ROLE OF ELECTRON SCATTERING BY PHONONS AND MAGNETIC INHOMOGENEITIES IN THE HALL EFFECT IN FERROMAGNETIC METALS

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A phenomenological theory of the Hall effect is proposed for ferromagnetic metals in fields corresponding to the initial region of magnetic saturation. Formulas are derived for the Hall constant as a function of electric resistivity and magnetization in a magnetic field. The anomalous Hall constant is expressed as the sum of the constants R_p and R_m which characterizes the parts of the anomalous Hall field corresponding to electron scattering by phonons and by magnetic inhomogeneities. A possible method is indicated for using experimental data to separate R_p and R_m. The values obtained in this way for nickel are commensurable in absolute magnitude but have opposite signs: $R_p < 0$ and $R_m > 0$, with $|R_p| > R_m$.

SEVERAL papers[1-4] concerning the theory of the Hall effect have discussed the causes of the anomalous part of the Hall field, which is proportional to the magnetization, in ferromagnetic metals. From theoretical evaluations of different contributions to this anomalous field it was concluded that above the Debye temperature Θ_D the largest contribution results from electron scattering by phonons and magnetic inhomogeneities. This Hall field can therefore be represented approximately as a sum:

$$E_a = E_p + E_m = (R_p + R_m) \cdot 4\pi I j = R_a \cdot 4\pi I j, \quad (1)$$

where E_p and E_m are the parts of the anomalous Hall field associated with electron scattering by phonons and by magnetic inhomogeneities, respectively; R_p and R_m are the corresponding parts of the Hall constant R_a. It follows from the theory that $R_p \sim \rho^2$ and $R_m \sim \rho_m$, where ρ is the total electric resistivity and ρ_m is the part associated with electron scattering by magnetic inhomogeneities.[1)]

The values of R_p and R_m cannot, unfortunately, be discriminated by means of the experimental temperature dependences of R_a, ρ, and ρ_m. As the temperature is varied from Θ_D to the Curie point Θ_C, the two quantities ρ^2 and ρ_m vary in an approximately identical manner, whereas R_a contains, in addition to R_p and R_m, a part R_{p0} which is associated with electron scattering on phonons and on impurities, and which, as calculations have shown,[6-8] is also temperature dependent. On the other hand, R_p and R_m can be separated by using the experimental data regarding the dependence of the Hall field on true magnetization and temperature in the initial region of magnetic saturation. In magnetic fields of the order $10^3 - 10^4$ Oe one observes a decrease of the resistivity ρ of ferromagnetic metals as the field increases, instead of the usual increase in high magnetic fields. This reduction is associated by the contemporary theory with the reduction of ρ_m due to the decrease of magnetic inhomogeneities as the magnetic field grows. In the same initial region of magnetic saturation we also observe Hall field changes which, as will be shown, can be accounted for by the dependence of ρ_m and R_m on the magnetic field.

In the present work a phenomenological theory is proposed for the anomalous Hall field in the initial magnetic saturation region at temperatures $T > \Theta_D$, and the theory is compared with experiment. It is found that in nickel R_p and R_m are commensurable but differ in sign, with $|R_p| > |R_m|$.

THEORY OF THE ANOMALOUS HALL FIELD IN THE INITIAL MAGNETIC SATURATION REGION

The magnetic saturation of ferromagnetic metals which accompanies the orientation of mag-

