HYDRODYNAMIC THEORY OF COLLECTIVE OSCILLATIONS OF VORTICES IN TYPE II SUPERCONDUCTORS

A. A. ABRIKOSOV, M. P. KEMOKLIDZE, and I. M. KHALATNIKOV

Institute for Physics Problems, Academy of Sciences, U.S.S.R., Institute of Physics, Academy of Sciences, Georgian S.S.R.

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RECENTLY, De Gennes and Matricon [1] considered the collective oscillations of vertices in type II superconductors. However, their approach gives rise to objections, in view of the fact that they started out from an assumption that there were no current oscillations, for which, in general, there is no basis.

We consider the case for which $\kappa \gg 1$, just as in [1]. Under these conditions, the superconducting correlation parameter $\xi = \delta/\kappa$ is small in comparison with the field penetration depth.[2] Because of this fact, the core of the vortex can be regarded as a singularity in the field equations. As is well known, the London equation in the presence of a vortex is changed to

$$\operatorname{rot} \mathbf{j} + \frac{c}{4\pi\delta^2}\mathbf{H} = \frac{\Phi_0 c}{4\pi\delta^2}\sum_n \mathbf{v}_n \delta(\boldsymbol{\rho} - \boldsymbol{\rho}_n), \qquad (1)^*$$

where $\Phi_0 = \pi\hbar c/e$ is the magnetic flux quantum and δ is the field penetration depth. In the case of an alternating field, it is necessary to add an additional equation. We write it down by starting from the analogy between the London superconductors and a superfluid liquid. Of course, this derivation is not rigorous. We hope that eventually such an equation can be obtained from the complete equations of superconductivity theory.

Let us consider not too high a temperature and not take the normal current into consideration. As is well known, the motion of the superfluid liquid is described by the equation

$$\partial \mathbf{v}_s / \partial t + (\mathbf{v}_s \nabla)\mathbf{v}_s + \nabla \mu = 0, \qquad (2)$$

where $\mathbf{v}_s$ is the superfluid velocity and μ is the chemical potential. If it is assumed that the liquid is charged, then it is necessary to add to the right hand side the contribution from the Lorentz force: $em^{-1}(\mathbf{E} + c^{-1}\mathbf{v}_s \times \mathbf{H})$. Furthermore, we add another term, which corresponds to the "friction," which we shall find below. Introducing

$$\mathbf{j} = Ne\mathbf{v}_s, \qquad \mathbf{Q} = \operatorname{rot}\mathbf{j} + (c/4\pi\delta^2)\mathbf{H},$$

we get, after simple transformations:

$$\frac{\partial \mathbf{j}}{\partial t} - \frac{1}{Ne}[\mathbf{jQ}] = \frac{Ne^2}{m}\mathbf{E} + \mathbf{f}, \qquad (3)^\dagger$$

where $\mathbf{f}$ is the term associated with the friction. Here N must be understood as the "effective number of electrons," connected with δ by the London relation $\delta = (mc^2/4\pi Ne^2)^{1/2}$. The term $(m/e)\nabla(\mu + v_s^2/2)$ is included in E. Here, as before, we can neglect the displacement current, and we shall not use the equation $\operatorname{div}\mathbf{E} = 4\pi\rho$, assuming that the electron density in the metal is unchanged and $\operatorname{div} j = 0$.

We now consider the force of friction. It follows from general considerations that it should be proportional to the current. In principle, it can have components along $\mathbf{Q}$, along $\mathbf{j} \times \mathbf{Q}$ and along $\mathbf{Q} \times (\mathbf{j} \times \mathbf{Q})$. The term $\mathbf{j} \times \mathbf{Q}$ cannot participate in the friction force, inasmuch as it leads to an incorrect value of the frequency of oscillations, as we shall see below. In view of this, there remain only two terms, which we shall write down in the form $(Q/Ne\{\alpha[\boldsymbol{\nu} \times (\mathbf{j} \times \boldsymbol{\nu})] + \beta\boldsymbol{\nu}(\mathbf{j}\cdot\boldsymbol{\nu})\}$, where $\boldsymbol{\nu} = \mathbf{Q}/Q$. From the law of conservation of energy and the law for entropy increase, one can show, just as in [2], that $\beta, \alpha > 0$. Thus the equation finally takes on the form

$$\frac{\partial \mathbf{j}}{\partial t} - \frac{1}{Ne}[\mathbf{jQ}] = \frac{Ne^2}{m}\mathbf{E} + \frac{Q}{Ne}\{\alpha[\boldsymbol{\nu}[\mathbf{j}\boldsymbol{\nu}]] + \beta\boldsymbol{\nu}(\mathbf{j}\boldsymbol{\nu})\}. \qquad (4)$$

We shall assume that the external field is in the range $H_{c1} \ll H \ll H_{c2}$, so that the distance between the vortices lies in the interval $\xi \ll d \ll \delta$ [2]. Assuming that the wavelength of the oscillations $\lambda \gg d$, we shall average Eq. (4) over a region large in comparison with d. Evidently Eq. (4) can be understood in this sense.

