ON THE THEORY OF A TURBULENT PLASMA

L. M. KOVRIZHNYKH

P. N. Lebedev Physics Institute, Academy of Sciences, U.S.S.R.

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Assuming the electromagnetic field energy density to be small and the phases of the electromagnetic waves to be random, we obtain a closed set of non-linear equations for the distribution function of the plasma particles and the electromagnetic field. For the case of a plasma without an external magnetic field, we find explicit expressions for all coefficients in those equations.

1. INTRODUCTION

A number of authors[1-16] have considered by different methods the problems of the non-linear interaction of waves in a plasma in several particular cases. An attempt is made in the present paper to construct systematically a non-linear theory of a weakly turbulent plasma and to obtain by a single method all necessary coefficients occurring in the basic equations.

For the derivation of the basic equations we use a semiquantum method of calculation which introduces the concept of number of quanta $N(\mathbf{k})$ (or plasmons) corresponding to any one branch of the dispersion equation, and the concept of the probabilities for a process in which particles and quanta take part (scattering processes) or in which only quanta take part (decay processes). Since the method of description given below uses the concept of number of quanta which clearly has sense only in the case of weakly damped waves, it allows us to take into account only the interaction through almost undamped waves, and does not take into account the interaction corresponding to strongly damped "waves." The latter, however, turns out to be important only when the noise intensity is close to the noise intensity of thermodynamic equilibrium and when emission and absorption processes for waves and "binary collisions" give a contribution of the same order of magnitude and, strictly speaking, can not be considered to be independent.[17]

However, in the case of a turbulent plasma in which the noise intensity is by many orders of magnitude larger than the thermal noise intensity, the main role is played by the interaction between the particles and the waves or between the waves with one another, and we can completely neglect "binary collisions." In other words, we may assume that the noise intensity is non-vanishing only in the region where the plasma is transparent and we can describe the processes in such a turbulent plasma by means of a set of equations obtained on the basis of assuming particle-wave and wave-wave interactions.

We must perhaps note that the method used in the following to obtain the equations, which take into account the non-linear interaction, is not unique. Indeed, a similar set of equations can be obtained also from Vlasov's self-consistent equations[1-5,9] or, more rigorously, using Bogolyubov's method of correlation functions[6,14,18,19]. Although the last method is in some sense more general, as it makes it possible in principle to take consistently into account also the "binary collisions" of the particles, it is considerably more complicated mathematically and less lucid from the physical point of view. In contrast to this, the semi-quantum method of consideration used by us is relatively simple and allows us on the one hand to obtain results valid in the range of large noise intensity which is of most interest and on the other hand - and that is also its undoubted advantage - it is easy to see the physical meaning of the different terms occurring in the equations we start from.

2. BASIC EQUATIONS

It is well known that the damping and build-up of waves obtained in a linear theory can also be easily obtained and elucidated in quantum mechanical language if one uses the concepts of the probability for the absorption and emission of quanta by the particles moving through the plasma.[1)] This

[1)]Of course, when it is meaningful to talk about the quanta of the electro-magnetic field, i.e., for an almost monochromatic wave, when $\mathrm{Re}\,\omega \gg \mathrm{Im}\,\omega$.

process, in which one particle and one quantum take part, corresponds to taking into account only terms linear in the number of quanta in the kinetic equation and could be called a first-order process. One sees easily, however, that higher-order processes are also possible, in which no longer two "objects" (a particle and a quantum) take part, but three, four, and so on. Taking these processes of higher and higher order consistently into account corresponds clearly to expanding the non-linear equations in power series in the energy density U of the electromagnetic field. If the energy density U is sufficiently small, we can thus in first approximation describe the non-linear effects by merely taking into account the first and second order processes.

In accordance with this we shall take into account in what follows only processes where a particle emits or absorbs a single quantum, scattering processes where two quanta take part in which the particle absorbs (or emits) one quantum and emits another quantum (or the other way round), and finally processes in which only three quanta take part but no particle, i.e., the so-called decay processes in which one quantum decays into two other quanta (or the other way around). All other higher-order processes will be neglected as quantities of a higher order of smallness.[2)]

Thus, we denote by $N_\alpha(\mathbf{k})$ the number of quanta of kind α with momentum $\mathbf{k}$ contained in unit wave number interval and by $F_q(\mathbf{p})$ the distribution function of particles of kind q in the coordinate-momentum space. We choose the normalization of the functions N_α and F_q in such a way that[3)]

$$\int d\mathbf{k}\frac{\Omega_\alpha N_\alpha}{(2\pi)^3} = U_\alpha, \qquad \int d\mathbf{p} F_q = n_q, \tag{2.1}$$

where $\Omega_\alpha(\mathbf{k})$ is the frequency (or energy) of a quantum of type α (with momentum $\mathbf{k}$), U_α the electromagnetic energy density in the plasma, i.e., the energy of quanta of type α in a unit volume $d\mathbf{r} = dx\,dy\,dz$, and n_q is the number density of particles of type q. Let, moreover, $w_q^\alpha(\mathbf{p}, \mathbf{k})$ be the probability for the emission by a particle of type q with momentum p of a quantum of type α with momentum k; $W_q^{\alpha\beta}(\mathbf{p}, \mathbf{k}, \mathbf{k}_1)$ the probability for scattering of particles of type q with momentum p, involving the absorption of a quantum of type β and momentum $\mathbf{k}_1$ and the emission of a quantum of type α and momentum k; $\widetilde{W}_q^{\alpha\beta}(\mathbf{p}, \mathbf{k}, \mathbf{k}_1)$ the probability for the emission by a particle of type q and momentum p of a quantum of type α and momentum k and a quantum of type β and momentum $\mathbf{k}_1$, and finally, $V^{\alpha\beta\gamma}(\mathbf{k}, \mathbf{k}_1, \mathbf{k}_2)$ the probability for the decay of a quantum of type α and momentum k into a quantum of type β and momentum $\mathbf{k}_1$ and a quantum of type γ and momentum $\mathbf{k}_2$.

Assuming the above-mentioned probabilities as given, we obtain easily equations for the particle and quanta distribution functions, which are nothing but the balance equations for the number of particles and number of quanta. In the classical case, of most interest, when the energy and the momentum of the quantum is much smaller than the energy and the momentum of the particles, these equations have the form [11,20]

$$\frac{dF_q}{dt} = \sum_{l,m}\frac{\partial}{\partial p_l} D_{l,m}^{(q)}(\mathbf{p})\frac{\partial F_q}{\partial p_m}, \tag{2.2}$$

$$\begin{aligned}
D_{l,m}(\mathbf{p}) &= \sum_\alpha \int d\mathbf{k}\, k_l k_m w_q{}^\alpha(\mathbf{p},\mathbf{k}) N_\alpha(\mathbf{k}) \\
&+ \frac{1}{2}\sum_{\alpha,\beta}\int d\mathbf{k}\, d\mathbf{k}_1[(k_l - k_{1l})(k_m - k_{1m}) W_q{}^{\alpha\beta}(\mathbf{p},\mathbf{k},\mathbf{k}_1) \\
&+ (k_l + k_{1l})(k_m + k_{1m})\widetilde{W}_q{}^{\alpha\beta}(\mathbf{p},\mathbf{k},\mathbf{k}_1)] N_\alpha(\mathbf{k}) N_\beta(\mathbf{k}_1);
\end{aligned}$$

$$\begin{aligned}
\frac{dN_\alpha}{dt} &= (2\pi)^3\sum_q \int d\mathbf{p}\left(\mathbf{k}\frac{\partial F_q}{\partial \mathbf{p}}\right) w_q{}^\alpha(\mathbf{p},\mathbf{k}) N_\alpha(\mathbf{k}) \\
&+ (2\pi)^3 \sum_q\sum_\beta \int d\mathbf{p}\, d\mathbf{k}_1\Big[(\mathbf{k}-\mathbf{k}_1)\frac{\partial F_q}{\partial \mathbf{p}} W_q{}^{\alpha\beta}(\mathbf{p},\mathbf{k},\mathbf{k}_1) \\
&+ (\mathbf{k}+\mathbf{k}_1)\frac{\partial F_q}{\partial \mathbf{p}}\widetilde{W}_q{}^{\alpha\beta}(\mathbf{p},\mathbf{k},\mathbf{k}_1)\Big] N_\alpha(\mathbf{k}) N_\beta(\mathbf{k}_1) \\
&+ \sum_{\beta,\gamma}\int d\mathbf{k}_1 d\mathbf{k}_2 \{V^{\alpha\beta\gamma}(\mathbf{k},\mathbf{k}_1,\mathbf{k}_2)[N_\beta(\mathbf{k}_1)N_\gamma(\mathbf{k}_2) \\
&- N_\alpha(\mathbf{k})N_\beta(\mathbf{k}_1) - N_\alpha(\mathbf{k})N_\gamma(\mathbf{k}_2)] + V^{\beta\gamma\alpha}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}) \\
&+ [N_\beta(\mathbf{k}_1)N_\gamma(\mathbf{k}_2) + N_\alpha(\mathbf{k})N_\beta(\mathbf{k}_1) - N_\alpha(\mathbf{k})N_\gamma(\mathbf{k}_2)] \\
&+ V^{\gamma\alpha\beta}(\mathbf{k}_2,\mathbf{k},\mathbf{k}_1)[N_\beta(\mathbf{k}_1)N_\gamma(\mathbf{k}_2) - N_\alpha(\mathbf{k})N_\beta(\mathbf{k}_1) \\
&+ N_\alpha(\mathbf{k})N_\gamma(\mathbf{k}_2)]\};
\end{aligned} \tag{2.3}$$

$l, m = 1, 2, 3, \mathbf{k} = \{k_1, k_2, k_3\}, \quad \mathbf{k}_1 = \{k_{11}, k_{12}, k_{13}\},$

$\mathbf{p} = \{p_1, p_2, p_3\}.$

These equations contain already classical probabilities and, as can be easily checked, they are interconnected by the following relations

$$W_q{}^{\alpha,\beta}(\mathbf{p},\mathbf{k},\mathbf{k}_1) = \widetilde{W}_q{}^{\alpha,\beta}(\mathbf{p},\mathbf{k},-\mathbf{k}_1)\,|_{\Omega_\beta\to-\Omega_\beta},$$

[2)]In principle they could easily be taken into account exactly in the same way as the first- and second-order processes.

