}{u}\right)\sum_i \Gamma(3/2 + r_i) A_{0i} u^{r_i}. \quad (1.19)$$

The dimensionless temperature $u = \Theta/T$ serves as the parameter. It follows from the expressions for $\sigma_p(u)$ that the current-voltage characteristic depends greatly on the boundary conditions.

2. EFFECTIVE TEMPERATURE OF AN ELECTRON GAS AND GALVANOMAGNETIC EFFECTS IN HIGH MAGNETIC FIELDS

In this section we shall consider the limiting case of high magnetic fields subject to the inequality $(e\tau H/mc)^2 \gg 1$. Asymptotic expressions for the conductivity tensor components are

$$\sigma_{11} = \frac{4Nmc^2}{3\sqrt{\pi}H^2}\sum_k \Gamma(5/2 - q_k)\tau_{0k}^{-1} u^{-q_k}, \quad \sigma_{12} = \frac{eNc}{H},$$

$$\sigma_{33} = \frac{4e^2N}{3\sqrt{\pi}m\Theta^{5/2}}\int_0^\infty \varepsilon^{3/2}\tau(\varepsilon) e^{-\varepsilon/\Theta} d\varepsilon. \quad (2.1)$$

For qualitative investigations σ_{33} can be approximated roughly by

$$\sigma_{33} = \frac{4e^2N}{3\sqrt{\pi}m}\left[\sum_k \{\Gamma(5/2 + q_k)\tau_{0k} u^{q_k}\}^{-1}\right]^{-1}. \quad (2.1')$$

When only one scattering mechanism is important, i.e., when only a single term need be retained in (2.1′), this formula is exact.

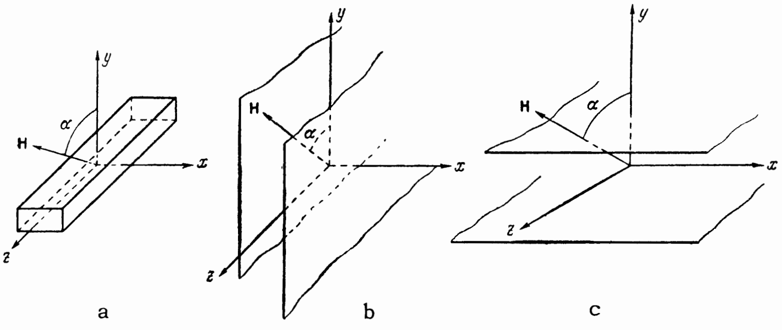

FIG. 1

We shall assume that in the given range of electric and magnetic fields only one scattering mechanism is important, and shall derive the temperature and the galvanomagnetic properties of a semiconductor for the different boundary conditions.

A. Substituting (2.1) into (1.8) and (1.11), and employing (1.17), we obtain

$$\frac{E_x}{E_z}=\frac{4\Gamma(^5/_2+q)}{3\sqrt{\pi}(\cos^2\alpha+a_q\sin^2\alpha)}\frac{eH\tau_0}{mc}u^q\sin\alpha,$$

$$\frac{E_y}{E_z}=\frac{(1-a_q)\sin\alpha\cos\alpha}{\cos^2\alpha+a_q\sin^2\alpha},$$

$$\sigma(u)=\frac{4\Gamma(^5/_2+q)}{3\sqrt{\pi}(\cos^2\alpha+a_q\sin^2\alpha)}\frac{e^2N\tau_0}{m}u^q. \quad (2.2)$$

Here $a_q = 16\Gamma(5/2 + q)\,\Gamma(5/2 - q)/9\pi$; this quantity has been tabulated in[14]. The equation for u has the form

$$u^{r-q-1}(u-1)=\mathscr{E}_A{}^2;$$

$$\mathscr{E}_A{}^2=\frac{E_z{}^2}{E_A{}^2},\qquad E_A{}^2=\frac{3\Gamma(^3/_2+r)(\cos^2\alpha+a_q\sin^2\alpha)}{2\Gamma(^5/_2+q)}\frac{mA_0}{e^2\tau_0T}. \quad (2.3)$$

The formulas (2.2) and (2.3) completely determine the dependence of the galvanomagnetic properties on electric and magnetic fields.