1)The temperature dependence of ρ can usually be used to determine ρ_m by means of linear extrapolation from the paramagnetic to the ferromagnetic region. However, this is a very crude procedure; considerably more reliable results are obtained when the method of Volkov and Kozlova is used to determine ρ_m.[5]

netic moments of domains parallel to the magnetic field takes place in fields for which, when $T > \Theta_D$, we have $\Omega\tau \ll 1$, where Ω is the cyclotron resonance frequency in the field $B = H + 4\pi I$ and τ is the relaxation time. Therefore, in the initial region of magnetic saturation the normal Hall field E_H associated with the displacement of electrons by the Lorentz force is approximately proportional to the magnetic induction B. In the given region the anomalous Hall field is proportional to the spontaneous magnetization. Then, for the current density $j = 1$ the Hall field is[2)]

$$E = R_a \cdot 4\pi I + R_H(H + 4\pi kI), \tag{2}$$

where R_H is the field-dependent Hall constant and $k \approx 1$. In the linear approximation with respect to the field we have

$$I = I_s + \varkappa_p H, \qquad R_a = R_s + \left(\frac{\partial R_a}{\partial H}\right)_T H,$$
$$R_s = \lim_{H\to 0}\frac{E}{4\pi I} - R_H, \tag{3}$$

where $\kappa_p = (\partial I/\partial H)_T$ is the differential magnetic susceptibility for true magnetization (paraprocess susceptibility). For the differential constant R_d characterizing the variable part of the anomalous Hall field in the initial magnetic saturation region, (2) and (3) yield[3)]

$$R_d = \frac{\partial}{\partial I}(R_a I) = R_s + \left(\frac{\partial R_a}{\partial H}\right)_T H + \left(\frac{\partial R_a}{\partial H}\right)_T \frac{I}{\varkappa_p}$$
$$= \frac{1}{4\pi\varkappa_p}\left[\left(\frac{\partial E}{\partial H}\right)_T - R_H(1 + 4\pi k\varkappa_p)\right], \tag{4}$$

whence

$$\left(\frac{\partial R_a}{\partial H}\right)_T = \frac{\varkappa_p}{I}\left(1 + \frac{\varkappa_p H}{I}\right)^{-1}(R_d - R_s). \tag{5}$$

In order to relate $(\partial R_a/\partial H)_T$ to the resistivity we represent R_a, as previously, by a sum:

$$R_a = R_p + R_m; \qquad R_p = c_p\rho^2, \qquad R_m = c_m\rho_m. \tag{6}$$

Then

$$\left(\frac{\partial R_a}{\partial H}\right)_T = \left(2\frac{R_p}{\rho} + c\frac{R_m}{\rho_m}\right)\left(\frac{\partial\rho}{\partial H}\right)_T, \tag{7a}$$

where

$$c = \left(\frac{\partial\rho_m}{\partial H}\right)_T \Big/ \left(\frac{\partial\rho}{\partial H}\right)_T. \tag{7b}$$

[2)]$H + 4\pi kI$ is the mean strength of the magnetic field acting on the conduction electrons.

[3)]Equations (4) and (5) hold true for $I_s \gg \kappa_p H$ or for $I_s = 0$. For $I_s \approx \kappa_p H$ we must introduce a term containing the second derivative of R_a with respect to the field.

It was shown above that $(\partial\rho/\partial H)_T < 0$ for ferromagnetic metals in the initial magnetic saturation region; in most other metals $(\partial\rho/\partial H)_T > 0$ in accordance with the theory. On the other hand, higher magnetic fields are accompanied by a reduced number of magnetic inhomogeneities; consequently, ρ_m diminishes, i.e., $(\partial\rho_m/\partial H)_T < 0$. It then follows from (7b) that $c > 0$. It can reasonably be assumed that the diminution of magnetic inhomogeneity is the principal cause of smaller ρ as the magnetic field is increased, i.e., that

$$|\partial\rho_m / \partial H|_T > |\partial(\rho - \rho_m) / \partial H|_T. \tag{7c}$$

In this case c is close to unity or equals a few times unity, because there is practically no possibility that $|\partial\rho_m/\partial H|_T$ and $|\partial(\rho - \rho_m)/\partial H|_T$ will agree to the second significant figure.