[3)]Here and everywhere in what follows we shall use a system of units in which Dirac's constant $\hbar = 1$ and the velocity of light $c = 1$.

$$W_q^{\alpha,\beta}(\mathbf{p},\mathbf{k},\mathbf{k}_1)=W_q^{\beta,\alpha}(\mathbf{p},\mathbf{k}_1,\mathbf{k});$$

$$V^{\alpha\beta\gamma}(\mathbf{k},\mathbf{k}_1,\mathbf{k}_2)=V^{\beta\gamma\alpha}(-\mathbf{k}_1,\mathbf{k}_2,-\mathbf{k})\mid_{\Omega_\alpha\to-\Omega_\alpha,\Omega_\beta\to-\Omega_\beta}$$

$$V^{\alpha\beta\gamma}(\mathbf{k},\mathbf{k}_1,\mathbf{k}_2)=V^{\alpha\gamma\beta}(\mathbf{k},\mathbf{k}_2,\mathbf{k}_1). \quad (2.4)$$

Moreover, one shows easily that the probabilities for processes of different order contain as a factor essentially different δ-functions which reflect the conservation laws for energy and momentum, namely:

$$w_q^\alpha(\mathbf{p},\mathbf{k})\sim\delta(\Omega_\alpha-\mathbf{kv}),\quad \Omega_\alpha=\Omega_\alpha(k),$$

$$W_q^{\alpha\beta}(\mathbf{p},\mathbf{k},\mathbf{k}_1)\sim\delta(\Omega_\alpha-\Omega_\beta-(\mathbf{k}-\mathbf{k}_1)\mathbf{v}),$$

$$\Omega_\beta=\Omega_\beta(k_1), \quad (2.5)$$

where v is the velocity of a particle of type q and, correspondingly

$$V^{\alpha\beta\gamma}(\mathbf{k},\mathbf{k}_1,\mathbf{k}_2)\sim\delta(\Omega_\alpha-\Omega_\beta-\Omega_\gamma)\delta(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2). \quad (2.6)$$

Taking Eqs. (2.4) to (2.6) into account one verifies easily that the set (2.2) and (2.3) possesses a number of first integrals which express the conservation laws for energy and momentum for the system particles + field quanta, and in a number of cases the conservation law for the total number of quanta.

We note also that decay processes clearly do not change the total energy of the electromagnetic field and lead only to a redistribution of the energy over the spectrum and to a transfer of the energy from quanta of one type to quanta of another type. This follows already from the definition itself of the decay processes and can easily be verified mathematically if we take into account Eqs. (2.5) and (2.6).

Thus, for given values of the probabilities w_q^α, $W_q^{\alpha\beta}$, $\widetilde{W}_q^{\alpha\beta}$ and $V^{\alpha\beta\gamma}$ Eqs. (2.2) and (2.3) are a closed system of self-consistent equations for the distribution functions for the particles, F_q, and for the quanta, N_α, which enable us to study the behavior of these functions in time and space, taking non-linear effects into account. In the particular case of rather weak noise intensity we can neglect in Eqs. (2.2) and (2.3) all terms corresponding to second-order processes (quadratic in the number of quanta). The system of equations is then considerably simplified and obviously goes over into the equations obtained in the quasi-linear approximation [21,22].

The problem is thus reduced to finding explicit expressions for the probabilities of different processes. However, since the classical probabilities enter already in Eqs. (2.2) and (2.3), we can find them without having recourse to quantum field theoretical methods, using only the formalism of classical electrodynamics. In general outline, the scheme of calculations consists of the following (see also [10,20]). The complete equation for the number of quanta N_α takes into account the change in the number of quanta occurring both as a result of processes of spontaneous emission and as a result of processes of absorption and induced emission. We now assume that the radiation intensity $N_\alpha(\mathbf{k})$ is so small that the emission is determined only by spontaneous transitions, and that we can completely neglect processes of absorption and induced emission. The equation determining the change in the average density U_α of the electromagnetic energy will then clearly be of the form

$$\frac{dU_\alpha}{dt}=\left(\frac{\delta U_\alpha}{\delta t}\right)_1+\left(\frac{\delta U_\alpha}{\delta t}\right)_2, \quad (2.7)$$

where

$$\left(\frac{\delta U_\alpha}{\delta t}\right)_1=\sum_q\int d\mathbf{p}\,\Omega_\alpha F_q w_q^\alpha\,d\mathbf{k}$$
$$+\sum_q\sum_\beta\int d\mathbf{p}d\mathbf{k}d\mathbf{k}_1\Omega_\alpha F_q[W_q^{\alpha\beta}+\widetilde{W}_q^{\alpha\beta}]N_\beta(\mathbf{k}_1) \quad (2.8)$$

is the change in the energy density of quanta of type α due to scattering processes while

$$\left(\frac{\delta U_\alpha}{\delta t}\right)_2=\frac{1}{(2\pi)^3}\sum_{\alpha,\beta}\int d\mathbf{k}d\mathbf{k}_1d\mathbf{k}_2\Omega_\alpha[V^{\alpha\beta\gamma}$$
$$+V^{\beta\gamma\alpha}+V^{\gamma\alpha\beta}]N_\beta(\mathbf{k}_1)N_\gamma(\mathbf{k}_2) \quad (2.9)$$

is its change due to decay processes.

On the other hand, the change in the energy of the field in the system can be found by solving directly the non-linear set of equations for the distribution functions for the particles and the electromagnetic field vectors. For the solution of such equations it is then necessary, in accordance with the above-made neglect of absorption and induced emission processes, to neglect the influence of the radiation field on the motion of the particles in the plasma. Comparing the expression for the change in the energy density of the quanta thus obtained with Eqs. (2.8) and (2.9) and using Eqs. (2.5) and (2.6) we can determine the required probabilities. The uniqueness of such a procedure is guaranteed by the essentially different structure of the arguments of the δ-functions occurring in the expressions for the probabilities for the different processes and for well-known a priori probabilities.

We must note that in Eq. (2.7) the probabilities for two kinds of processes occur, namely decay and scattering processes, the physical nature of which is completely different. Indeed, decay processes are determined clearly only by parameters characterizing the properties of quanta, i.e., by the wave vector **k** and the frequency $\Omega(\mathbf{k})$, which in turn are determined by such "over-all" plasma parameters as its density, velocity dispersion, and so on, and are completely independent of the individual properties of some separately chosen particle or a small group of such particles. In contrast to this, the scattering processes depend not only on the properties of the quanta, i.e., the average characteristics of the medium in which the process takes place, but also on the parameters of the particle taking part in the scattering, i.e., its charge, mass, and velocity. In accordance with this the methods of determining the scattering and decay probabilities are also somewhat different. Thus, to find the scattering probabilities it is sufficient to find the energy emitted by the plasma when some "test particle" moves through it and to compare the expression obtained with (2.8). However, to determine the decay probabilities, on the other hand, we can completely neglect the influence of resonating particles and consider the plasma as a medium the properties of which are completely determined by its average characteristics (for instance, the density and the temperature).

We must emphasize that we can use such a procedure only in the case when the resonating particles form a small group which does not appreciably change the average parameters of the plasma. One sees easily that this requirement is the same as the condition, used above, of weakly damped or weakly intensified waves: $\mathrm{Re}\,\omega \gg \mathrm{Im}\,\omega$.

3. PROBABILITIES OF SCATTERING PROCESSES

We start now with finding expressions for the scattering probabilities. We restrict ourselves here to the case of a non-relativistic plasma and we assume that the external magnetic field is equal to zero. It is well known that in that case three kinds of waves can propagate in the plasma: transverse, Langmuir, and sound waves. The dependence of the frequency Ω_α on the wave vector is given for them by the following formulae

$$\Omega_t^2 = \omega_{0e}^2 + k^2, \qquad v_{Tq}^2 \ll 1,$$

$$\Omega_l^2 = \omega_{0e}^2 + 3k^2 v_{Te}^2, \qquad k^2 v_{Te}^2 \ll \omega_{0e}^2,$$

$$\Omega_s^2 = \frac{\omega_{0i}^2 k^2 v_{Te}^2}{\omega_{0e}^2 + k^2 v_{Te}^2} + 3k^2 v_{Ti}^2, \qquad k^2 v_{Ti}^2 \ll \omega_{0i}^2, \quad (3.1)$$

where $v_{Tq}^2 = T_q/m_q$ is the mean square of the velocity, T_q the temperature, m_q the mass of a particle of kind q, $\omega_{0q}^2 = 4\pi e_q^2 n_q/m_q$ the corresponding Langmuir frequency, n_q the density, while the indices t, l, and s indicate that the corresponding quantities refer to transverse, Langmuir, and sound waves.