B. We have here

$$\frac{E_x}{E_z}=\frac{3\sqrt{\pi}}{4\Gamma(^5/_2-q)}\frac{e\tau_0H}{mc}u^q\sin\alpha,$$

$$\sigma(u)=\frac{3\sqrt{\pi}(\sin^2\alpha+a_q\cos^2\alpha)}{4\Gamma(^5/_2-q)}\frac{e^2N\tau_0}{m}u^q. \quad (2.4)$$

In this case the equation for u coincides with (2.3) when we replace $\mathscr{E}_A^2$ by $\mathscr{E}_B^2$, where

$$E_B{}^2=\frac{8\Gamma(^5/_2-q)}{3\pi\Gamma(^3/_2+r)(\sin^2\alpha+a_q\cos^2\alpha)}\frac{mA_0}{e^2\tau_0T}. \quad (2.4a)$$

In cases C and D the results exhibit angular dependence.

C. For the case $\tan^2\alpha \gg (e\tau_0Hu^q/mc)^{-2} \ll 1$ we have

$$\frac{E_y}{E_z}=-\cot\alpha,\qquad \sigma(u)=\frac{4\Gamma(^5/_2-q)}{3\sqrt{\pi}}\frac{e^2Nmc^2}{H^2\tau_0\sin^2\alpha}u^{-q}. \quad (2.5)$$

The equation for u is

$$u^{r+q-1}(u-1)=\mathscr{E}_B^2;$$

$$\mathscr{E}_C{}^2=\frac{E_z{}^2}{E_C{}^2},\qquad E_C{}^2=\frac{3\Gamma(^3/_2+r)}{2\Gamma(^5/_2-q)}\frac{H^2\tau_0A_0}{e^2mc^2T}\sin^2\alpha. \quad (2.6)$$

C′. For the case $\tan^2\alpha \ll (eH\tau_0u^q/mc)^{-2} \ll 1$ ($\alpha \approx 0$) we have

$$\frac{E_y}{E_z}=-\frac{\Gamma(^5/_2+q)}{\Gamma(^5/_2-q)}\left(\frac{eH\tau_0}{mc}u^q\right)^2\sin\alpha. \quad (2.7)$$

The expression for $\sigma(u)$ and the equation for u are obtained from (2.2) and (2.3) after setting $\alpha = 0$. We note that under the given hypotheses the ratio E_y/E_z in the last case can be of the order of unity or even considerably larger.

D. For the case $\tan^2\alpha \gg (eH\tau_0u^q/mc)^2 \gg 1$ ($\alpha \approx \pi/2$) all results are obtained from (2.6) after setting $\alpha = \pi/2$.

D′. For the case $\tan^2\alpha \ll (eH\tau_0u^q/mc)^2 \gg 1$ we obtain

$$\sigma(u)=\frac{4\Gamma(^5/_2+q)}{3\sqrt{\pi}}\frac{e^2N\tau_0}{m}u^q\cos^2\alpha. \quad (2.8)$$

The equation for u coincides with (2.3) after $\mathscr{E}_A^2$ is replaced by $\mathscr{E}_{D'}^2 = E_z^2/E_{D'}^2$:

$$E_{\Gamma'}{}^2=\frac{3\Gamma(^3/_2+r)}{2\Gamma(^5/_2+q)}\frac{mA_0}{e^2\tau_0T\cos^2\alpha}. \quad (2.8a)$$

It is interesting to note that for high magnetic fields cases C and D are realized in intrinsic semiconductors also for the boundary conditions A and B, because, as shown in[9], the Hall field in an intrinsic semiconductor approaches zero as H approaches infinity.

The foregoing formulas show that the angle between the magnetic field and the current is an important factor in determining the electronic temperature. In cases A, B, C′, and D′ the temperature is of the same order as in the absence of a magnetic field and does not depend on the magnitude of the latter. (In the absence of a magnetic field the temperature is obtained from the formulas of case A with $\alpha = 0$.) Cases C and D are different, involving strong anisotropy. For $u \gg 1$ it follows from (2.3) and (2.6) that

$$\frac{u_D}{u_{D'}}\sim\frac{u_C}{u_{C'}}\sim\left(\frac{eH\tau_0}{mc}u^q\right)^{-2(r+q)}\ll 1\ \text{ for }\ r+q>0, \quad (2.9)$$

which implies that in these cases the presence of a magnetic field results in strong cooling of the electron gas.

We note that in case C cooling by the magnetic field occurs for all angles except in a narrow cone with an aperture of the order $\sim (eH\tau_0u_C^q/mc)^{-1}$ around the direction $\alpha = 0$. In case D the magnetic field cools the electron gas within a narrow cone having an aperture of the order $(eH\tau_0u_D^q/mc)^{-1}$ around the direction $\alpha \approx \pi/2$.

