EFFECT OF THE FINITE SIZE OF THE NUCLEUS ON THE INTENSITY OF CHARACTERISTIC X-RAYS

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An expression is derived for the probability of a relativistic radiative electric multipole transition. Oscillator strengths for the $K \to L_{II}$ and $K \to L_{III}$ transitions in heavy elements and the relative intensity of the K_{α_2} line in the characteristic x-radiation are calculated. The effect of the finite size of the nucleus on the oscillator strength and relative intensity of the $K \to L_{II}$ transition is taken into account.

In earlier papers the author derived an expression for the probability of multipole radiative transitions in the relativistic analysis, with[1] and without[2] account of delay. In deriving the formula for the probability of electric multipole radiation, use was made of the approximate expression of Akhiezer and Berestetskiĭ[3] for the photon potential, which is valid for small r. If we forego this approximation, then the potential of the proton of the electric type, with momentum L and projection M, can be written in the form

$$\hat{A}_{LM} = i^{L-1}\left(\frac{\omega}{R}\frac{L}{L+1}\right)^{1/2}\left\{\gamma\frac{j_{L-1}(\omega r)}{L}\times\left(r\nabla + L\frac{\mathbf{r}}{r}\right)Y_{LM} + \beta j_L(\omega r)\,Y_{LM}\right\}. \qquad (1)$$

Formula (1) can be readily obtained from the exact expression of Akhiezer and Berestetskiĭ with the usual gauge

$$C = -(L/(L+1))^{1/2}, \qquad (2)$$

if we introduce the spherical Bessel function $j_L(\omega r)$ and use the well known formulas for expansions in terms of spherical vectors for $r\nabla Y_{LM}$ and $\mathbf{r}Y_{LM}$.

We choose as wave functions for the initial and final states the relativistic Dirac functions. By using standard quantum-mechanical calculation procedures we obtain the following expression for the probability of relativistic radiative electric multipole transition (in atomic units):

$$W(EL)_{1\to 2} = e^2\omega\frac{2L(2L+1)(2j_2+1)(2l_1+1)}{(L+1)} \times C^2(l_1Ll_2;\ 00)\, W^2\left(l_1j_1l_2j_2;\ \frac{1}{2}L\right) \times\left|R_1+R_2+R_3-R_4+\frac{\varkappa_1-\varkappa_2}{L}(R_3+R_4)\right|^2, \qquad (3)$$

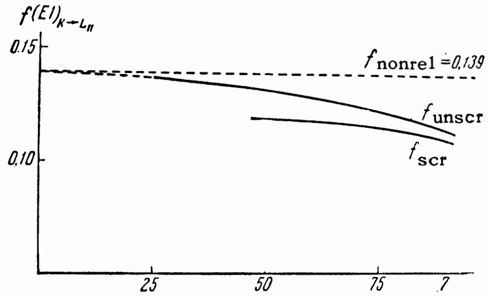

FIG. 1

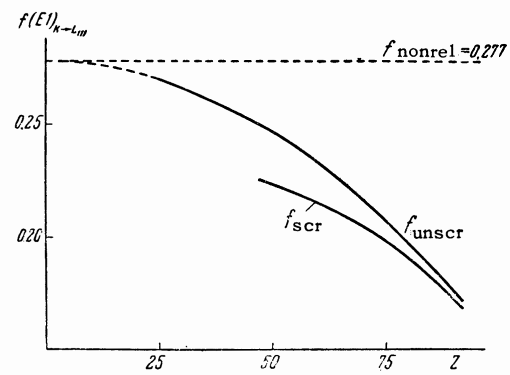

FIG. 2

where the radial integrals are

$$R_1 = \int_0^\infty g_2 j_L(\omega r) g_1 r^2\,dr, \qquad R_2 = \int_0^\infty f_2 j_L(\omega r) f_1 r^2\,dr,$$

$$R_3 = \int_0^\infty g_2 j_{L-1}(\omega r) f_1 r^2\,dr, \qquad R_4 = \int_0^\infty f_2 j_{L-1}(\omega r) g_1 r^2\,dr. \qquad (4)$$