Not having the possibility to expound in more or less detail the complete procedure of calculations, and referring for details to another paper by the author [20], we give here only the final expressions for the required probabilities, indicating the limits of their applicability. In order not to complicate the formulae we assume everywhere that the electron temperature is not much higher than the ion temperature so that quantities of the order $m_e T_e/m_i T_i$ can be neglected compared to unity. We also neglect small non-linear corrections to the probabilities for first-order processes.

In accordance with this, the expressions for w_q^α will have the well-known form:

$$w_q^t(\mathbf{p}, \mathbf{k}) = 0,$$

$$w_q^l(\mathbf{p}, \mathbf{k}) = \frac{e_q^2}{2\pi}\frac{\omega_{0e}}{k^2}\delta(\Omega_l - \mathbf{kv}),$$

$$w_q^s(\mathbf{p}, \mathbf{k}) = \frac{e_q^2}{2\pi}\frac{\Omega_s^3}{k^2\omega_{0i}^2}\delta(\Omega_s - \mathbf{kv}). \quad (3.2)$$

For the probabilities of the second order we find the following expressions:

tt-scattering. Restricting ourselves to the non-relativistic case and neglecting quantities of order kv/Ω_t as compared to unity, we get[4)]

$$W_e^{tt_1}(\mathbf{p}, \mathbf{k}, \mathbf{k}_1) = \left(\frac{e_e^2}{m_e}\right)^2 \frac{\delta(\Omega_t - \Omega_{t_1} - (\mathbf{k} - \mathbf{k}_1)\mathbf{v})}{(2\pi)^3 |\Omega_t \Omega_{t_1}|} \times \left|\frac{\varepsilon_i^l(\Omega_2 k_2)}{\varepsilon^l(\Omega_2 k_2)}\right|^2 \frac{1 + \cos^2 \widehat{\mathbf{kk}_1}}{2}$$

$$W_i^{tt_1}(\mathbf{p}, \mathbf{k}, \mathbf{k}_1) = \left(\frac{e_e e_i}{m_e}\right)^2 \frac{\delta(\Omega_t - \Omega_{t_1} - (\mathbf{k} - \mathbf{k}_1)\mathbf{v})}{(2\pi)^3 |\Omega_t \Omega_{t_1}|} \times \left|\frac{\varepsilon_e^l(\Omega_2 k_2) - 1}{\varepsilon^l(\Omega_2 k_2)}\right|^2 \times \frac{1 + \cos^2 \widehat{\mathbf{kk}_1}}{2}, \quad (3.3)$$

$$\Omega_t = \Omega_t(k), \qquad \Omega_{t_1} = \Omega_t(k_1),$$

$$\Omega_2 = \Omega_t - \Omega_{t_1}, \qquad \mathbf{k}_2 = \mathbf{k} - \mathbf{k}_1.$$

The expression for the probability of the electron scattering is valid only under the condition

4)Here $\epsilon_e^{l,t}$ and $\epsilon_i^{l,t}$ are the longitudinal (or transverse) dielectric constants of the electron and ion plasma with distribution functions F_e and F_i, so that the total dielectric constant $\epsilon^{l,t}$ of the electron plasma is equal to $\epsilon^{l,t} = \epsilon_e^{l,t} + \epsilon_i^{l,t} - 1$.

$$\left|\frac{\varepsilon_i^l(\Omega_2 k_2)}{\varepsilon^l(\Omega_2 k_2)}\right| > \frac{\Omega_2 k}{\Omega_t k_2}$$

In analogy with first-order processes (see (3.2)) the probability $\widetilde{W}_q^{tt_1}(\mathbf{p}, \mathbf{k}, \mathbf{k}_1) = 0$.

<u>l,t-scattering</u>. In that case we get easily the expression for the probability $W_q^{tl_1}(\mathbf{p}, \mathbf{k}, \mathbf{k}_1)$ for arbitrary velocities, and we have:

$$W_e^{tl_1}(\mathbf{p},\mathbf{k},\mathbf{k}_1) = \left(\frac{e_e^2}{m_e}\right)^2 \frac{\delta(\Omega_t - \Omega_{l_1} - (\mathbf{k}-\mathbf{k}_1)\mathbf{v})}{(2\pi)^3|\Omega_t|}\omega_{0e} \times \left|\frac{[\mathbf{k}(\mathbf{A}+\mathbf{B}_l)]}{k}\right|^2, \tag{3.4}^*$$

where[5]

$$W_i^{tl_1}(\mathbf{p},\mathbf{k},\mathbf{k}_1) = \left(\frac{e_e e_i}{m_e}\right)^2 \frac{\delta(\Omega_t - \Omega_{l_1} - (\mathbf{k}-\mathbf{k}_1)\mathbf{v})}{(2\pi)^3|\Omega_t|}\omega_{0e}\left|\frac{[\mathbf{k}\mathbf{B}_l]}{k}\right|^2,$$

$$\mathbf{A}(\mathbf{k}, \mathbf{k}_1) = \frac{1}{k_1 x}\left\{[1-v^2]^{1/2}\left[\mathbf{k}_1 - \frac{\mathbf{v}}{x}(\mathbf{k}\mathbf{k}_1 - \Omega_t(\mathbf{k}_1\mathbf{v}))\right]\right\},$$

$$x = \Omega_t - \mathbf{k}\mathbf{v}; \tag{3.5}$$

$$\mathbf{B}_l(\mathbf{k},\mathbf{k}_1) = \frac{\omega_{0e}^2}{\Omega_t\Omega_{l_1}\Omega_2}\left\{\frac{\mathbf{k}_1}{k_1}\frac{[1-\Omega_{l_1}\Omega_2(\varepsilon_e^l(\Omega_2 k_2)-1)/\omega_{0e}^2]}{\varepsilon^l(\Omega_2 k_2)} + \frac{\Omega_t\Omega_2}{\Omega_{l_1}}\frac{k_1}{k_2^2}\left[\frac{\mathbf{k}_2}{\Omega_2\varepsilon^l(\Omega_2 k_2)} - \frac{k_2^2\mathbf{v} - \mathbf{k}_2\Omega_2}{k_2^2 - \Omega_2^2\varepsilon^t(\Omega_2 k_2)}\right]\right\},$$

$$\mathbf{k}_2 = \mathbf{k} - \mathbf{k}_1, \qquad \Omega_2 = \Omega_t - \Omega_{l_1}.$$

We note that if the frequency of the transverse waves $\Omega_t \gg \Omega_{l_1} \approx \omega_{0e}$, then only above-thermal particles take part in the scattering. The expression for the probabilities W^{tl_1} is the same as the one found earlier in a paper by Gaĭlitis and Tsytovich [23] and used in the papers by Tsytovich and the author [11].

If, on the other hand, the frequency of the transverse waves is close to that of the longitudinal waves, subthermal particles with $w \lesssim v_{Te}$ can take also part, and their number exceeds by far the number of above-thermal particles. In that case the expressions for $W_q^{tl_1}$ simplify and take the form[6]

$$W_e^{tl_1}(\mathbf{p}, \mathbf{k}, \mathbf{k}_1) = \left(\frac{e_e^2}{m_e}\right)^2 \frac{\delta(\Omega_t - \Omega_{l_1} - (\mathbf{k}-\mathbf{k}_1)\mathbf{v})}{(2\pi)^3\omega_{0e}^2} \times \left|\frac{\varepsilon_i^l(\Omega_2 k_2)}{\varepsilon^l(\Omega_2 k_2)}\right|^2 \sin^2\widehat{\mathbf{k}\mathbf{k}_1},$$

$$W_i^{tl_1}(\mathbf{p}, \mathbf{k}, \mathbf{k}_1) = \left(\frac{e_e e_i}{m_e}\right)^2 \frac{\delta(\Omega_t - \Omega_{l_1} - (\mathbf{k}-\mathbf{k}_1)\mathbf{v})}{(2\pi)^3\omega_{0e}^2} \times \left|\frac{\varepsilon_e^l(\Omega_2 k_2) - 1}{\varepsilon^l(\Omega_2 k_2)}\right|^2 \sin^2\widehat{\mathbf{k}\mathbf{k}_1}. \tag{3.6}$$

