ON THE ANOMALOUS TEMPERATURE DEPENDENCE OF THE RESISTIVITY OF NON-MAGNETIC METALS WITH A WEAK CONCENTRATION OF MAGNETIC IMPURITIES

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THE phenomenon of the increase of the electrical resistivity with decreasing temperature in non-magnetic metals with a small admixture of paramagnetic atoms has long attracted attention. In a recent paper Kondo [1] showed that if one takes into account the lowest-order correction to the Born-approximation expression for the magnetic scattering of an electron, there appears in the electrical resistivity an extra term of relative order of magnitude $(J/\epsilon_F)\ln(\epsilon_F/T)$ where J is the exchange interaction and ϵ_F is the Fermi energy. If J is negative, i.e., the interaction of the electron with the impurity is antiferromagnetic in nature, this leads to an increase of the resistivity with decreasing T.

The applicability of the Kondo theory is limited on the low temperature side by the temperature at which correlations between the spins of the impurity atoms become important, that is, the ferromagnetic or antiferromagnetic Curie temperature. However, the Curie temperature is proportional to the atomic concentration of the impurities. Therefore, for sufficiently small impurity concentrations we reach the temperature at which $(J/\epsilon_F)\ln(\epsilon_F/T)$ becomes larger than unity before we reach the Curie point. Then the perturbation theory used by Kondo is no longer applicable and it is necessary to sum the whole series.

The main difficulty is that the appearance of logarithmic terms is connected with the quantum nature of the impurity spin operators. This makes it impossible to use the usual diagram technique, for it is impossible to express averages of the form $\langle T(\hat{S}_{i1}(t_1)\hat{S}_{i2}(t_2)\ldots\hat{S}_{in}(t_n))\rangle$ in terms of a sum of products of pair averages of the form

$$\langle T(\hat{S}_{i1}(t_1)\hat{S}_{i2}(t_2))\rangle\langle T(\hat{S}_{i3}(t_3)\hat{S}_{i4}(t_4))\rangle+\ldots$$

However, this problem has been successfully solved with the help of a generalization of the technique originally devised for the case $S = 1/2$ by I. E. Dzyaloshinskiĭ and the author. Here we shall not give the details of this technique; we just remark that it is based on representing the impurity spin operator in the form

$$\hat{S}(t)=a_\alpha^+(t)\hat{S}_{\alpha\beta}a_\beta(t),$$

where $S_{\alpha\beta}$ are spin matrices and a_α are the operators of a fictitious fermion field. Next we obtain more or less general finite-temperature techniques for two fermion fields with the interaction Hamiltonian

$$H_{int}=-(J/N)\sum_n\psi_\alpha^+(r_n)\sigma_{\alpha\beta}{}^i\psi_\beta(r_n)a_{n\gamma}{}^+S_{\gamma\delta}{}^ia_{n\delta}. \quad (1)$$

where N is the atomic density, and σ^i are the Pauli matrices. (The interaction is assumed to have a δ-function form.) In the averaging process supplementary techniques are applied in order to exclude "nonphysical" states (states with no $a_{n\alpha}$-fermion, or more than one, in the sum over all states α).

As a result of the calculations (of which we omit the details here) we get the following results for the impurity part of the resistivity:

1. As previously, the resistivity is made up of two independent terms: one connected with the usual interaction, the other connected with the exchange interaction of the electrons with the magnetic impurity atoms.

2. The magnetic part of the resistivity, if we take into account only the leading logarithmic terms, has the form

$$\frac{\rho_M}{\rho_{M0}}=\left(1+\frac{3J_1z}{2\varepsilon_F}\ln\frac{q\varepsilon_F}{T}\right)^{-2},$$
$$\rho_{M0}=\frac{3\pi mJ_0S(S+1)c}{2N\varepsilon_Fe^2\hbar} \quad (2)$$

where J_1 is the exchange scattering amplitude of a free electron on an impurity, which is assumed isotropic [in the Born approximation J_1 is the same as J in (1)]; c is the concentration of impurities; $q\sim 1$; and z is the number of electrons per atom.