Table I

No. of variant	R_p	R_m	$\lvert R_p\rvert - \lvert R_m\rvert$	$2\frac{\lvert R_p\rvert}{\rho} - c\frac{\lvert R_m\rvert}{\rho_m}$	R_a	$\left(\frac{\partial R_a}{\partial H}\right)_T$
1	+	+	±	±	+	−
2	−	−	±	±	−	+
3	+	−	+	+	+	−
4	+	−	+	−	+	+
5	+	−	−	−	−	+
6	−	+	+	+	−	+
7	−	+	+	−	−	−
8	−	+	−	−	+	−

Table I gives all possible combinations of signs for the differences between the absolute values of R_p and R_m and the signs of $(\partial R_a/\partial H)_T$, assuming $\rho - \rho_m > \rho_m$, $(\partial\rho/\partial H)_T < 0$, and $c > 0$. It is seen that the signs of R_a and $(\partial R_a/\partial H)_T$ are both positive or both negative in only two of the eight variants. When this agreement of signs appears experimentally, we can determine uniquely the signs of R_p and R_m and the ratio of their absolute values (see the note at the end of this article).

The value of the derivative $(\partial\rho_m/\partial H)_T$ in (7a) and (7b) can be related to I. It follows from both experiment [5] and theory [9] that

$$\rho_m = \alpha(I_0^2 - I^2), \tag{8}$$

where I_0 is the spontaneous magnetization at $T = 0$, and α is a coefficient which is independent of T and H. From (8) we obtain

$$(\partial\rho / \partial H)_T = c^{-1}(\partial\rho_m / \partial H)_T = -2\alpha c^{-1}\varkappa_p I. \tag{9}$$

Substituting (9) into (7a) and using (6), we obtain

$$(\partial R_a / \partial H)_T = -2\alpha c_m \varkappa_p I(1 + 2\rho_m R_p / \rho c R_m). \tag{10}$$

In the special case when R_p and R_m are of the same order, $\rho_m \ll \rho$, and $\kappa_p H \ll I$, Eqs. (5) and (10) yield the relation

$$R_d - R_s \approx -2\alpha c_m I_s^2. \quad (11)$$

The derived equations are also valid for the paramagnetic region in fields where $\Omega\tau \ll 1$ is fulfilled. Assuming $\kappa_p = \kappa$ and $I = \kappa H$ in (4) and (10), we obtain

$$R_a = R_s + \left(\frac{\partial R_a}{\partial H}\right)_T H = R_s - 2\alpha c_m \varkappa^2 H^2 \left(1 + 2\frac{\rho_m R_p}{\rho c R_m}\right), \quad (12)$$

$$R_d = R_s + 2\left(\frac{\partial R_a}{\partial H}\right)_T H = R_s - 4\alpha c_m \varkappa^2 H^2 \left(1 + 2\frac{\rho_m R_p}{\rho c R_m}\right), \quad (13)$$

$$(\partial E / \partial H)_T = (1 + 4\pi k \varkappa) R_H + 4\pi\varkappa R_d. \quad (14)$$

Neglecting the terms containing κ^2 in (12) and (13), for the paramagnetic region we obtain

$$R_a \approx R_d \approx R_s. \quad (15)$$

The second term in (14) characterizes the anomalous part of the Hall field. The existence of this term in the paramagnetic region was first established by Kikoin.[10] It was shown in [10,11] that $(\partial E/\partial H)_T$ and $1/(T - \Theta_C) \sim \kappa$ are related linearly above the Curie temperature Θ_C.

SIGNS AND RATIO OF THE ABSOLUTE VALUES OF R_p AND R_m IN NICKEL

It follows from the foregoing theory that when R_a and $(\partial R_a/\partial H)_T$ agree in sign, the signs and absolute value ratio of R_p and R_m can be determined uniquely from (1) and (6). The sign of R_a can be determined directly from experiment. In order to determine the sign of $(\partial R_a/\partial H)_T$ the quantities in (4) and (5) must be known as a basis for determining $(\partial E/\partial H)_T$, I_s, and κ_p from experimental results at different values of T.