3. THE CURRENT-VOLTAGE CHARACTERISTIC AND NEGATIVE RESISTANCE

The most prominent scattering mechanism within some range of electric and magnetic fields and lattice temperatures will be called the principal mechanism; all other scattering mechanisms will be designated as secondary. The current-voltage characteristic for cases A, B, C′, and D′ can be put into the form

$$J_p^2 = u^{r+q-1}(u-1) + \varphi_J(u),$$

$$\mathscr{E}_p^2 = u^{r-q-1}(u-1) + \varphi_{\mathscr{E}}(u). \qquad (3.1)$$

In this equation the exponents r and q pertain to the principal scattering mechanism; φ_J and $\varphi_{\mathscr{E}}$ represent the effects of the secondary mechanisms. J_p denotes the quantity j/j_p (p = A, B, C′, D′):

$$j_A^2 = \frac{8\Gamma(^3/_2 + r)\Gamma(^5/_2 + q)e^2N\tau_0A_0}{3\pi(\cos^2\alpha + a_q\sin^2\alpha)mT},$$

$$j_B^2 = \frac{3\Gamma(^3/_2 + r)(\sin^2\alpha + a_q\cos^2\alpha)e^2N\tau_0A_0}{2\Gamma(^5/_2 - q)mT}$$

$$j_{C'}^2 = j_A^2 \quad \text{for} \quad \alpha = 0,$$

$$j_{D'}^2 = \frac{\Gamma(^3/_2 + r)\Gamma(^5/_2 + q)\cos^2\alpha e^2N\tau_0A_0}{3\pi mT} \qquad (3.1a)$$

It is interesting that the current-voltage characteristic in the absence of a magnetic field is also represented by (3.1), but with $\alpha = 0$ in the formulas for j_p and $\mathscr{E}_p$.

Neglecting the secondary scattering mechanisms, Eq. (3.1) leads to the following asymptotic dependences of u and J on $\mathscr{E}$:
for $u - 1 \ll 1$

$$u_p = 1 + \mathscr{E}_p^2, \qquad J_p = \mathscr{E}_p, \qquad (3.2)$$

for $u \gg 1$

$$u_p = \mathscr{E}_p^{2(r-q)}, \qquad J_p = \mathscr{E}_p^{(r+q)(r-q)}, \qquad (3.3)$$

The current-voltage characteristic for cases C′ and D′ is

$$J_s^2 = u^{r-q-1}(u-1) + \psi_J(u),$$

$$\mathscr{E}_s^2 = u^{r+q-1}(u-1) + \psi_{\mathscr{E}}(u). \qquad (3.4)$$

Here ψ_J and $\psi_{\mathscr{E}}$ represent the contributions of the secondary scattering mechanisms; s = C, D,

$$J_s = \frac{j}{j_s}, \qquad j_C^2 = \frac{8\Gamma(^3/_2 + r)\Gamma(^5/_2 - q)e^2N^2A_0mc^2}{3\pi H^2\tau_0T\sin^2\alpha},$$

$$j_D = j_C \quad \text{for} \quad \alpha = \pi/2. \qquad (3.4a)$$

For the asymptotic behavior in cases C and D with $u - 1 \ll 1$ we have formulas analogous to (3.2), while for $u \gg 1$ we obtain

$$u_s = \mathscr{E}_s^{2/(r+q)}, \qquad J_s = \mathscr{E}_s^{(r-q)/(r+q)}. \qquad (3.5)$$

In deriving (3.2) and (3.5) the secondary mechanisms were neglected, and it was assumed that $r \pm q \neq 0$.

It can easily be seen that when secondary scattering mechanisms are neglected (3.4) becomes (3.1) if J and $\mathscr{E}$ are exchanged. It follows that if $J_p = F(\mathscr{E}_p)$, then $J_s = F'(\mathscr{E}_s)$, where F′ is the inverse of the function F.