<u>s,t-scattering</u>. Since the frequency of the sound waves is much smaller than the frequency of the transverse waves, the subthermal electrons can take part in the scattering only when $k \sim \omega_{0e}$ and $k_1 \gtrsim \omega_{0e}/v_{Te}$. In the opposite case of long-wavelength sound waves $k_1 \ll \omega_{0e}/v_{Te}$ only above-thermal electrons with $v \gg v_{Te}$ can take part in scattering. As far as scattering ions are concerned, they are always clearly above-thermal (i.e., $v \gg v_{Ti}$). Taking into account that $\Omega_{s_1} \ll k_1 v_{Te}$, we find

$$W_e^{ts_1}(\mathbf{p}, \mathbf{k}, \mathbf{k}_1) = \left(\frac{e_e^2}{m_e}\right)^2 \frac{\delta(\Omega_t - \Omega_{s_1} - (\mathbf{k}-\mathbf{k}_1)\mathbf{v})}{(2\pi)^3\omega_{0i}^2}\left|\frac{\Omega_{s_1}^3}{\Omega_t}\right| \times \left|\frac{[\mathbf{k}(\mathbf{A}+\mathbf{B}_s)]}{k}\right|^2,$$

$$W_i^{ts_1}(\mathbf{p},\mathbf{k},\mathbf{k}_1) = \left(\frac{e_e e_i}{m_e}\right)^2 \frac{\delta(\Omega_t - \Omega_{s_1} - (\mathbf{k}-\mathbf{k}_1)\mathbf{v})}{(2\pi)^3\omega_{0i}^2} \times \left|\frac{\Omega_{s_1}^3}{\Omega_t}\right|\left|\frac{[\mathbf{k}\mathbf{B}_s]}{k}\right|^2 \tag{3.7}$$

where $\mathbf{A}$ is defined by Eq. (3.5) while[7]

$$\mathbf{B}_s(\mathbf{k}, \mathbf{k}_1) = -\frac{1}{\Omega_t}\left\{\frac{\mathbf{k}_1}{k_1}\frac{[\varepsilon_e^l(\Omega_2 k_2) - 1]}{\varepsilon^l(\Omega_2 k_2)} + \frac{\Omega_2}{k_2^2}\frac{\omega_{0e}^2}{k_1 v_{Te}^2} \times \left[\frac{\mathbf{k}_2}{\Omega_2\varepsilon^l(\Omega_2 k_2)} - \frac{k_2^2\mathbf{v} - \mathbf{k}_2\Omega_2}{k_2^2 - \Omega_2^2\varepsilon^t(\Omega_1 k_2)}\right]\right\}$$

$$\mathbf{k}_2 = \mathbf{k} - \mathbf{k}_1, \quad \Omega_2 = \Omega_t - \Omega_{s_1}. \tag{3.8}$$

As a rule most interest lies in the probability for the interaction through subthermal particles, the number of which is relatively large. In that case the expression for the probability of scattering by electrons takes the form

$$W_e^{ts_1}(\mathbf{p}, \mathbf{k}, \mathbf{k}_1) \approx \left(\frac{e_e^2}{m_e}\right)^2 \frac{\delta(\Omega_t - \Omega_{s_1} - (\mathbf{k}-\mathbf{k}_1)\mathbf{v})}{(2\pi)^3} \times \frac{\omega_{0i}}{|\Omega_t^3|}\sin^2\widehat{\mathbf{k}\mathbf{k}_1}. \tag{3.9}$$

<u>ll-scattering</u>. When particles interact with Langmuir oscillations, the main role is played by the process of the emission of an l-plasmon with momentum $\mathbf{k}$ and the absorption of an l_1-plasmon with momentum $\mathbf{k}_1$. The process of simultaneous emission (or absorption) of two l-plasmons (cor-

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*$[\mathbf{k}\mathbf{B}_l] = \mathbf{k} \times \mathbf{B}_l$.

5) We note that we have written the expression for $\mathbf{B}_l$ in such a form only in order to take at once into account both above-thermal and subthermal particles. In particular, if we are interested in scattering by subthermal particles with $v \lesssim v_{Te}$ we must neglect all terms except the one proportional to $\epsilon_e^l(\Omega_2 k_2) - 1$.

6) Like in the case of tt-scattering, the expression for $W_e^{tl_1}(\mathbf{p},\mathbf{k},\mathbf{k}_1)$ is valid under the condition

$$\left|\frac{\varepsilon_i^l(\Omega_2 k_2)}{\varepsilon^l(\Omega_2 k_2)}\right| > \max\left\{\frac{\Omega_2 k_1}{\Omega_{l_1} k_2}, \frac{\Omega_2 k}{\Omega_t k_2}\right\}.$$

7) We note that when $\Omega_2 \gg k_2 v_{Te}$ the first term in (3.8), proportional to $\epsilon_e^l(\Omega_2 k_2) - 1$, must be neglected compared to the others.

responding probability $\widetilde{W}_q^{ll_1}$) plays, generally speaking a less important role. Indeed, since the frequency of the Langmuir oscillations depends always weakly on the wavelength, it follows from the energy and momentum conservation laws that the process corresponding to the probability $W_q^{ll_1}$ takes place with participation of slow particles with $v \approx v_{Te}$, $kv_{Te}/\omega_{0e} \ll v_{Te}$. At the same time, in the process of simultaneous emission (absorption) of two l-plasmons only fast, above-thermal particles, the number of which is much smaller than the number of subthermal particles, can take part. On the other hand, in the corresponding first-order process, i.e., in the process of emission (absorption) by a particles of one l-plasmon also only above-thermal particles, with velocities of the same order of magnitude as the velocity of particles taking part in the process of simultaneous emission of two l-plasmons, take part. Therefore, in the framework of the approximation used here, taking into account the process of the simultaneous emission (absorption) of two plasmons in many cases leads only to small corrections to the first-order process, whereas taking into account the process of the simultaneous absorption and emission of two plasmons may turn out to be very important since the number of particles taking part in this process may by far exceed the number of particles taking part in the first-order scattering process.

In accordance with this we give only the expression for the probability $W_q^{ll_1}$:

$$W_e^{ll_1}(\mathbf{p}, \mathbf{k}, \mathbf{k}_1) = \left(\frac{e_e^2}{m_e}\right)^2 \frac{\delta(\Omega_l - \Omega_{l_1} - (\mathbf{k} - \mathbf{k}_1)\mathbf{v})}{(2\pi)^3 \omega_{0e}^2} \times \left| \frac{2\mathbf{kv}}{\Omega_l} + \frac{\varepsilon_i^l(\Omega_2 k_2)}{\varepsilon^l(\Omega_2 k_2)} \right|^2 \cos^2 \widehat{\mathbf{k}\mathbf{k}_1},$$

$$W_i^{ll_1}(\mathbf{p}, \mathbf{k}, \mathbf{k}_1) = \left(\frac{e_e e_i}{m_e}\right)^2 \frac{\delta(\Omega_l - \Omega_{l_1} - (\mathbf{k} - \mathbf{k}_1)\mathbf{v})}{(2\pi)^3 \omega_{0e}^2} \times \left| \frac{\varepsilon_e^l(\Omega_2 k_2) - 1}{\varepsilon^l(\Omega_2 k_2)} \right|^2 \cos^2 \widehat{\mathbf{k}\mathbf{k}_1},$$

$$\mathbf{k}_2 = \mathbf{k} - \mathbf{k}_1, \qquad \Omega_2 = \Omega_l - \Omega_{l_1}. \tag{3.10}$$

<u>sl-scattering</u>. The probability for scattering in this case has exactly the same form as the probability for the st-scattering with the difference that the vector product is replaced in this case by the scalar product and instead of the frequencies Ω_t of the transverse waves we have the frequency Ω_l of the longitudinal waves. Depending on the wavelength of the s-plasmons both subthermal and above-thermal electrons can take part in the sl-scattering. In contrast to the ll-scattering, the process of the simultaneous emission of an l-plasmon and an s-plasmon can then also take place with the participation of subthermal particles. Ions, however, taking part in sl-scattering always have a velocity much larger than the thermal velocity v_{Ti}.

We can accordingly neglect the scattering by ions and take into account only the scattering by subthermal electrons. Recognizing that the latter takes place only for relatively short-wavelength s-plasmons with $k_1 \geq \omega_{0e}/v_{Te}$ when $\Omega_{s_1} \approx \omega_{0i}$ and $k_1 \gg k$, we get

$$W_e^{ls_1}(\mathbf{p}, \mathbf{k}, \mathbf{k}_1) = \left(\frac{e_e^2}{m_e}\right)^2 \frac{\delta(\Omega_l - \Omega_{s_1} - (\mathbf{k} - \mathbf{k}_1)\mathbf{v})}{(2\pi)^3} \times \frac{\omega_{0i}}{\omega_{0e}^3} \cos^2 \widehat{\mathbf{k}\mathbf{k}_1}. \tag{3.11}$$

<u>ss-scattering</u>. We now consider the last scattering process in which, in contrast to all processes considered above, an important role is played no longer by the electronic but by the ionic component of the plasma current. As in the simultaneous emission of two l-plasmons, when we consider the interaction of particles with s-plasmons we can neglect the scattering of s-plasmons by electrons, the velocity of which is of the same order of magnitude or less than the velocity of electrons taking part in the Cerenkov emission of a single s-plasmon. In other words, non-linear effects will be small for an electron and the total interaction with sound waves will be mainly determined by the first-order process. For ions, however, the situation is different. Indeed, whereas only above-thermal ions with $v \gg v_{Ti}$ take part in the first order processes, in second-order processes already subthermal ions the number of which is considerably larger can take part. In particular, such a situation occurs for the process of the simultaneous absorption and emission of two short-wavelength s-plasmons with $k, k_1 > \omega_{0e}/v_{Te}$. We can then, however, neglect also processes of scattering of s-plasmons by above-thermal ions.

