PROPAGATION OF ELECTROMAGNETIC WAVES IN A METAL WITH ALLOWANCE FOR "FERMI-LIQUID" INTERACTIONS

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The energy spectrum and damping of the electromagnetic excitations in a metal in a strong magnetic field are calculated taking into account the Fermi-liquid interactions. In some cases investigation of weakly damped waves allows one to determine the value of the correlation function for the Fermi liquid formed by the electrons in a metal.

1. We know that the electrons in a metal form a Fermi liquid in the sense discussed by Landau [1]. The Fermi-liquid properties of a metal may show up, in particular, in various effects connected with the penetration of an electromagnetic field into the interior. The effect of the Fermi liquid properties on the impedance in the infrared region has been considered by Silin [2] and Pitaevskiĭ [3], while Gor'kov and Dzyaloshinskiĭ [4] have investigated the possibility of exciting zero sound in a metal by a high-frequency field. Conditions are especially favorable for the investigation of Fermi-liquid effects in metals in the presence of a magnetic field, thanks to the existence of various resonance effects. As shown by Azbel', [5] the cyclotron resonance region is well suited to this purpose.

Obviously investigations of weakly damped electromagnetic waves in metals, which have been developed intensively in recent years[1)], can provide a source of extra information about the energy spectrum and interaction of the current carriers. In this connection it is of some interest to consider the effect of Fermi-liquid correlations on the propagation of weakly damped waves in a metal. This will be the subject of this paper.

2. As shown by a number of authors (see the references in [6]), in metals with unequal carrier concentrations in a strong magnetic field there may be propagated weakly damped electromagnetic waves with a quadratic dispersion law (helicon waves). In a metal with equal carrier concentrations magnetohydrodynamic waves are possible. The dispersion and damping of these waves are sensitive to the topology and form of the energy surface and also depend strongly on the orientation of the field relative to the crystal axes of symmetry [7].

For the existence of these types of excitation it is necessary that the radius of the orbit of an electron in the magnetic field be small compared to the wavelength of the electromagnetic wave and the effective mean free path,

$$kR \ll 1, \qquad R \ll l^*. \qquad (1)$$

Here $\mathbf{k}$ is the wave vector, $R = cp_F/eH$ is the orbit of the carrier in a magnetic field, where p_F is the Fermi momentum and H the magnitude of the constant magnetic field; $l^* = v/|\nu - i\omega|$ is the effective mean free path, where v is a characteristic velocity of an electron on the Fermi surface, ν is the collision frequency and ω the frequency of the alternating field. Note that (1) presupposes the inequality

$$\Omega \gg |\nu - i\omega|, kv.$$

The dispersion and damping of the characteristic vibrations of the electromagnetic field may be obtained from the homogeneous system formed by Maxwell's equations, in which the relation between the current and the field is given by

$$j_i = \sigma_{ik} E_k. \qquad (2)$$

To find the electrical conductivity tensor $\sigma_{ik}(\omega, \mathbf{k})$ taking temporal and spatial dispersion into account, we must solve the kinetic equation

$$dn / dt + \hat{J}(n) = 0, \qquad (3)$$

where $\hat{J}(n)$ is the collision operator.

We write the deviation of the distribution function from its equilibrium value in the form

[1)]References to experimental and theoretical work on electromagnetic waves in metals can be found in the article by Kaner and Skobov[6].

$$n - n_0(\varepsilon_0) \equiv \delta n = -\frac{\partial n_0}{\partial \varepsilon} n_1, \quad n_0 = \left[\exp\frac{\varepsilon_0 - \mu}{T} + 1\right]^{-1}, \tag{4}$$

where n_0 is the equilibrium Fermi distribution function. We can also write the energy of the quasi-particles in an analogous form:

$$\varepsilon = \varepsilon_0(\mathbf{p}) + \delta\varepsilon(\mathbf{r}, \mathbf{p}, t). \tag{5}$$

In formulae (4) and (5), the function $\epsilon_0(\mathbf{p})$ corresponds to the equilibrium state of the electronic Fermi liquid with the distribution function $n_0(\mathbf{p})$. According to Landau [1] the quasi-particle energy is in general a complex functional of n, while $\delta\epsilon$ is related to δn by the formula

$$\delta\varepsilon(\mathbf{p}) = \int \Phi(\mathbf{p}, \mathbf{p}')\, \delta n(\mathbf{p}')\, d\mathbf{p}' \equiv \hat{\Phi} n_1. \tag{6}$$

In Fermi-liquid theory the collision integral $\hat{J}(n)$ has the same form as the corresponding expression in the gas model, except that it refers to the combination $\delta n - \delta\epsilon(\partial n_0/\partial\epsilon)$. [2]