The determination of the field-dependent Hall constant R_H is a more complicated problem. In many articles this constant for ferromagnetic materials has been taken to equal $(\partial E/\partial B)_T$ in the magnetic saturation region and has been determined from the slopes of E(B) curves relative to the B axis; in high magnetic fields these slopes become straight lines. However, (4) shows that the approximate equality $R_H \approx (\partial E/\partial B)_T$ exists only subject to the condition that

$$\varkappa_p^{-1} |\partial E / \partial H|_T = |\partial E / \partial I|_T \gg 4\pi |R_d|.$$

Volkov's experimental results[12] show that for several ferromagnetic alloys the last inequality is not fulfilled, since $(\partial E/\partial I)_T$ and $4\pi R_a$ are commensurable in these alloys. The indicated procedure for determining R_H is therefore unreliable and can apparently be employed only at relatively low temperatures where R_a and R_d are small and $\Omega\tau \ll 1$ still holds true.

Kevane and Legvold[13] have proposed a procedure for determining R_H by the linear extrapolation of $(\partial E/\partial H)_T$ measured by temperatures $T > \Theta_C$ in the region of zero magnetic susceptibility κ. The correctness of this procedure, despite the fact that it was based on an approximate equation, can be shown by means of (14), from which it follows that in the paramagnetic region

$$\lim_{\varkappa \to 0} (\partial E/\partial H)_T = R_H.$$

It is reasonable to assume, furthermore, that at temperatures not very far from the Curie point, R_H in the ferromagnetic region has values like those found when $T > \Theta_C$.

For the purpose of calculating $(\partial R_a/\partial H)_T$ we used the value $R_H = 4.5 \times 10^{-13}$ V-cm/A-Oe obtained for nickel by Volkov and Kozlova using the method described in [13]. Values of $(\partial E/\partial H)_T$ and I for nickel at different temperatures from 550 to 610°K have been measured by Cheremushkina [see [8]], to whom the author is greatly indebted for making the additional data available. A calculation of $(\partial R_a/\partial H)_T$ from (5) yielded values in the given temperature range for nickel from -0.1×10^{-15} to -0.7×10^{-15} V-cm/A-G-Oe, which therefore has the same sign as R_a in this interval. We note that $(\partial E/\partial H)_T \gg R_H$ and $4\pi k\kappa_p < 1$ for nickel at the given temperatures. In this case (4) and (5) show that the sign of $(\partial R_a/\partial H)_T$ is the same as that of $(\partial E/\partial H)_T - 4\pi\kappa_p R_s$. We have thus practically excluded the possibility that an erroneous sign would be obtained for this difference, whose absolute value is commensurable with the absolute values of its two terms.

It can be seen from Table I that the signs of R_a and $(\partial R_a/\partial H)_T$ will both be negative only if $R_p < 0$, $R_m > 0$, and $|R_p| > |R_m|$. It follows from $|R_p| > |R_m|$ that R_p and R_a are of the same order of magnitude. Assuming $R_p \approx R_a$ and $c \approx 1$ in (7) and taking the signs of R_p and R_m into account, we obtain for nickel:

$$R_m \approx \frac{\rho_m}{\rho}\left[\left(\frac{\partial R_a}{\partial H}\right)_T \Big/ \frac{1}{\rho}\left(\frac{\partial \rho}{\partial H}\right)_T + 2|R_a|\right]. \quad (16)$$

Since all quantities in the right-hand side of (16) are measurable, the order of magnitude of R_m can be assessed.