In our subsequent discussion the differential conductivity of a sample is a very essential quantity:

$$\eta_{p,s} = \frac{dJ_{p,s}}{d\mathscr{E}_{p,s}} = \frac{dJ_{p,s}/du}{d\mathscr{E}_{p,s}/du}. \qquad (3.6)$$

For the current-voltage characteristic represented by (3.1), which we shall hereinafter designate as type A, we have

$$\eta_p = u^{-q}\frac{r+q}{r-q}\left[u - \left(1 - \frac{1}{r+q}\right)\right] \Big/ \left[u - \left(1 - \frac{1}{r-q}\right)\right]. \qquad (3.7)$$

For the characteristic (3.4), designated as type B, we have

$$\eta_s = u^{q}\frac{r-q}{r+q}\left[u - \left(1 - \frac{1}{r-q}\right)\right] \Big/ \left[u - \left(1 - \frac{1}{r+q}\right)\right]. \qquad (3.8)$$

Equations (3.7) and (3.8) yield the obvious relations

$$\eta_p\eta_s = 1, \qquad \operatorname{sign}\eta_p = \operatorname{sign}\eta_s. \qquad (3.7')$$

An important role will be played by the temperatures

$$u_{1,2} = 1 - 1/(r \pm q), \qquad (3.9)$$

at which the signs of η_p and η_s are reversed; of course, these points have a physical meaning only when $u_{1,2} > 1$. The currents and fields corresponding to these temperatures are given by the formulas

$$J_{p;1,2} = \mathscr{E}_{s;1,2} = \frac{1}{|r \mp q|^{1/2}}\left[1 - \frac{1}{r \mp q}\right]^{(r+q-1)/2};$$

$$\mathscr{E}_{p;1,2} = J_{s;1,2} = \frac{1}{|r \mp q|^{1/2}}\left[1 - \frac{1}{r \mp q}\right]^{(r-q-1)/2}. \qquad (3.10)$$

It follows from the foregoing that η_p vanishes when η_s becomes infinite, and conversely, η_s vanishes when η_p becomes infinite.

We shall now consider the effect of a secondary scattering mechanism. It follows from (1.19) that for large u we have

$$\varphi_{\mathscr{E}}(u) \sim u^{\gamma_1}, \qquad \varphi_J(u) \sim u^{\gamma_2},$$

$$\psi_{\mathscr{E}}(u) \sim u^{\gamma_3}, \qquad \psi_J(u) \sim u^{\gamma_4}. \qquad (3.11)$$

The only important secondary mechanisms for our purposes will be those for which $\gamma_1, \gamma_2, \gamma_3, \gamma_4 > 0$, since only these can insure a state with a positive differential current-voltage characteristic (a stationary state) at very high temperatures. The table shows that secondary scattering mechanisms of this type exist in all kinds of semiconductors.

We can consider either (3.1) or (3.4); we choose the former. When $r - q > 0$, a secondary mechanism can begin to be significant for large u only if $\gamma_1 > r - q$. When $r - q \le 0$, then for $u \to \infty$ the first term in the given formula is bounded and the secondary mechanism sooner or later becomes the principal mechanism.

The order of magnitude of the temperature at which a secondary mechanism becomes important can be estimated by equating the two terms in (3.1) and (3.4) $[u^{r+q-1}(u-1) = \varphi g(u)]$; it is then easy to evaluate the corresponding fields and currents. The temperature, field, and current at which the secondary mechanism begins to be important will be denoted by primes.

It follows from (3.7) and (3.8) that the sign of the differential conductivity is governed by the quantities $r - q$ and $r + q$. In virtue of (3.7′) the secondary scattering mechanism is here neglected.

We shall now consider individual special cases.

1. $r - q > 0$, $r + q > 0$ or, equivalently, $r > 0$, $-r < q < r$. These conditions are satisfied by the scattering mechanisms I1, I2, I3, I6, I7, I8, II1, II6, II8, IV6. It follows from (3.7) and (3.8) that the differential conductivity is positive, because the dimensionless temperatures $u_{1,2}$ at which the sign of the differential conductivity is reversed are smaller than unity, whereas $u_{1,2} \ge 1$ is required for physical meaning (the temperature of the electron gas cannot be lower than the lattice temperature), and for $u \ge 1$, η_p and η_s are positive. The temperature of the electron gas in this case can also rise monotonically as the electric field increases.

2. $r - q = 0$, $r + q > 0$ ($r = q > 0$). These conditions are satisfied by the scattering mechanisms I7, IV4, IV8. All relations can here be obtained explicitly. For the type A current-voltage characteristic we have

$$u_p = \frac{1}{1-\mathscr{E}_p^2}, \qquad J_p = \frac{\mathscr{E}_p}{(1-\mathscr{E}_p^2)^q},$$

$$\eta_p = \frac{\mathscr{E}_p}{(1-\mathscr{E}_p^2)^q}\left\{\frac{1}{\mathscr{E}_p} + \frac{2q\mathscr{E}_p}{1-\mathscr{E}_p^2}\right\} > 0. \qquad (3.12)$$

It follows herefrom that u_p, $\eta_p \to \infty$ when $\mathscr{E}_p \to 1$.