The collision integral vanishes for $n = n_0(\epsilon)$. Linearizing the kinetic equation (3) relative to $n - n_0(\epsilon) = -\chi(\partial n_0/\partial\epsilon)$ and setting

$$n_1 = (1 + \hat{\Phi})^{-1}\chi, \qquad \hat{\Phi}(1 + \hat{\Phi})^{-1} = \hat{G}, \tag{7}$$

we write (3) in the form [5]

$$(1 - \hat{G})\frac{\partial\chi}{\partial t} + \mathbf{v}\frac{\partial\chi}{\partial\mathbf{r}} + \Omega\frac{\partial\chi}{\partial\tau} + \nu\chi = e\mathbf{v}\mathbf{E}, \tag{8}$$

where τ is the dimensionless orbital revolution time of an electron and $\Omega = eH/mc$ is the cyclotron frequency.

It is evident from (8) that the effect of Fermi-liquid effects can lead to a significant change in the dispersion and damping of electromagnetic excitations only in the high-frequency case

$$\omega \gg \nu_s \quad (s = 1, 2), \tag{9}$$

so we shall consider this case. (In (9) the index 1 refers to electrons, the index 2 to "holes.")

For a monochromatic plane wave ($\sim \exp(-i\omega t + i\mathbf{k}\cdot\mathbf{r})$) the Fourier transform of (8) with respect to τ takes the form

$$\sum_{m=-\infty}^{\infty} [(\nu - i\omega + in\Omega)\,\delta_{nm} + i\mathbf{k}\mathbf{v}_{n-m} + i\omega\hat{G}_{nm}]\,\chi_m = e\mathbf{E}\mathbf{v}_n. \tag{10}$$

Here

$$f_n(\varepsilon, p_z) = \frac{1}{2\pi}\int_0^{2\pi} f(\varepsilon, p_z, \tau)\, e^{-in\tau}\, d\tau,$$

δ_{nm} is the Kronecker symbol, and

$$\hat{G}_{nm} = (2\pi)^{-2}\iint \hat{G}(\mathbf{p}, \mathbf{p}')\, e^{-im\tau - in\tau'}\, d\tau\, d\tau'$$

is an operator with respect to p_z. The z axis is taken parallel to the magnetic field H, and the x axis perpendicular to the vectors k and H; the angle between k and H will be denoted by φ.

Let us define a vector ρ_n by $\chi_n = e\mathbf{E}\cdot\rho_n$. The solution of (10) for ρ_n up to and including terms of order H^{-2} has the following form: for n = 0

$$\rho_0 = [\nu - i\omega(1 - \hat{G}_{00}) + i\mathbf{k}\mathbf{v}_0]^{-1}\left\{\mathbf{v}_0 - \sum_{m\neq 0}(\mathbf{k}\mathbf{v}_{-m} + \omega\hat{G}_{0m})\frac{\mathbf{v}_m}{m\Omega}\right\} \tag{11a}$$

while for $n \neq 0$,

$$\rho_n = \frac{\mathbf{v}_n}{in\Omega} - \frac{1}{in\Omega}\left\{(\nu - i\omega)\frac{\mathbf{v}_n}{in\Omega} + i(\mathbf{k}\mathbf{v}_n + \omega\hat{G}_{n0})\rho_0 + \sum_{m\neq 0}(\mathbf{k}\mathbf{v}_{n-m} + \omega\hat{G}_{nm})\frac{\mathbf{v}_m}{m\Omega}\right\}. \tag{11b}$$

Using the definition of the current density

$$\mathbf{j} = \frac{2e}{h^3}\int \mathbf{v}\chi\frac{\partial n_0}{\partial\varepsilon}\, d\mathbf{p}, \tag{12}$$

we can write the electrical conductivity tensor in the form

$$\sigma_{ik} = \frac{4\pi e^2}{h^3}\int_{\varepsilon=\mu} m\, dp_z\left[v_0^{*i}\rho_0^k + \sum_{n\neq 0} v_n^{*i}\rho_n^k\right]E^k, \tag{13}$$

where $\mathbf{v}_0 = \langle\mathbf{v}\rangle$, the sign $\langle\ \rangle$ denoting an average over τ.