We now consider the collective oscillations. If it is assumed that the external field is directed along the z axis, then, in zeroth approximation, we have $j = 0$, $\overline{Q}_z = cB/4\pi\delta^2$, where B is the mean field, which is practically identical with the external field H for $H \gg H_{c1}$. If the oscillations are propagated along the z axis, then it is easy to obtain the dispersion law from (4) and Maxwell's equations:

$$\omega = (\pm 1 - i\alpha)\Omega_0\delta^2 k^2/(1 + \delta^2 k^2), \qquad (5)$$

where $\Omega_0 = eH/mc$. These are circularly polarized waves. The quantities j_x, j_y, Q_x, and Q_y are different from zero. The density of the vortices, i.e., Q_z, does not change. Equation (5) differs from what was obtained in [1]. For $k \ll 1/\delta$ and $\alpha \ll 1$, it goes over into $\omega = \Omega_0$ and corresponds to the ordinary Larmor precession of the Cooper pairs. Thus the oscillations of the vortices and the Larmor precession correspond to different parts of a single branch of the excitation spectrum. We note that the presence of a term proportional to $j \times Q$ in the friction force would lead to the appearance in ω of a coefficient different from unity.

If the waves are propagated perpendicular to the vortices, then it is necessary to take into account the condition div $j = 0$ and the following expansion is used for $j \times Q$:

$$[\overline{j_0 Q_1}] = \left[\overline{(j_0 x)}, \frac{\partial Q_1}{\partial x}\right] + \frac{1}{2}\left[\overline{(j_0 x^2)}, \frac{\partial^2 Q_1}{\partial x^2}\right] + \ldots$$

The expression for j_0 is taken from [2]. If the centers of the vortices form a quadratic lattice in the perpendicular cross section, we then get

$$\omega_1 = -i\Omega_0 [Ck^4 d^4 \sin 4\varphi + \alpha k^2 \delta^2 / (1 + k^2 \delta^2)],$$

$$\omega_2 = -i\Omega_0 [Ck^4 d^4 \sin 4\varphi + \beta k^2 \delta^2 / (1 + k^2 \delta^2)], \qquad (6)$$

where C is a constant of the order of unity. For a trigonal lattice, in place of the term $k^4 d^4$, there is a term $k^6 d^6$. The first of these solutions refers to a plane polarized wave with f_y and Q_z, while the second, to a plane wave with f_z and Q_y. Thus, in contrast with the conclusion of De Gennes and Matricon,[1] the propagation of transverse undamped waves is not possible. It also follows from (6) that the lattice of the vortices is unstable in the absence of dissipation ($\alpha, \beta = 0$). In view of the strong dependence of the coefficients α and β on k for a small number of macroscopic defects (see below), this can lead to shortwave "jitter" of the lattice.

In the recently published work of Vinen and coworkers,[4] the motion of vortices was studied by the relaxation of the magnetization of a cylindrical specimen upon change in the external field. From Eq. (4) for the relaxation time, we get

$$\tau = R^2 / 5.8 \alpha \delta^2 \Omega_0, \qquad (7)$$

where R is the radius of the cylinder. Comparison with the results of the given research shows that $\alpha \sim 1000$. At first glance, it then follows that the frequency of oscillation (5) is essentially an imaginary quantity, i.e., the oscillations do not exist. However, it is necessary here to take it into consideration that the fundamental mechanism of dissipation at low temperature is the interaction with macroscopic inhomogeneities. If the number of such inhomogeneities is not large, so that the distance between them $l \gg d$, then one can consider the wavelength $\lambda \ll l$. For such oscillations, α will be small. If one is concerned with the motion of the filaments as a whole, which is the case in the work of Vinen et al., then one must naturally expect large values of α. In other words, on the basis of the data of Vinen and coworkers, one must not expect the absence of oscillations, but only a strong dependence of α on the wave vector.

*rot = curl.
†[jQ] = j × Q.

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METHOD OF INVESTIGATING ELASTIC pp SCATTERING AT HIGH ENERGIES BY MEANS OF SEMICONDUCTOR COUNTERS

Yu. K. AKIMOV, A. I. KALININ, V. A. NIKITIN, V. S. PANTUEV, V. A. SVIRIDOV, A. I. SIDOROV, and M. N. KHACHATURYAN

Joint Institute for Nuclear Research

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IN the present work we have experimentally demonstrated the possibility of studying elastic scattering of high energy protons in the small-momentum-transfer region

$$1.5 \cdot 10^{-3}\,\mathrm{GeV}^2/c^2 \leqslant -t \leqslant 1.5 \cdot 10^{-1}\,\mathrm{GeV}^2/c^2$$

by means of semiconductor nuclear particle detectors. The nuclear emulsion method [1] which has previously been used for this purpose has the disadvantage of a low rate of collecting statistics. Semiconductor counters are free from this difficulty, possess good energy resolution ($\sim 1\%$), are compact, and are insensitive to magnetic fields. The fact that the sensitive layer in the semiconductor detector can begin immediately at the surface permits counting protons of very low energies, down to several tens of keV.[2] This means that it is possible to study scattering in