At sufficiently large values of u_p a secondary scattering mechanism begins to participate, leading to finite values of u_p, J_p, and η_p for finite $\mathscr{E}_p$. However, if the secondary mechanism begins to participate at very high temperatures the electron gas can become very highly heated in relatively low fields.

In the case of the type B current-voltage characteristic it is convenient to represent the temperature and field in terms of the current:

$$u_s = \frac{1}{1-J_s^2}, \qquad \mathscr{E}_s = \frac{J_s}{(1-J_s^2)^q}. \qquad (3.13)$$

It follows therefrom that for $\mathscr{E}_s \to \infty$ the quantity $u_s \sim \mathscr{E}_s^{1/q}$ approaches infinity, while the current approaches unity according to $J_s \sim 1 - \frac{1}{2}\mathscr{E}_s^{-1/q}$.

3. $r + q > 0$, $r - q < 0$ ($q > 0$, $-q < r < q$). We now have the following relations:

$$\frac{du_p}{d\mathscr{E}_p}, \quad \eta_p \begin{cases} > 0 & \text{for} \quad \mathscr{E}_p > \mathscr{E}_{p1} \\ = \pm\infty & \text{for} \quad \mathscr{E}_p = \mathscr{E}_{p1}. \\ < 0 & \text{for} \quad \mathscr{E}_p < \mathscr{E}_{p1} \end{cases} \qquad (3.14)$$

It is seen from (3.3) that when $\mathscr{E}_p \to 0$ we have $J_p \to \infty$ and $u_p \to \infty$ (Fig. 2, curve III). We can expect that at some value u′ a secondary scattering mechanism will begin to operate, leading to finite current and temperature in finite electric fields.

The shape of the current-voltage characteristic depends considerably on whether u′ is smaller or larger than u_1. If $u' < u_1$ a negative resistance region is not realized (Fig. 2, curve I), and the current-voltage characteristic is a monotonic function of the field. When $u' > u_1$ (Fig. 2, curve II) a negative resistance region does appear on the current-voltage curve, which is S-shaped.

In the case of the type B current-voltage characteristic the sign of η_s varies as follows:

$$\eta_s > 0 \text{ for } \mathscr{E} > \mathscr{E}_{s1}, \quad \eta_s = 0 \text{ for } \mathscr{E} = \mathscr{E}_{s1}, \quad \eta_s < 0 \text{ for } \mathscr{E} < \mathscr{E}_{s1}. \qquad (3.15)$$

It follows from (3.3) that for $\mathscr{E} \to \infty$ we have $u \to \infty$ and $J \to 0$, while the temperature increases monotonically with the field. A type B current-voltage characteristic is shown in Fig. 3, where curve III corresponds to the absence of a secondary mechanism.

Two cases can exist here, as for type A, when a secondary mechanism operates. For $u' < u_1$ the characteristic is a monotonically increasing function of the field (Fig. 3, curve I). For $u' > u_1$ a negative resistance region appears and the curve becomes N-shaped (Fig. 3, curve II).

It is interesting that for the transition of the type A curve from positive to negative differential conductivity, at the transition point from $\eta > 0$ to $\eta < 0$ we find that η becomes $\pm\infty$, whereas for type B we find that η vanishes.

4. $r + q = 0$, $r - q < 0$ ($r < 0$, $q = -r > 0$). These relations are satisfied by scattering mechanisms III3 and III8. The type A current-voltage characteristic is represented by

$$\mathscr{E}_p = J_p(1 - J_p^2)^q. \qquad (3.16)$$

For $u' < u_1$ in the presence of a "restraining" mechanism, J as a function of $\mathscr{E}$ has the form represented by curve I in Fig. 2, while for $u' > u_1$ it is represented by curve II. For $\mathscr{E}_p \to 0$ we have $J_p \to 1$ (Fig. 2, curve IV) and $u \sim \mathscr{E}^{-1/q}$.

The type B current-voltage curve is represented by

$$J_s = \mathscr{E}_s(1 - \mathscr{E}_s^2)^q. \quad (3.17)$$

The temperature of the electron gas is given by (3.12). In this case the current-voltage curve has the same form as in case 3, except that in the absence of a restraining mechanism the current vanishes rigorously when $\mathscr{E}_s = 1$ (Fig. 3, curve IV).