3. We shall be interested in the asymptotic expressions for the electrical conductivity tensor in various limiting cases:

A) The case of weak spatial dispersion

$$k_z v \ll \omega. \tag{14}$$

Neglecting spatial dispersion and scattering of the carriers, we obtain the following values for the elements of the conductivity tensor for an arbitrary convex Fermi surface with central symmetry:

$$\sigma_{\alpha\beta} = -\frac{Nec}{H}\varepsilon_{\alpha\beta} - i\frac{4\pi c^2}{H^2h^3}\omega\int m\, dp_z \langle\!\langle g_\alpha(\tau)\,[1 - \hat{G}(\mathbf{p}(\tau), \mathbf{p}'(\tau'))]\, g_\beta(-\tau')\rangle\!\rangle,$$

$$\sigma_{\alpha z} = -\frac{4\pi ec}{Hh^3}\int m\, dp_z \langle\!\langle g_\alpha(\tau)\,\hat{G}(\mathbf{p}(\tau), \mathbf{p}'(\tau'))\,[1 - \langle\!\langle\hat{G}\rangle\!\rangle]^{-1}\langle v_z\rangle\rangle\!\rangle,$$

$$\sigma_{zz} = i\frac{4\pi e^2}{\omega h^3}\int m\, dp_z \langle v_z\rangle\,[1 - \langle\!\langle\hat{G}\rangle\!\rangle]^{-1}\langle v_z\rangle. \tag{15}$$

Here N is the difference in the carrier concentrations, $\epsilon_{\alpha\beta}$ is the antisymmetric tensor of the second rank and $g_x = p_y - \langle p_y\rangle$, $g_y = -(p_x - \langle p_x\rangle)$.

B) The case of strong spatial dispersion:

$$k_z v \gg \omega. \tag{16}$$

The diagonal elements of the electrical conductivity tensor (which are the ones we shall need subsequently) have the following form for arbitrary dependence of the energies of the carriers on momentum:

$$\sigma_{xx} = \frac{2\pi c^2}{h^3H^2}\sum_{s=1,2}\left|\frac{m}{\langle k_z v_z'\rangle}\right|_{\langle v_z\rangle=0}|\langle \mathbf{kv}p_y\rangle|^2 + \frac{4\pi c^2}{h^3H^2}\sum_s\int_{\varepsilon=\mu} mdp_z \langle\!\langle p_y(\tau) \times [\nu - i\omega(1-\hat{G}(\mathbf{p}(\tau),\mathbf{p}'(\tau')))]\,p_y(\tau')\rangle\!\rangle,$$

$$\sigma_{yy} = \frac{4\pi c^2}{h^3H^2}\sum_{s=1,2}\int_{\varepsilon=\mu} mdp_z \langle\!\langle p_x(\tau) \times [\nu - i\omega(1-\hat{G}(\mathbf{p}(\tau),\mathbf{p}'(\tau')))]\,p_x(\tau')\rangle\!\rangle,$$

$$\sigma_{zz} = \frac{\omega k_D^2}{ik_z^2}\left[1 - \frac{4\pi e^2}{k_D^2h^3}\sum_{s=1,2}\int mdp_z\langle\!\langle \hat{G}\rangle\!\rangle\right], \qquad (17)$$

where $v_z' = \partial v_z/\partial p_z$ and

$$k_D^2 = 4\pi e^2\int mdp_z = 4\pi e^2\sum_s\frac{dn}{d\mu}$$

is the square of the reciprocal Debye radius. The summation is everywhere to be taken over the different groups of carriers. In the calculations we have used the fact that it follows from the conditions (16) and (9) that

$$[\nu - i\omega(1-\langle\!\langle \hat{G}\rangle\!\rangle) + i\langle k_z v_z\rangle]^{-1} = \pi\delta(\langle k_z v_z\rangle) + iP[\omega(1-\langle\!\langle \hat{G}\rangle\!\rangle) - \langle k_z v_z\rangle]^{-1},$$

where P denotes the principal value.

4. Let us now consider the propagation of helicon waves. The helicon dispersion relation is determined by the Hall conductivity $\sigma_{xy} = -Nec/H$ and has the form

$$\omega = c^2k^2|\cos\varphi| / 4\pi\sigma_{xy}. \qquad (18)$$

Obviously in the approximation considered the dispersion relation (18) is independent of the correlation function Φ. In the general case ($\varphi \neq 0$) the damping of the helicons is due to spatial dispersion (Landau damping) and can only be sensitive to the Fermi-liquid interactions when ω is of order kv. In the special case $\varphi = 0$ there is no spatial dispersion, the damping of helicons is determined by collisions and the Fermi-liquid interaction is unimportant. The low-frequency case $\omega \ll \nu$ was discussed above.

5. Next we investigate the effect of the Fermi-liquid interactions on the electromagnetic excitations in a metal with equal concentrations of carriers ($n_1 = n_2 = n$) under conditions of weak spatial dispersion ($k_z\nu \ll \omega$).