In accordance with this we give here only the expressions for the probability of the scattering of an s-plasmon by a subthermal ion:

$$W_i^{ss_1}(\mathbf{p}, \mathbf{k}, \mathbf{k}_1) = 4\left(\frac{e_i^2}{m_i}\right) \times \frac{\delta(\Omega_s - \Omega_{s_1} - (\mathbf{k} - \mathbf{k}_1)\mathbf{v})}{(2\pi)^3} \frac{(\mathbf{kv})^2}{\omega_{0i}^4} \cos^2 \widehat{\mathbf{k}\mathbf{k}_1}. \tag{3.12}$$

4. PROBABILITIES FOR DECAY PROCESSES

We turn now to the probabilities for decay processes.[8)] When there are three different types of waves present, there are, generally speaking, ten different types of processes possible, namely ($\alpha \neq \beta \neq \gamma$):

(α, α, α), (α, α, β), (α, α, γ), (β, β, β), (β, β, γ), (γ, γ, γ), (γ, γ, α), (γ, γ, β), (β, β, α), (α, β, γ).

The symbol (...) indicates here the totality of decays in which a quantum of one, arbitrary, type (of the ones indicated in the brackets) decays into the other two.[9)]

Taking into account the dispersion relation (3.1) we can easily check however, that in the case of an isotropic plasma, considered in the present paper (when, let us say, α = t, β = l, γ = s) the processes (t, t, t), (l, l, l), (s, s, s), (s, s, t), and (s, s, l) violate the energy and momentum conservation laws (2.6) and can thus not take place. We need thus only determine the probabilities for the following types of decay: (t, t, l), (l, l, t), (t, t, s), (l, l, s), (t, l, s). The method for evaluating the probabilities $V^{\alpha\beta\gamma}(\mathbf{k}, \mathbf{k}_1, \mathbf{k}_2)$ is formally the same as the method of calculating scattering probabilities. The only difference lies in the fact that whereas to find the scattering probability we must evaluate the change in the field energy caused by the presence of a test particle in the plasma, to find the decay probabilities we must evaluate the change in the electromagnetic energy connected only with the nonlinear corrections to the plasma current, completely neglecting the test-particle current which takes into account the presence of resonating particles (for more details see [20]).

Omitting intermediate steps, we give the final results.[10)]

(l, t, t) decays:[11)]

$$V^{l,t,t_1}(\mathbf{k}, \mathbf{k}_1, \mathbf{k}_2) = \frac{e_e^2}{64\pi m_e^2}\frac{\omega_{0e}k^2}{|\Omega_t\Omega_{t_1}|}[1+\cos^2\widehat{\mathbf{k}_1\mathbf{k}_2}] \times \delta(\Omega_{t_1}+\Omega_{t_2}-\Omega_l)\,\delta(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k}); \quad (4.1)$$

(t, l, l) decays:

$$V^{t,l_1,l_2}(\mathbf{k}, \mathbf{k}_1, \mathbf{k}_2) = \frac{e_e^2}{16\pi m_e^2}\frac{\omega_{0e}^2}{k_2|\Omega_t|}\left(\frac{k_1^2}{\Omega_{l_1}} - \frac{k_2^2}{\Omega_{l_2}}\right)^2 \times \sin^2\widehat{\mathbf{k}_1\mathbf{k}_2}\,\delta(\Omega_{l_1}+\Omega_{l_2}-\Omega_t)\,\delta(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k}); \quad (4.2)$$

(s, t, t) decays:

$$V^{s,t_1,t_2}(\mathbf{k}, \mathbf{k}_1, \mathbf{k}_2) = \frac{1}{64\pi}\left(\frac{e_e}{m_e v_{Te}^2}\right)^2 \frac{|\Omega_s|^3\omega_{0e}^4}{|\Omega_{t_1}\Omega_{t_2}|k^2\omega_{0i}^2} \times [1+\cos^2\widehat{\mathbf{k}_1\mathbf{k}_2}]\,\delta(\Omega_{t_1}+\Omega_{t_2}-\Omega_s)\,\delta(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k}); \quad (4.3)$$

(s, l, l) decays:

$$V^{s,l_1,l_2}(\mathbf{k}, \mathbf{k}_1, \mathbf{k}_2) = \frac{1}{16\pi}\left(\frac{e_e}{m_e v_{Te}^2}\right)^2 \frac{\omega_{0e}^2|\Omega_s|^3}{\omega_{0i}^2k^2} \times \cos^2\widehat{\mathbf{k}_1\mathbf{k}_2}\,\delta(\Omega_{l_1}+\Omega_{l_2}-\Omega_s)\,\delta(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k}); \quad (4.4)$$

(s, l, t) decays:

$$V^{s,l_1,t_2}(\mathbf{k}, \mathbf{k}_1, \mathbf{k}_2) = \frac{1}{32\pi}\left(\frac{e_e}{m_e v_{Te}^2}\right)^2 \frac{\omega_{0e}^3|\Omega_s|^3}{\omega_{0i}^2k^2|\Omega_{t_2}|} \times \sin^2\widehat{\mathbf{k}_1\mathbf{k}_2}\,\delta(\Omega_{l_1}+\Omega_{t_2}-\Omega_s)\,\delta(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k}). \quad (4.5)$$

5. SCATTERING CROSS SECTIONS

For given values of the scattering probabilities w_q^α, $W_q^{\alpha\beta}$, $\widetilde{W}_q^{\alpha\beta}$, and the decay probabilities $V^{\alpha\beta\gamma}$ the set of Eqs. (2.2) and (2.3) completely determines the evolution of the "field-particles" system in space and time. All the same, the solution of this set of non-linear equations is a very complicated problem. One must, however, point out that in a number of cases which are of practical interest these equations can be reduced to a simpler form. Indeed, taking into account that according to our initial assumptions, on the one hand, the distribution of resonating particles is mainly determined by first-order processes while, on the other hand, the energy contained in the electromagnetic field is much smaller than the kinetic energy of the random motion of the particles, we can in Eq. (2.2) for the distribution function neglect all terms corresponding to second-order processes.[12)] In accordance with this and bearing in mind that the number of subthermal particles by far exceeds the number of above-thermal ones, we can in Eqs. (2.3) replace the probabilities for second-order scattering

8)Similar processes have also been considered by other authors[12,13] who studied the scattering and transformation of a wave in a plasma by fluctuations.

9)For instance, (α,α,β) indicates all decays of a quantum of type α into quanta of type α and β and of a quantum of type β into two quanta of type α.

10)We give here only expressions for one probability of a given type of decay, bearing in mind that the others can easily be obtained using Eq.(2.4).

11)For the case of hf fields, $\Omega_t \gg \omega_{0e}$, the expression for V^{l,t_1,t_2} was also given in[11].

12)Exceptions may be processes occurring with the participation of transverse waves, since the corresponding first-order probabilities w_q^r vanish identically. In that case we must take into account in the equation for the particle and quantum distribution functions second-order scattering processes occuring when above-thermal particles also participate.

processes by their values for subthermal particles and afterwards integrate over the momenta assuming, say, a Maxwell velocity distribution for the particles. However, in the terms corresponding to first-order scattering processes we can not, generally speaking, replace the distribution function by a Maxwellian one and we must retain them in their previous form.

The system (2.2) and (2.3) thus takes the form

$$\frac{dF_q}{dt} = \sum_{l,m} \frac{\partial}{\partial p_l} D_{l,m}^{(q)} \frac{\partial F_q}{\partial p_m},$$

$$D_{l,m}^{(q)} = \sum_{\alpha} \int d\mathbf{k}\, k_l k_m w_q^{\alpha}(\mathbf{p}, \mathbf{k}) N_{\alpha}(\mathbf{k}), \tag{5.1}$$

$$\frac{dN_\alpha}{dt} = (2\pi)^3 \sum_q \int d\mathbf{p}\, w_q^{\alpha}(\mathbf{p}, \mathbf{k}) N_\alpha(\mathbf{k}) \left(\mathbf{k}\frac{\partial F_q}{\partial \mathbf{p}}\right)$$
$$+ \sum_\beta \int d\mathbf{k}_1 \hat{\sigma}^{\alpha\beta}(\mathbf{k}, \mathbf{k}_1) N_\alpha(\mathbf{k}) N_\beta(\mathbf{k}_1) + \sum_{\beta,\gamma} \int d\mathbf{k}_1\, d\mathbf{k}_2 \{N_\beta N_\gamma$$
$$\times [V^{\alpha\beta\gamma} + V^{\beta\gamma\alpha} + V^{\gamma\alpha\beta}] - N_\alpha N_\beta [V^{\alpha\beta\gamma} - V^{\beta\gamma\alpha} + V^{\gamma\alpha\beta}]$$
$$- N_\alpha N_\gamma [V^{\alpha\beta\gamma} + V^{\beta\gamma\alpha} - V^{\gamma\alpha\beta}]\}. \tag{5.2}$$