3. According to formula (2), for $J_1 > 0$ (ferromagnetic interaction) ρ_M decreases with decreasing temperature and tends to zero for $T = 0$. On the other hand, if $J_1 < 0$, then according to Eq. (2) ρ_M tends to infinity for $T = T_r$, where

$$T_r=q\varepsilon_F\exp(-2\varepsilon_F/3|J_1|z). \quad (3)$$

Actually, more detailed investigation shows that in this case the scattering amplitude of an electron on an impurity has a resonance character and owing to the finite width of the resonance ρ_M remains finite. We have not succeeded in obtaining an exact expression for ρ_M in the region of the maximum; we can only give the order-of-magnitude estimate:

$$\rho_{M\,max} \sim \rho_{M0}(\varepsilon_F / J_1)^2. \quad (4)$$

This means that if magnetic impurities are important, the magnetic part of the resistivity ρ_M is comparable in order of magnitude to the non-magnetic part at the maximum. With further decrease in temperature ρ_M tends to zero.

4. The hypothesis that owing to collective effects a resonance might occur in the scattering of electrons in the neighborhood of the Fermi surface has been previously advanced in the literature [3]. This hypothesis, then, is to some extent confirmed; however, it has turned out, in contradiction to the ideas of Korringa and Gerritsen, that the resonance occurs only as a result of the exchange interaction of the electrons with impurity atoms and only when this interaction is of antiferromagnetic sign. The resonance energy and the temperature T_r (which are of the same order of magnitude) do not depend on the impurity concentration for small concentrations.

Details of this calculation will be published subsequently.

I should like to take this opportunity to express my gratitude to I. E. Dzyaloshinskiĭ for numerous discussions.

[1] J. Kondo, Prog. Theor. Phys. 32, 37 (1964).

[2] Abrikosov, Gor'kov and Dzyaloshinskiĭ, Metody kvantovoĭ teorii polya v statisticheskoĭ fizike (Methods of Quantum Field Theory in Statistical Physics) Fizmatgiz, 1962. (Translation: Prentice-Hall, Englewood Cliffs, N. J., 1963)

[3] J. Korringa and A. N. Gerritsen, Physica 19, 457 (1953).

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RADIATIVE CORRECTIONS TO PAIR PHOTOPRODUCTION[1)]

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IN a paper by E. G. Vekshteĭn a statement was made concerning the anomalously strong energy dependence of relativistic radiative corrections to the cross section for pair photoproduction in the limiting case

$$\varepsilon_+ = \varepsilon_- = \omega / 2 \gg m, \quad \theta_+ = \theta_- = m / \omega \ll 1$$

$$(\theta_\pm = \widehat{\mathbf{k}\mathbf{p}_\pm}) \quad (1)$$

where the momenta $\mathbf{k}$, $\mathbf{p}_+$ and $\mathbf{p}_-$ lie in a plane. In particular it is asserted that

$$\delta \equiv d\sigma^{(4)} / d\sigma^{(2)} = A\alpha\omega^2 / \pi m^2, \quad A \sim 1 \quad (2)$$

However, as will be shown below, this conclusion is in error and in fact

$$\delta = B\alpha / \pi, \quad B \sim 1. \quad (3)$$

The source of this error is the following. Of the ten diagrams describing the radiative corrections to photoproduction (diagrams a-k in Fig. 3 of [1]), only the first eight are taken into account in [1]. Thus it was found that the contributions of diagrams c-d gave

$$\delta^{c}{}_{+}{}^{d} = 0.54\, \alpha\omega^2 / \pi m^2. \quad (4)$$

while diagrams a-b and e-h did not make a contribution of order ω^2/m^2. The contributions from diagrams j-k were not obtained because of the difficulty of the calculation. It was assumed instead that they could not compensate for the result in (4) because of the "independence" of diagrams j-k and c-d.

However, this assumption is incorrect. First, the indicated diagrams are mutually dependent and, second, the contributions of diagrams j-k cancel the result in (4). Both these assertions are proven below.

The existence of a relationship between diagrams j-k and c-d follows immediately from the fact that under the gradient transformation of the Coulomb potential

$$a_\mu(q) \to a_\mu(q) + iq_\mu f(q) \quad (5)$$

the contribution from each of them changes, but the sum of all four remains invariant. Indeed, putting $S(p) = (i\hat{p} + m)^{-1}$ and using the obvious relations

$$\bar{u}_2 i\hat{q} S(p_2 + q) = \bar{u}_2, \quad S(p_1 - q) i\hat{q} u_1 = -u_1,$$
$$S(p) i\hat{q} S(p + q) = S(p) - S(p + q),$$