Owing to the assumption (9), the Hermitian part of the conductivity tensor (15), which is responsible for dissipation, is negligibly small in comparison with the antihermitian part. We write the antihermitian part of (15) in the form

$$\sigma_{ik} = \frac{nec}{H}\begin{pmatrix} -i\omega\Omega^{-1}a_{11} & -i\omega\Omega^{-1}a_{12} & a_{13} \\ -i\omega\Omega^{-1}a_{12} & -i\omega\Omega^{-1}a_{22} & -a_{32} \\ -a_{13} & a_{32} & i\Omega\omega^{-1}a_{33}\end{pmatrix}, \qquad (19)$$

where $\Omega = eH/(m_1+m_2)c$ and the quantities a_{ik} are expressed in an obvious way in terms of the matrix elements in (15).

As shown in [6], the dispersion relation of weakly damped waves in this case has a linear character:

$$\omega = kv_\pm,$$

$$v_\pm = v_a(2\det A_{\alpha\beta})^{-1/2}[A_{11} + A_{22} \pm \{(A_{11}-A_{22})^2 + 4A_{12}^2\}^{1/4}], \qquad (20)$$

where $v_a = H/(4\pi nm)^{1/2}$ is the Alfvén velocity, and the matrix elements A_{ik} are given by

$$A_{ik} = \begin{pmatrix} a_{11} + a_{13}^2/a_{33} & (a_{12} + a_{13}a_{32}/a_{33})|\cos\varphi|^{-1} \\ (a_{12} + a_{13}a_{32}/a_{33})|\cos\varphi|^{-1} & (a_{22} + a_{23}^2/a_{33})\cos^{-2}\varphi \end{pmatrix}.$$

In an anisotropic metal the excitations form two branches of magnetohydrodynamic (magnetoplasma) oscillations. Using (20), we can put the conditions (9) and (14) for the existence of these waves in the form

$$\nu_s \ll \omega \ll \Omega_s, \qquad v_s \ll v_\pm. \qquad (21)$$

In most metals (where $n \sim 10^{22}$) the second equation in (21) is satisfied only in fields of the order of several million oersteds. In metals such as bismuth with small carrier concentrations ($n \sim 10^{17}$) this inequality is satisfied in fields of the order of 10^3 Oe. On the other hand, as shown by Abrikosov [8] in metals of the bismuth type the correlation between electrons is weak (up to energies of the order of 1 eV), so that the gas model is valid for these metals.

6. In the case of strong spatial dispersion ($k_z v \gg \omega$) and an anisotropic carrier energy spectrum, Kaner and Skobov [6] have shown that the electromagnetic excitations are weakly damped only if the magnetic field H is oriented parallel to an axis of symmetry of high order; this is connected with the fact that the Landau damping vanishes in such directions. This condition is equivalent to an isotropic dependence of carrier energy on momentum In this region (magnetic fields subject to the condition $\nu_s \ll \omega \ll kv_s \ll \Omega$, $v_a \ll v_s$) there exist two oscillations: an Alfvén wave and a slow magnetosonic wave. Their frequencies are defined respectively by the elements σ_{yy} and σ_{xx} in formula (17) [6], viz:

$$k^2 = 4\pi i c^{-2}\omega\sigma_{\alpha\alpha}\cos^{-2}\varphi \quad (\alpha = x, y). \tag{22}$$

For non-longitudinal propagation only the Alfvén wave is weakly damped, while for **k** parallel to **H** the first term in expression (17) for σ_{xx} vanishes and the spectrum of slow magnetosonic waves coincides with that of Alfvén waves:

$$k = \frac{\omega}{H\cos\varphi}\left(4\pi\sum_s\int_{\varepsilon=\mu}\frac{m\,dp_z}{h^3}\langle\langle p_x(\tau)\,[1-\hat{G}(\tau,\tau')]\,p_x(\tau')\rangle\rangle\right)^{1/2}. \tag{23}$$

It is clear that the correction to the spectrum due to Fermi-liquid interactions is not in general small. For $G \sim 1$ (large Φ) the spectrum is displaced to higher frequencies.

In the case of strong spatial dispersion in an anisotropic metal the propagation of electromagnetic waves polarized along the direction of the magnetic field is also possible, owing to the fact that σ_{zz} is small in comparison with σ_{xx} and σ_{yy}. In this case the inequality $|\sigma_{\alpha z}\sigma_{z\rho}| \ll |\sigma_{\alpha\beta}\sigma_{zz}|$ must be satisfied The dispersion relation for these waves can be written in the form (cf. [7])

$$\omega = (\hbar k^2/2M)|\sin 2\varphi|,$$

$$M = \hbar c^{-1}k_D\left[1 - \frac{4\pi e^2}{h^3 k_D^2}\sum_s\int m\,dp_z\langle\langle\hat{G}\rangle\rangle\right]^{1/2}. \tag{24}$$

Investigation of waves with the spectrum (24) will allow us to determine the function G, since the density of states $dn/d\mu$ can be calculated directly, e.g., from the experimental heat capacity.

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