Equation (5.2) now no longer contains the scattering probabilities, but some other quantities $\hat{\sigma}^{\alpha\beta}$ connected with the usual scattering cross sections $\sigma^{\alpha\beta}$ and $\tilde{\sigma}^{\alpha\beta}$ by the following relations:

$$\hat{\sigma}^{\alpha\beta}(\mathbf{k}, \mathbf{k}_1) = -\sum_q \left[\frac{\Omega_\alpha - \Omega_\beta}{T_q} \sigma_q^{\alpha\beta}(\mathbf{k}, \mathbf{k}_1) + \frac{\Omega_\alpha + \Omega_\beta}{T_q} \tilde{\sigma}_q^{\alpha\beta}(\mathbf{k}, \mathbf{k}_1)\right],$$

$$\sigma^{\alpha\beta} = \sum_q \sigma_q^{\alpha\beta}, \quad \tilde{\sigma}^{\alpha\beta} = \sum_q \tilde{\sigma}_q^{\alpha\beta}, \quad \sigma_q^{\alpha\beta} = (2\pi)^3 \int d\mathbf{p}\, W_q^{\alpha\beta} F_q^{(0)},$$

$$\tilde{\sigma}_q^{\alpha\beta} = (2\pi)^3 \int d\mathbf{p}\, \tilde{W}_q^{\alpha\beta} F_q^{(0)},$$

$$F_q^{(0)} = \frac{n_q}{(2\pi m_q T_q)^{3/2}} \exp\left(-\frac{p^2}{2m_q T_q}\right). \tag{5.3}$$

One finds these cross sections easily if one uses the expressions given above for the probabilities for the scattering of quanta by subthermal particles. Bearing Eq. (5.3) in mind we give only the expressions for $\sigma^{\alpha\beta}$ and $\tilde{\sigma}^{\alpha\beta}$. They have the following form:

$$\sigma_e^{t,t_1}(\mathbf{k}, \mathbf{k}_1) = \frac{e_e^2}{m_e} \frac{\omega_{0e}^2}{\Omega_t \Omega_{t_1}} \left|\frac{\varepsilon_i^l(\Omega_2 k_2)}{\varepsilon^l(\Omega_2 k_2)}\right|^2 \frac{1 + \cos^2 \widehat{\mathbf{k}\mathbf{k}_1}}{4(2\pi)^{3/2} k_2 v_{Te}} \times \exp\left[-\frac{(\Omega_t - \Omega_{t_1})^2}{2k_2^2 v_{Te}^2}\right],$$

$$\sigma_i^{t,t_1}(\mathbf{k}, \mathbf{k}_1) = \frac{|e_e e_i|}{m_e} \frac{\omega_{0e}^2}{\Omega_t \Omega_{t_1}} \left|\frac{\varepsilon_e^l(\Omega_2 k_2) - 1}{\varepsilon^l(\Omega_2 k_2)}\right|^2 \frac{1 + \cos^2 \widehat{\mathbf{k}\mathbf{k}_1}}{4(2\pi)^{3/2} k_2 v_{Ti}} \times \exp\left[-\frac{(\Omega_t - \Omega_{t_1})^2}{2k_2^2 v_{Ti}^2}\right], \tag{5.4}$$

$$\tilde{\sigma}_e^{t,t_1}(\mathbf{k}, \mathbf{k}_1) = \tilde{\sigma}_i^{t,t_1}(\mathbf{k}, \mathbf{k}_1) = 0, \quad \mathbf{k}_2 = \mathbf{k} - \mathbf{k}_1, \quad \Omega_2 = \Omega_t - \Omega_{t_1};$$

$$\sigma_e^{t,l_1}(\mathbf{k}, \mathbf{k}_1) = \frac{e_e^2}{m_e} \left|\frac{\varepsilon_i^l(\Omega_2 k_2)}{\varepsilon^l(\Omega_2 k_2)}\right|^2 \frac{\sin^2 \widehat{\mathbf{k}\mathbf{k}_1}}{2(2\pi)^{3/2} k_2 v_{Te}} \times \exp\left[-\frac{(\Omega_t - \Omega_{l_1})^2}{2k_2^2 v_{Te}^2}\right],$$

$$\sigma_i^{t,l_1}(\mathbf{k}, \mathbf{k}_1) = \frac{|e_e e_i|}{m_e} \left|\frac{\varepsilon_e^l(\Omega_2 k_2) - 1}{\varepsilon^l(\Omega_2 k_2)}\right|^2 \frac{\sin^2 \widehat{\mathbf{k}\mathbf{k}_1}}{2(2\pi)^{3/2} k_2 v_{Ti}} \times \exp\left[-\frac{(\Omega_t - \Omega_{l_1})^2}{2k_2^2 v_{Ti}^2}\right], \tag{5.5}$$

$$\tilde{\sigma}_e^{t,l_1}(\mathbf{k}, \mathbf{k}_1) = \tilde{\sigma}_i^{t,l_1}(\mathbf{k}, \mathbf{k}_1) = 0, \quad \mathbf{k}_2 = \mathbf{k} - \mathbf{k}_1, \quad \Omega_2 = \Omega_t - \Omega_{l_1};$$

$$\sigma_e^{t,s_1}(\mathbf{k}, \mathbf{k}_1) = \tilde{\sigma}_e^{t,s_1}(\mathbf{k}, \mathbf{k}_1) = \frac{e_e^2}{m_e} \frac{\omega_{0e}^2 \omega_{0i}}{\Omega_t^3} \frac{\sin^2 \widehat{\mathbf{k}\mathbf{k}_1}}{2(2\pi)^{3/2} k_2 v_{Te}} \times \exp\left(-\frac{\Omega_t^2}{2k_2^2 v_{Te}^2}\right), \tag{5.6}$$

$$\sigma_i^{t,s_1}(\mathbf{k}, \mathbf{k}_1) = \tilde{\sigma}_i^{t,s_1}(\mathbf{k}, \mathbf{k}_1) = 0, \quad \mathbf{k}_2 = \mathbf{k} - \mathbf{k}_1;$$

$$\sigma_e^{l,l_1}(\mathbf{k}, \mathbf{k}_1) = \frac{e_e^2}{m_e} \left[\left|\frac{\varepsilon_i^l(\Omega_2 k_2)}{\varepsilon^l(\Omega_2 k_2)}\right|^2 + \frac{4v_{Te}^2}{\omega_{0e}^2} \frac{[\mathbf{k}\mathbf{k}_2]^2}{k_2^2}\right] \times \frac{\cos^2 \widehat{\mathbf{k}\mathbf{k}_1}}{2(2\pi)^{3/2} k_2 v_{Te}} \exp\left[-\frac{(\Omega_l - \Omega_{l_1})^2}{2k_2^2 v_{Te}^2}\right],$$

$$\sigma_i^{l,l_1}(\mathbf{k}, \mathbf{k}_1) = \frac{|e_e e_i|}{m_e} \left|\frac{\varepsilon_e^l(\Omega_2 k_2) - 1}{\varepsilon^l(\Omega_2 k_2)}\right|^2 \frac{\cos^2 \widehat{\mathbf{k}\mathbf{k}_1}}{2(2\pi)^{3/2} k_2 v_{Ti}} \times \exp\left[-\frac{(\Omega_l - \Omega_{l_1})^2}{2k_2^2 v_{Ti}^2}\right], \tag{5.7}$$

$$\tilde{\sigma}_e^{l,l_1}(\mathbf{k}, \mathbf{k}_1) = \tilde{\sigma}_i^{l,l_1}(\mathbf{k}, \mathbf{k}_1) = 0, \quad \mathbf{k}_2 = \mathbf{k} - \mathbf{k}_1, \quad \Omega_2 = \Omega_l - \Omega_{l_1};$$

$$\sigma_e^{l,s_1}(\mathbf{k}, \mathbf{k}_1) = \tilde{\sigma}_e^{l,s_1}(\mathbf{k}, \mathbf{k}_1) = \frac{e_e^2}{m_e} \frac{\omega_{0i}}{\omega_{0e}} \frac{\cos^2 \widehat{\mathbf{k}\mathbf{k}_1}}{2(2\pi)^{3/2} k_1 v_{Te}} \times \exp\left(-\frac{\Omega_l^2}{2k_1^2 v_{Te}^2}\right), \tag{5.8}$$

$$\sigma_i^{l,s_1}(\mathbf{k}, \mathbf{k}_1) = \tilde{\sigma}_i^{l,s_1}(\mathbf{k}, \mathbf{k}_1) = 0;$$

$$\sigma_i^{s,s_1}(\mathbf{k}, \mathbf{k}_1) = \frac{e_i^2}{m_i} \frac{k^2 k_1^2 v_{Ti}^2}{k_2^2 \omega_{0i}^2} \frac{2 \sin^2 \widehat{\mathbf{k}\mathbf{k}_1} \cos^2 \widehat{\mathbf{k}\mathbf{k}_1}}{(2\pi)^{3/2} k_2 v_{Ti}} \times \exp\left[-\frac{(\Omega_s - \Omega_{s_1})^2}{2k_2^2 v_{Ti}^2}\right], \tag{5.9}$$

$$\sigma_e^{s,s_1}(\mathbf{k}, \mathbf{k}_1) = \tilde{\sigma}_e^{s,s_1}(\mathbf{k}, \mathbf{k}_1) = \tilde{\sigma}_i^{s,s_1}(\mathbf{k}, \mathbf{k}_1) = 0, \quad \mathbf{k}_2 = \mathbf{k} - \mathbf{k}_1.$$