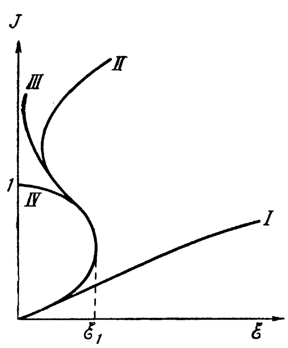

FIG. 2

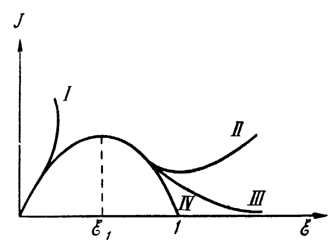

FIG. 3

5. $r - q < 0$, $r + q < 0$ ($r < 0$, $r < q < -r$). This corresponds to scattering mechanism III6, i.e., $q = 0$. The current is governed by Ohm's law; $u \to \infty$ for $\mathscr{E} \to 0$. If we had $q \neq 0$, then in the absence of a secondary scattering mechanism the current-voltage characteristic would possess points with both $\eta = 0$ and $\eta = \pm\infty$.

The remaining three possible cases are of purely academic interest, since they do not represent any real scattering objects. An investigation, completely analogous to the foregoing discussion, shows that for $r = q = 0$ Ohm's law applies, while in the cases of $r - q > 0$, $r + q = 0$ and $r - q > 0$, $r + q < 0$ the type A and type B characteristics coincide with the type B and type A characteristics of cases 2 and 3.

It has thus far been assumed that the electron concentration does not depend on the electron gas temperature. However, as has been shown in[5] and in our earlier work[15], if impact ionization plays an important part the electron concentration is governed by the effective temperature of the electron gas. We shall assume, for simplicity, that the principal process, the inverse of impact ionization, is a triple collision. The formula for the concentration is then

$$N(u) = N_0 e^{-\delta/u}. \quad (3.18)$$

In deriving this equation it was assumed that δ (the activation energy in units of T) is much greater than u and therefore much greater than unity.

Let us consider the influence of impact ionization on negative resistance. The equations of the current-voltage characteristic taking impact ionization into account are obtained by inserting the additional factor $e^{-2\delta/u}$ in the right-hand side of the expression for J^2. We note that the relations (3.7′) then become invalid. As already shown, negative resistance appears when the condition $r - q < 0$ is fulfilled.

Impact ionization thus does not affect the occurrence of negative resistance in a type A curve, since in this case η_p has the same sign as $d\mathscr{E}/du$, but the latter is not affected by impact ionization. For a type B curve η_s has the same sign as dJ/du; a direct calculation shows that this quantity will always exceed zero when impact ionization is taken into account. Thus the existence of an impact ionization excludes negative resistance from the current-voltage characteristic of type B. This result is explained physically by the fact that a current increase due to impact ionization overcomes the current decrease in a growing electric field as a result of electron mobility.

We have confined ourselves to investigating the dependence of the current-voltage characteristic and of the electron gas temperature on the electric field. We can also employ the formulas given in the preceding section to investigate similarly the conductivity tensor σ_{ik}, the conductivity σ, and the fields E_x and E_y in a sample.

4. ON THE RELATIONSHIP BETWEEN NEGATIVE RESISTANCE AND ELECTRON RUNAWAY

The shape of the current-voltage characteristic is closely related to the electron energy distribution. The effective temperature method cannot be used to investigate this relationship; the explicit form of the distribution function is now required. From the kinetic equation for the distribution we obtain[10]

$$f(w) = C \exp\left\{-\int_0^w dw \times \left[1 + \frac{2e^2T}{3m} \frac{w\tau(w)[E^2 + (e\tau(w)H/mc)^2(\mathbf{Eh})^2]}{A(w)[1 + (e\tau(w)H/mc)^2]}\right]^{-1}\right\}, \quad (4.1)$$

where $w = \epsilon/T$, $\mathbf{h} = \mathbf{H}/H$, C is the normalizing constant, and A(w) and $\tau(w)$ are determined from (1.5) and (1.6).

We introduce the concept of "electron runaway" and use the moments M_k of the distribution:

$$M_k = \int_0^\infty w^k f(w)\,dw, \quad k > -1. \tag{4.2}$$

It is easily seen that all macroscopic quantities—the conductivity tensor, the mean energy etc.—can be reduced to this type of integral.