Table II

Majority carriers of anomalous Hall current	Sign of $\overline{\Delta M_z}$	Sign of R_p	Sign of R_m	Majority carriers of anomalous Hall current	Sign of $\overline{\Delta M_z}$	Sign of R_p	Sign of R_m
Electrons	+	+	—	Holes	+	—	+
Electrons	—	—	—	Holes	—	+	+

The foregoing values of $(\partial R_a/\partial H)_T$ for nickel, calculated from experimental data by means of (5), show that in the given temperature interval 550—600°K we have $(\partial R_a/\partial H)_T \approx -0.3 \times 10^{-15}$ V-cm/A-G-Oe. In the same temperature interval, $R_a \approx -3 \times 10^{-11}$ V-cm/A-G[8] and $\rho_m \approx -2.5 \times 10^5\, d\rho_m/dH$ for $\rho \approx 25 \times 10^{-6}\ \Omega$-cm.[4)] In the temperature range of present interest $\rho^{-1}(\partial\rho/\partial H)_T \approx -0.8 \times 10^{-6}$ Oe^{-1} according to measurements for nickel by Galkina.[14] The cited experimental data show that

$$\left(\frac{\partial R_a}{\partial H}\right)_T \Big/ \frac{1}{\rho}\left(\frac{\partial \rho}{\partial H}\right)_T \gg R_a,$$

from which it follows that $(\partial R_a/\partial H)_T \approx (\partial R_m/\partial H)_T$ in the given temperature interval. Using the foregoing estimates of $(\partial R_a/\partial H)_T$, $\rho^{-1}(\partial\rho/\partial H)_T$, and ρ_m, we obtain from (16):

$$R_m \approx \frac{\rho_m}{\rho}\left(\frac{\partial R_a}{\partial H}\right)_T \Big/ \frac{1}{\rho}\left(\frac{\partial \rho}{\partial H}\right)_T \approx 10^{-10}\frac{\text{V}\cdot\text{cm}}{\text{A}\cdot\text{G}}. \quad (17)$$

Thus, R_a, R_p, and R_m are of the same order of magnitude.

MAJORITY CARRIERS OF THE ANOMALOUS HALL CURRENT IN NICKEL

The conditions determining the signs of R_p and R_m have been considered in several articles.[7,8,4] We have shown in [7] that the sign of R_p depends on the following factors: 1) the sign of the effective mass of majority anomalous Hall current carriers, i.e., whether the carriers are electrons or holes; 2) the parameter $\overline{\Delta M_z}$ [see Eqs. (8) and (10) in [7]], characterizing the degree of localization of the principal carriers of the magnetic moment in the ferromagnetic state (for $\overline{\Delta M_z} > 0$ the principal carriers of the magnetic moment are unlocalized electrons participating in conduction, and for $\overline{\Delta M_z} < 0$ they are localized electrons bound to ions). It was shown in [4] that the sign of R_m depends on the character of the majority carriers of the anomalous Hall current.

[4)]The coefficient 2.5×10^5 was calculated using (8), into which, after differentiating with respect with H, experimental values of κ_p and I (from the data of Weiss and Forrer) were substituted.

Table II gives the signs of R_p and R_m, derived from the theory presented in [7] and [4], for different cases. It is seen that, according to this theory, for nickel with $R_p < 0$ and $R_m > 0$ the principal carriers of the magnetic moment are unlocalized electrons ($\overline{\Delta M_z} > 0$), which, since they possess negative effective mass, are the majority carriers of the anomalous Hall (hole) current. We note that the derived positive sign of $\overline{\Delta M_z}$ for nickel was also derived by the present author and Vasil'eva in [15], where results obtained in an investigation of the Nernst-Ettingshausen effect are discussed.

The conclusion that the anomalous Hall current is carried by holes is also arrived at by reasoning based on the derivation of a positive sign for $\overline{\Delta M_z}$ from an investigation of the Nernst-Ettingshausen effect. Indeed, since for nickel $|R_p| > |R_m|$, it follows immediately that R_p and R_a have the same sign, i.e., $R_p < 0$; also, from $\overline{\Delta M_z} > 0$ and $R_p < 0$ it follows that the anomalous Hall current is a hole current. This conclusion was reached by the present author in collaboration with Cheremushkina and Kurbaniyazov[8] on the basis of the assumption that the principal role in the anomalous Hall field belongs to electron scattering by phonons and impurities, i.e., assuming $|R_p| > |R_m|$; the scattering of electrons by magnetic inhomogeneities was not considered in [8].