We note that Eq. (5.4) for the tt-scattering cross section was obtained by a different method in[24,25]. On the other hand, the expression (5.7) for the cross section for ll-scattering by electrons goes over into the expression found by a number of authors [14,15,26] if we neglect in the square brackets the term $|\epsilon_i^l(\Omega_2 k_2)/\epsilon^l(\Omega_2 k_2)|^2$. This is, however, only possible for sufficiently short waves, when

$$\frac{\omega_{0e}}{kv_{Te}} \ll \max\left\{\left(\frac{m_i}{m_e}\right)^{1/3}, \quad \frac{T_i}{T_e}\right\}.$$

A similar term must also be taken into account in the cross sections (and, of course, in the corresponding probabilities) σ_e^{t,t_1}, σ_e^{t,l_1}; it was, however omitted in [15,26].[13])

6. STUDY OF THE EQUATIONS IN SOME PARTICULAR CASES

Using the expressions obtained above for the scattering and decay processes, we can study different processes occurring in a plasma, taking into account the non-linear interaction of the different types of waves with one another. Such a study was made in some particular cases by the present author and Tsytovich [11] (see also [2,14,15,27]). We shall briefly dwell here upon other very simple particular cases and we shall try to estimate from their example, albeit qualitatively, the role of the non-linear effects.

1. We consider first of all the case of the interaction of transverse waves with an isothermal Maxwellian plasma. We shall then assume that the intensity of the transverse waves, excited, say, by some external source, is by far larger than the intensity of the Langmuir waves. Moreover, we assume that the transverse noise is non-vanishing only in a small region near the points $k = k_1^0$ and $k = k_2^0$ and vanishes outside the vicinity of these points. Such a situation may arise, for instance, when the plasma is bombarded by two "beams" of transverse waves with $k = k_1^0$ and $k = k_2^0$, generated by an external source.

We assume initially that k_1^0 and k_2^0 are such that the decay conditions (2.6) are not satisfied. If we, furthermore, assume that the wave packets are sufficiently narrow, we can neglect the interaction of the waves with one another inside each packet and take into account only the interaction between the packets. We can in this case write the equations for the amplitude of the packets in the form

$$\frac{dX_1}{dt} = a\Omega_1 X_1 X_2, \qquad \frac{dX_2}{dt} = -a\Omega_2 X_1 X_2, \tag{6.1}$$

where

$$a(\mathbf{k}_1^0, \mathbf{k}_2^0) = (2\pi)^3 \frac{n_e T_e}{\Omega_1 \Omega_2} \hat{\sigma}^{tt}(\mathbf{k}_1^0, \mathbf{k}_2^0), \qquad \Omega_1 = \Omega_t(\mathbf{k}_1^0), \qquad \Omega_2 = \Omega_t(\mathbf{k}_2^0), \tag{6.2}$$

while $X_{1,2}(t)$ are the ratios of the electromagnetic energy densities in the "beams" to the average kinetic energy density $n_e T_e$ of the particles in the plasma.

The solution of Eqs. (6.1) can be found elementarily. It has the form

$$X_1(t) = \frac{X_1(0)[\Omega_2 X_1(0) + \Omega_1 X_2(0)]}{\Omega_2 X_1(0) + \Omega_1 X_2(0) \exp\{-[\Omega_2 X_1(0) + \Omega_1 X_2(0)]at\}},$$

$$X_2(t) = \frac{X_2(0)[\Omega_2 X_1(0) + \Omega_1 X_2(0)]}{\Omega_1 X_2(0) + \Omega_2 X_1(0) \exp\{[\Omega_2 X_1(0) + \Omega_1 X_2(0)]at\}} \tag{6.3}$$

From this it follows that taking the non-linearity into account leads to a transfer of energy from the region of larger k to the region of smaller k. The velocity of transfer is determined both by the quantity $a(k_1^0, k_2^0)$ and by the initial values $X_1(0)$ and $X_2(0)$ of the amplitudes. A part of the electromagnetic field energy, equal to

$$\Delta X(t) = X_1(0) + X_2(0) - X_1(t) - X_2(t), \tag{6.4}$$

goes then over into the energy of the kinetic motion of the particles in the plasma. Depending on the relation between the quantities k_1^0 and k_2^0 this energy can be transferred either to the electrons only, or to the ions only, or to both simultaneously.

It follows from (6.3) and (6.4) that the maximum possible magnitude of this energy is equal to

$$\max \Delta X = \begin{cases}(\Omega_2 - \Omega_1) X_2(0)/\Omega_2 \text{ when } k_2^0 > k_1^0 \\ (\Omega_1 - \Omega_2) X_1(0)/\Omega_1 \text{ when } k_2^0 < k_1^0\end{cases}, \tag{6.5}$$

and the characteristic transfer time is

$$\tau_0 = [(\Omega_1 X_2(0) + \Omega_2 X_1(0)) |a(\mathbf{k}_1^0, \mathbf{k}_2^0)|]^{-1}. \tag{6.6}$$

In particular it follows from this that, other conditions being equal, it is a minimum when the "beams" meet head on.

We turn now to the case when the centers k_1^0 and k_2^0 of the wave packets are such that the decay conditions $\Omega_2 - \Omega_1 = \Omega_l(k_2^0 - k_1^0)$ turn out to be satisfied. Assuming for concreteness that $k_2^0 > k_1^0$, and writing $N_t(k)$ in the form

$$N_t(\mathbf{k}) = \frac{N_1(t)}{(2\pi\Delta_1^2)^{3/2}} \exp\left[-\frac{(\mathbf{k} - \mathbf{k}_1^0)^2}{2\Delta_1^2}\right] + \frac{N_2(t)}{(2\pi\Delta_2^2)^{3/2}} \times \exp\left[-\frac{(\mathbf{k} - \mathbf{k}_2^0)^2}{2\Delta_2^2}\right], \tag{6.7}$$

where Δ_1 and Δ_2 are constants, we find

[13])Earlier, Gorbunov and Silin[14] have found an expression for the cross section for the scattering of Langmuir waves by ions, σ_i^{l,l_1}. However, the region of applicability indicated there is incorrect; this is connected with an incorrect expression found for the cross section σ_e^{l,l_1}.

$$\frac{dX_1}{dt}=b\Omega_1X_1X_2,\quad \frac{dX_2}{dt}=b\Omega_2X_1X_2,$$

$$\frac{dX_l}{dt}=\gamma_0X_l+b\omega_{0e}X_1X_2. \qquad (6.8)$$

Here $\gamma_0 = \gamma(\mathbf{k}_2^0 - \mathbf{k}_1^0)$ is the damping decrement of the linear theory,

$$X_{1,2}=\frac{\Omega_{1,2}N_{1,2}(t)}{(2\pi)^3n_eT_e},\quad X_l=\omega_{0e}\int\frac{d\mathbf{k}N_l(\mathbf{k})}{(2\pi)^3n_eT_e}$$

are dimensionless field energy densities, and

$$b(\mathbf{k}_1^0,\mathbf{k}_2^0)=\frac{\pi(1+\cos^2\widehat{\mathbf{k}_1^0\mathbf{k}_2^0})(\mathbf{k}_1^0-\mathbf{k}_2^0)^2\omega_{0e}^3T_e}{16m_e\Omega_1\Omega_2[(\Omega_2k_1^0\Delta_1)^2+(\Omega_1k_2^0\Delta_2)^2]^{1/2}}. \qquad (6.9)$$

Clearly, the solutions X_1 and X_2 are determined by Eqs. (6.3), in which we must replace a by b, while the intensity of the longitudinal waves is

$$X_l(t)=X_l(0)e^{\gamma_0t}+b\omega_{0e}e^{\gamma_0t}\int_0^t e^{-\gamma_0t'}X_1(t')X_2(t')\,dt'. \qquad (6.10)$$

Decays, like scattering, lead to a transfer of energy from the region of larger k to the region of smaller k. The excess energy goes now, however, into exciting longitudinal waves. If the damping decrement is $\gamma_0 = 0$, then

$$X_l(t)=X_l(0)+\frac{\omega_{0e}}{\Omega_2}X_2(0)\left[1-\frac{X_2(t)}{X_2(0)}\right].$$

From this it follows that the maximum noise intensity is reached in a time of order $\tau_0 = [b(\Omega_2X_1(0)+\Omega_1X_2(0))]^{-1}$ and is equal to

$$X_{l\,max}=X_l(0)+\omega_{0e}X_2(0)/\Omega_2.$$

In the case of finite values of γ_0 the amplitude of longitudinal noise initially increases, then reaches a maximum value, after which it starts to diminish and tends to zero as $t\to\infty$. Here, both the time to reach the maximum value and the magnitude of $X_{l\max}$ itself decrease with increasing γ_0.