Equation (4.1) shows that the distribution is greatly dependent on the character of electron scattering. We shall follow Levinson[16] in our classification of the scattering mechanisms. A scattering mechanism is called completely restraining if $M_k < \infty$ for all $k > -1$. A mechanism for which $M_k < \infty$ with $-1 < k < k'$ and $M_k = \infty$ with $k \geq k'$ is called partially restraining. The value of k' depends on the external fields and the lattice temperature. The scattering mechanism is called completely nonrestraining if all moments $M_k = \infty$ for all k. It will be shown that this classification does not exhaust all possible cases. Electrons will be designated as "runaway" if the moments M_k diverge beginning with some value k'.

We shall assume that only one scattering mechanism is important in the considered electric and magnetic fields. In high magnetic fields we have $(e\tau H/mc)^2 \gg 1$, and the distribution is given by

$$f(w) = C \exp\left\{-\int_0^w dw\left[1 + \frac{2e^2T}{3m}\left(\frac{E^2m^2c^2}{e^2H^2A_0\tau_0}w^{1-r-q} + \frac{(\mathbf{Eh})^2\tau_0}{A_0}w^{1-r+q}\right)\right]^{-1}\right\}. \tag{4.3}$$

In view of the fact that $E_x \sim H$, for an arbitrary distribution and for the boundary conditions A and B[10] the first term in the square brackets of (4.3) is not, as a general rule, small compared with the second term. We note that the distribution in the absence of a magnetic field is obtained from (4.3) by dropping the first term within the square brackets, and substituting E_z for $\mathbf{E}\cdot\mathbf{h}$. On the other hand, when $\mathbf{E} \perp \mathbf{H}$ only the first term remains within the square brackets.

We shall now consider some special cases.

1. $r + q > 0$, $r - q < 0$. It follows from (4.1) and (4.3) that the scattering mechanism is completely restraining both in the absence and in the presence of a magnetic field. Thus the current-voltage characteristic with positive differential conductivity corresponds to a completely restraining scattering mechanism.

2. $r + q > 0$, $r - q = 0$. For the boundary conditions A and B we obtain different results, depending on whether E and H are, or are not, mutually perpendicular. When $\mathbf{E}\cdot\mathbf{h} \neq 0$, then with $w \gg 1$ the second term in the square brackets of (4.3) is the principal term. The asymptotic form of the distribution for large w is

$$f(w) \sim w^{-\xi}, \qquad \xi = \frac{2e^2T\tau_0}{3mA_0}(\mathbf{Eh})^2. \tag{4.4}$$

For $\mathbf{E}\cdot\mathbf{h} \neq 0$ the given scattering mechanism is obviously partially restraining, with $k' = \xi - 1$. If $\mathbf{E}\cdot\mathbf{h} = 0$ the distribution is

$$f(w) = C \exp\left\{-\int_0^w dw\left[1 + \frac{2Tmc^2E^2}{3H^2A_0\tau_0}w^{1-r-q}\right]^{-1}\right\}. \tag{4.5}$$

It might seem at first glance that we have here a completely restraining scattering mechanism, since the asymptotic distribution for large w is

$$f(w) = C \exp\left\{-\frac{3w^{r+q}H^2A_0\tau_0}{2(r+q)Tmc^2E^2}\right\} \tag{4.5$'$}$$

and all moments of the distribution are finite. In actuality this is not the case. We have for the electric field $E^2 \approx E_x^2$, since it follows from (1.8) and (1.12) that in high magnetic fields $E_x \gg E_y, E_z$.

In high magnetic fields we must therefore replace E^2 by E_x^2 in (4.5). The Hall field E_x^2 will be expressed in terms of the initial parameters E_z and H by means of (1.8) or (1.12).[10] Specifically, let us consider scattering by piezo-acoustic vibrations, for which $r + q = 1$. The distribution is here Maxwellian with the effective temperature

$$u = 1 + \frac{2Tmc^2}{3H^2A_0\tau_0}E_x^2.$$

Using (1.8) or (1.12) in conjunction with (1.19) to express E_x^2 in terms of u and making a substitution in (2.3), we finally obtain

$$u = \left[1 - \frac{3\pi e^2T\tau_0}{8mA_0}E^2\right]^{-1}.$$

The distribution function is now

$$f(w) = C \exp\left\{-\left(1 - \frac{3\pi e^2T\tau_0}{8mA_0}E^2\right)w\right\}. \tag{4.6}$$

All moments of this distribution are finite for $E^2 < 8mA_0/3\pi e^2T\tau_0$ and all moments are infinite for $E^2 \geq 8mA_0/3\pi e^2T\tau_0$. This is a new type of electron runaway, caused by the presence of a magnetic field and the corresponding boundary conditions.