Formula (3) was used to calculate the oscillator strengths and x-ray transition intensities. The oscillator strengths of the K-series lines are

$$f_{1s\to np_{1/2}} = \frac{1}{2}\frac{mc^2}{\hbar\omega}|R_1+R_2-R_3-3R_4|^2,$$

$$f_{1s\to np_{3/2}} = \frac{mc^2}{\hbar\omega}|R_1+R_2+2R_3|^2. \qquad (5)$$

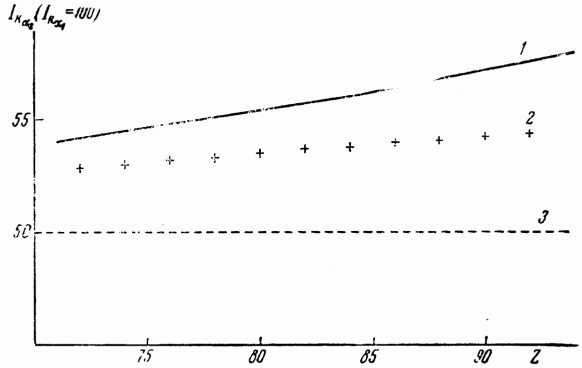

FIG. 3. Relative intensity of K_{α_2} line: curve 1 – theoretical (allowance for screening after Slater leads to the same values of the relative intensity of the K_{α_2}, as without account of screening, so that this curve pertains to both cases); 2 – values of the smooth curve plotted by Wapstra et al.[4] on the basis of experimental data (recalculated in terms of the ratio of the energy fluxes); curve 3 – nonrelativistic value of the relative intensity of the K_{α_2} line.

The oscillator strengths were calculated with and without allowance for screening. The screening was taken into account by the Slater rule. The results of calculations are shown in Figs. 1 and 2. For comparison, the figures show the nonrelativistic values of the oscillator strengths, which are constant for all the elements. From these diagrams we see that the relativistic values of the oscillator strengths decrease with increasing Z and, furthermore, the influence of screening also decreases with increasing Z.

Using the obtained values of the oscillator strengths, we calculated the relative intensity of the K_{α_2} line:

$$\frac{I_{\alpha_2}}{I_{\alpha_1}} = \left(\frac{\omega_{\alpha_2}}{\omega_{\alpha_1}}\right)^3 \frac{f_{\alpha_2}}{f_{\alpha_1}}. \qquad (6)$$

The intensity of the K_{α_1} line was assumed, as usual, to be 100. The calculated relative intensity of the K_{α_2} line is plotted in Fig. 3. In the calculations we used the experimental values of the characteristic-radiation frequencies ω (see [5]). The same Fig. 3 shows the smooth curve plotted by Wapstra, Nijgh, and Lieshout[4] on the basis of experimental data. It must be noted that these data pertain to the relative intensity of the quanta and that for comparison with the energy flux ratio they must be multiplied by the frequency ratio. We see that the theoretical and experimental data differ somewhat from each other. This discrepancy increases with increasing Z.

What is the role played in the manifestation of this discrepancy by the fact that we do not take into account the finite dimensions of the nucleus in our calculations? The finite dimensions of the nucleus influence the probabilities of those transitions, which are expressed in terms of wave functions that differ from zero near the nucleus. The functions that are singular at the origin are the relativistic Dirac functions for the states $ns_{1/2}$ and $np_{1/2}$. Consequently, the finite dimensions of the nucleus will be manifest primarily in the probability of the transition $1s_{1/2} \rightarrow 2p_{1/2}$ (compared with $1s_{1/2} \rightarrow 2p_{3/2}$).

The wave functions with account of the finite dimensions of the nucleus, for the external ($r \geq R_{nuc}$) and internal ($r \leq R_{nuc}$) regions were obtained by the author earlier[6].

Allowance for the finite dimensions of the nucleus leads to the calculation of integrals over the volume of the nucleus and the adjacent space, in which

$$j_1(\omega r) \ll j_0(\omega r). \qquad (7)$$

This also causes the influence of the finite dimensions of the nucleus to be manifest primarily in the calculation of the integrals R_3 and R_4. The latter can be expressed in the form of sums of five integrals over the region inside and outside the nucleus, in complete analogy with the procedure used by Sliv[7].

The results of calculations for $_{92}U$ are given in the table. The calculations show that allowance for the finite dimensions of the nucleus changes very insignificantly the intensity of the x-ray lines. We note that at the present experimental accuracy we cannot obtain any additional information concerning the dimensions of the nucleus and the charge distribution in the nucleus, since these data are contained in integrals taken over the volume of the nucleus, which are vanishingly small, and the corrections introduced are connected with integrals over a region outside the nucleus, in which account is taken of the difference between the wave functions and the Coulomb functions at a certain distance from the nucleus.

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	Oscillator strength $f_{1s\rightarrow 2p_{1/2}}$	Intensity I_{α_2}, %
Without account of finite dimensions of the nucleus	0.109075	57.63
With account of finite dimensions of the nucleus	0.108658	57.41

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Translated by J. G. Adashko
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