2. We now consider effects which result from the non-linear interaction of longitudinal Langmuir waves with one another. Assuming that the intensity of the Langmuir waves is much larger than the intensity of the transverse waves and that there is no plasma sound (i.e., $T_e \gtrsim T_i$), we find from Eqs. (2.2)

$$\frac{dN_l}{dt}=\gamma(\mathbf{k})N_l(\mathbf{k})+N_l(\mathbf{k})\int d\mathbf{k}_1N_l(\mathbf{k}_1)[\hat{\sigma}^{l,l_1}(\mathbf{k},\mathbf{k}_1)$$
$$-2\int d\mathbf{k}_2V^{t_2ll_1}(\mathbf{k}_2,\mathbf{k},\mathbf{k}_1)], \qquad (6.11)$$

$$\frac{dN_t(\mathbf{k})}{dt}=\int d\mathbf{k}_1d\mathbf{k}_2N_l(\mathbf{k}_1)N_l(\mathbf{k}_2)V^{tl_1l_2}(\mathbf{k},\mathbf{k}_1,\mathbf{k}_2),$$

where the cross sections $\hat{\sigma}^{l,l_1}$ are determined by Eqs. (5.3) and (5.7), the probability $V^{tl_1l_2}$ by Eq. (4.2), while $\gamma(\mathbf{k})$ is the increment (decrement) of the linear theory.

The study of Eqs. (6.11) is in a well-defined sense a more complicated problem than the study of the analogous equations for transverse waves. Indeed, while for transverse waves the assumption of almost monochromatic behavior in a number of cases corresponds to the situation occurring in practice, in the present case the analogous assumption is rather far removed from reality as the "longitudinal" noise arising, say, due to the instability of the beam has a very broad spectrum. In accordance with this we restrict ourselves here merely to a qualitative analysis of Eqs. (6.11) and we give a number of estimates which enable us to understand, albeit in general terms, the character of the phenomena to which the inclusion of the non-linear interaction of the waves leads.

First of all we note that it follows from the second of Eqs. (6.11) that the presence of an intensive "longitudinal" noise leads to the generation of transverse waves of frequency $\sim 2\,\omega_{0e}$. Here a necessary condition for the generation is the presence in the spectrum of the "longitudinal" noise of waves satisfying the condition $|\mathbf{k}_1 + \mathbf{k}_2| = \sqrt{3}\,\omega_{0e}$. One can easily estimate the order of magnitude of the intensity of the generated transverse waves. Assuming, say, that the value of $N_l = \mathrm{const}$ for $0 < k < \omega_{0e}/v_{Te}$, we find

$$dU_t/dt=U_l2\pi\omega_{0e}v_{Te}{}^5U_l/n_eT_e. \qquad (6.12)$$

The presence of the factor v_{Te}^5 leads to the fact that the power of the generated transverse noise is very small for a low-temperature plasma and increases steeply when the electron temperature is increased.

We turn now to the equation for the longitudinal waves. Simple estimates show that in the majority of cases scattering processes rather than decay processes play the main role in Eq. (6.11) for N_l. Changing in (6.11) to dimensionless variables,

$$\varphi(\mathbf{x},\tau)=\frac{\omega_{0e}k_m{}^3N_l(\mathbf{k})}{(2\pi)^3n_eT_e},$$

$$\tau=\omega_{0e}t,\quad \Gamma=\frac{\gamma}{\omega_{0e}},\quad \mathbf{x}=\frac{\mathbf{k}}{k_m},\quad k_m=\frac{\omega_{0e}}{v_{Te}},$$

we find

$$\frac{d\varphi}{d\tau}=\varphi(\mathbf{x},\tau)[\Gamma(\mathbf{x},\tau)-\int d\mathbf{x}_1K(\mathbf{x},\mathbf{x}_1)\varphi(\mathbf{x}_1,\tau)],$$

$$K(\mathbf{x},\mathbf{x}_1)=\frac{3n_e}{2\omega_{0e}}(2\pi)^6(x^2-x_1{}^2)$$
$$\times\sum_q\int F_q{}^{(0)}(\mathbf{p})\,d\mathbf{p}\,W^{ll_1}(\mathbf{p},\mathbf{k},\mathbf{k}_1)\Big|_{\substack{\mathbf{k}=k_m\mathbf{x}\\ \mathbf{k}_1=k_m\mathbf{x}_1}} \qquad (6.13)$$

We note that in the particular case of relatively long-wavelength vibrations with

$$K < \frac{\omega_{0e}}{v_{Te}}\left(\frac{m_e T_i}{m_i T_e}\right)^{1/2},$$

when the ions give the main contribution to the scattering cross section while the scattering by electrons can be neglected, the kernel $K(\mathbf{x}, \mathbf{x}_1)$ takes on a relatively simple form (see also [14]):

$$K(\mathbf{x}, \mathbf{x}_1) = \frac{3\sqrt{\pi}}{4\sqrt{2}}\left[\frac{m_i T_i}{m_e T_e}\right]^{1/2}$$

$$\frac{z T_e^2}{(T_i + z T_e)^2}\ \frac{x^2 - x_1^2}{|\mathbf{x} - \mathbf{x}_1|}\cos^2 \widehat{\mathbf{x}\mathbf{x}_1}, \qquad z = \left|\frac{e_i}{e_e}\right|.$$

On must, in general, solve Eq. (6.13) together with the equation for the distribution function determining the increment $\Gamma(\mathbf{x}, \tau)$. Not aiming here at a rigorous solution of these equations we restrict ourselves merely to a qualitative study and indicate a number of physical consequences flowing from them.

First of all we note that as it follows from (2.4) that the kernel $K(\mathbf{x}, \mathbf{x}_1)$ is odd with respect to an interchange of its argument and positive for $x > x_1$, it is clear from (6.13) that taking into account second-order scattering effects leads to a transfer of the waves from the short-wavelength to the long-wavelength range of the spectrum. The excess energy occurring then goes, depending on the character of the spectrum, to heating either the electronic or the ionic component of the plasma, or both in equal degree.[14)] In accordance with this waves with low wave numbers are damped more slowly than would follow from the linear theory, while for a relatively intensive noise (or in the region $k > \omega_{0e}$, where $\gamma(k) = 0$) they can even get intensified. On the other hand, in the range of large wave numbers taking non-linear effects into account leads to an increase of the damping decrement for a stable (in the linear approximation) plasma and a decrease in the increment for a plasma, unstable compared with the linear theory. Furthermore, when rather intensive noise is present in the long-wavelength part of the spectrum situations may arise in which, for instance, a beam propagating in the plasma turns out to be stable with respect to the excitation of Langmuir waves. A sufficient condition for this is clearly the inequality

$$\Gamma(\mathbf{x}_0, \tau) < \varepsilon \int d\mathbf{x}_1 K(\mathbf{x}_0, \mathbf{x}_1)\varphi(\mathbf{x}_1, \tau),$$

where $\mathbf{x}_0$ is the value of the wavenumber corresponding to the maximum value of the increment of intensification.

In conclusion we dwell briefly upon the problem of the propagation of an electron beam in a plasma. Denoting the particle number density in the beam by n_1 and the velocity dispersion in the beam by Δv_0 one shows easily that the intensification increment

$$\Gamma(\mathbf{x}, \tau) \leqslant \Gamma_0 = \pi\frac{n_i}{n_e}\left(\frac{v_0}{\Delta v_0}\right)^2.$$

On the other hand, for a beam freely moving in an unbounded plasma, the maximum possible noise intensity U_l produced by the beam does not exceed the magnitude

$$U_l^{(0)} \approx \int \varphi d\mathbf{x} \leqslant \frac{n_1 v_0^2}{n_e v_{Te}^2}.$$

From this it follows that in the case of a sufficiently monochromatic beam ($\Delta v_0 \ll v_{Te}$) the second term in Eq. (6.11) is always much smaller than the first one and non-linear effects can only play an appreciable role in the later stages of the process when the beam is already rather spread out ($\Gamma \ll \Gamma_0$) while the noise has almost reached its maximum value.

The initial stage of the process, however, can be rather well described by the quasi-linear theory [21,22]. We must, perhaps, draw only attention to one effect which may occur as a result of the non-linear interaction between waves, already in the initial stage of the process. The spectrum is only broadened in the quasi-linear theory in the wavenumber range $k > \omega_{0e}/v_0$, and hence only the rear end of the beam ($v < v_0$) will spread out. The front end, however, remains practically unchanged. Taking non-linear effects into account, however, can be shown to lead to a broadening of the noise spectrum in the long-wavelength region. As a consequence, particles with velocities $v > v_0$ start to diffuse into the region of even larger velocities and some spreading out occurs also for the front end of the beam, which increases as the noise intensity in that region increases.

It is necessary to emphasize, however, that even the initial stage of the process of the interaction of a beam with a plasma can by far not always be described by a quasi-linear theory. Indeed, in a number of cases, for instance, when a continuous beam passes through a bounded plasma, the noise intensity in the plasma can appreciably exceed the particle energy density in the beam.

[14)]We emphasize that these two statements are valid in the very general case, regardless of the concrete form of the kernel $K(x,x_1)$ and that they are a direct consequence of Eqs.(2.4) and (5.3).

In that case it is clearly no longer possible to neglect the non-linear terms in Eq. (6.11), the non-linear interaction processes play an essential part, and it is necessary to start from the complete set of non-linear equations.

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