It is easily seen that the distribution (4.5′) obtained on the basis of the hypothesis

$$(2Tmc^2/3H^2A_0\tau_0)E^2w^{1-r-q} \gg 1 \tag{4.7}$$

has no meaning. It can be shown that similar results are obtained for $r + q \neq 1$.

For the boundary conditions C and D a similar examination shows that when $\mathbf{E}\cdot\mathbf{h} \neq 0$ the given scattering mechanism is partially restraining, and that when $\mathbf{E}\cdot\mathbf{h} = 0$ it is completely restraining. It follows from the foregoing that the asymptotic relations $\eta_p \to \infty$ for $\mathscr{E}_p \to 1$ and $\eta_s \to 0$ for $\mathscr{E}_s \to \infty$

are associated with the partial "restraint" of electrons.

3. $r - q < 0$, $r + q > 0$. In this case we proceed as for case 1. For the boundary conditions A, B, and C with $\mathbf{E} \cdot \mathbf{h} \neq 0$ the scattering mechanism is absolutely nonrestraining, while for $\mathbf{E} \cdot \mathbf{h} = 0$ it is absolutely restraining.

4. $r - q < 0$, $r + q > 0$. When $\mathbf{E} \cdot \mathbf{h} \neq 0$ the scattering mechanism is nonrestraining, while for $\mathbf{E} \cdot \mathbf{h} = 0$ it is partially restraining.

We note that in cases 3 and 4, as has already been shown, negative resistance is observed, in association with the condition $r - q < 0$. On the other hand, this relation causes electron runaway when $\mathbf{E} \cdot \mathbf{h} \neq 0$.

Therefore, when total electron runaway occurs a semiconductor can have a current-voltage characteristic, including negative resistance, which is either N-shaped or S-shaped. The latter occurs only in the presence of a magnetic field and in the absence of a Hall field. These conditions can be realized for a selected semiconductor with suitable boundary conditions.

The physical reason for the negative resistance is the reduced probability that electrons will be scattered as their energy increases in the case of either momentum transfer or energy transfer. In the first case, although the electron gas temperature increases with the energy, the conductivity diminishes as the temperature rises. In the second case the temperature decreases as the field grows, whereas the conductivity increases with the temperature. If the probability of electron scattering accompanied by energy loss or momentum loss decreases sufficiently rapidly as the energy rises, the current also decreases as the field increases.

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[1] A. V. Gurevich, JETP **36**, 624 (1959), Soviet Phys. JETP **9**, 434 (1959).

[2] J. Bok, Ann. radioelec. **79**, 120 (1960).

[3] I. Adawi, J. Appl. Phys. **32**, 1101 (1961).

[4] Sh. M. Kogan, FTT **4**, 2474 (1962), Soviet Phys. Solid State **4**, 1813 (1962).

[5] R. F. Kazarinov and V. G. Skobov, JETP **42**, 1047 (1962), Soviet Phys. JETP **15**, 726 (1962).

[6] R. F. Kazarinov and V. G. Skobov, JETP **44**, 1368 (1963), Soviet Phys. JETP **17**, 921 (1963).

[7] L. E. Gurevich and I. Ya. Korenblit, JETP **44**, 2150 (1963), Soviet Phys. **17**, 1444 (1963).

[8] B. I. Davydov, JETP **7**, 1069 (1937).

[9] F. G. Bass, FMM **3**, 550 (1956).

[10] F. G. Bass, FMM **6**, 961 (1958).

[11] H. Fröhlich and B. V. Paranjape, Proc. Phys. Soc. (London) **B69**, 21 (1956).

[12] V. D. Ginzburg, Rasprostranenie elektromagnitnykh voln v plazme (Electromagnetic Wave Propagation in Plasmas), Fizmatgiz, 1960.

[13] V. L. Ginzburg and A. V. Gurevich, UFN **70**, 201 (1960), Soviet Phys. Uspekhi **3**, 115 (1960).

[14] F. G. Bass and I. M. Tsidil'kovskiĭ, ZhTF **24**, 1834 (1954).

[15] F. G. Bass, JETP **32**, 863 (1957), Soviet Phys. JETP **5**, 705 (1957).

[16] I. B. Levinson, FTT **6**, 2113 (1964), Soviet Phys. Solid State **6**, 1665 (1965).

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