Note added October 20, 1964. Unlike our earlier articles[6-8] where it was shown that $R_p \sim \rho^2$, or Luttinger's article,[1] which also leads to the indicated proportionality, Gurevich and Yassievich point out in their first article[2] the possibility of a different proportionality, $R_p \sim |\rho - \rho_m|$. In the latter case, instead of (7a) we would obtain a similar formula with a different coefficient of R_p:

$$\left(\frac{\partial R_a}{\partial H}\right)_T = \frac{2R_p}{\rho - \rho_m}\left[\frac{\partial(\rho - \rho_m)}{\partial H}\right]_T + \frac{R_m}{\rho_m}\left(\frac{\partial \rho_m}{\partial H}\right)_T. \quad (7c)$$

Since in the considered range of fields $\rho - \rho_m$ and $R_p \sim (\rho - \rho_m)^2$ are slightly dependent on the field, the first term in the right-hand side of (7c) is always smaller in absolute value than the second term, which thus determines the sign of $(\partial R_a/\partial H)_T$. Since $(\partial\rho_m/\partial H)_T < 0$, the sign of

$(\partial R_a/\partial H)_T$ in the given case is opposite to that of R_m. Keeping this in mind and considering all possible combinations of signs and absolute values of R_p, R_m, and $(\partial R_a/\partial H)_T$, we arrive at the conclusion that in the present case we obtain no new variants different from those listed in Table I, but that the third and sixth variants will now be excluded. Thus our subsequent conclusions regarding the signs and absolute values of R_p and R_m will be true independently of whether $R_p \sim \rho^2$ or $R_p \sim (\rho - \rho_m)^2$ is assumed.

The absolute value of the first term in the right-hand side of (7c) would exceed that of the second term upon fulfillment of the condition

$$|R_p| > \frac{\rho - \rho_m}{2\rho_m}\left(\left|\frac{\partial \rho_m}{\partial H}\right|_T \Big/ \left|\frac{\partial(\rho - \rho_m)}{\partial H}\right|_T\right)|R_m|, \qquad (18)$$

from which with $\rho - \rho_m > \rho_m$ and

$$|\partial\rho_m/\partial H|_T > |\partial(\rho - \rho_m)/\partial H|_T$$

we would have

$$|R_m| < |R_p| \approx |R_a|$$

(the symbol $\approx$ being used henceforth to indicate equality in order of magnitude). Then, remembering (7c) and (18) and excluding the unlikely cases of identical absolute values of the first and second terms in the right-hand side of (7c) or the agreement of $|\partial\rho_m/\partial H|_T$ and $|\partial(\rho - \rho_m)/\partial H|_T$ to the second significant figure, we would conclude that a second condition is required:

$$\left|\frac{\partial(\rho - \rho_m)}{\partial H}\right|_T \approx \frac{\rho}{|R_a|}\left|\frac{\partial R_a}{\partial H}\right|_T$$

with

$$\left|\frac{\partial(\rho - \rho_m)}{\partial H}\right|_T \lessapprox \left|\frac{\partial\rho}{\partial H}\right|_T.$$

The experimental data for nickel (already given), gadolinium, and iron-nickel alloys when $T > \Theta$ show that in these cases

$$\left|\frac{\partial\rho}{\partial H}\right|_T \ll \frac{\rho}{|R_a|}\left|\frac{\partial R_a}{\partial H}\right|_T,$$

i.e., the aforesaid second condition, to the effect that the first term in the right-hand side of (7c) is greater in absolute value than the second term, is clearly not fulfilled.

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