fact of absence of an asymmetry in the decays $\Sigma^+ \to n\pi^+$ and $\Sigma^- \to n\pi^-$. SU(6) symmetry distinguishes between the two possibilities dictated by experimental evidence, predicting pure p-wave in the decay $\Sigma^+ \to n\pi^+$. This can be checked experimentally [9]. Since expression (4) contains only one undetermined constant we find the following relationships between the S-amplitudes of all hadronic decays of baryons ($b \to b\pi$, $d \to d\pi$):

$$(\Lambda \to p\pi^-)_S = -(\Xi^- \to \Lambda\pi^-)_S = \sqrt{3/2}(\Sigma^- \to n\pi^-)_S$$
$$= -\sqrt{3}(\Sigma^+ \to p\pi^0)_S = \frac{1}{\sqrt{2}}(\Omega^- \to \Xi^{0*}\pi^-)_S. \quad (5)$$

From (4) naturally, there also follow the relations connected with the rule $|\Delta I| = 1/2$

$$(\Xi^- \to \Lambda\pi^-) = -\sqrt{2}(\Xi^0 \to \Lambda\pi^0),$$
$$(\Lambda \to p\pi^-) = -\sqrt{2}(\Lambda \to n\pi^0),$$
$$(\Omega^- \to \Xi^0\pi^-) = -\sqrt{2}(\Omega^- \to \Xi^{-*}\pi^0).$$

Equations (5) satisfy the triangle relation between the amplitudes for Λ^-, Ξ^- and Σ^- decays found in several articles [10,11,7] and agree with experiment [12] 2).

SU(6) symmetry with the help of (5) fixes a relation between the projections of this triangle on the S axis. Within the limit of experimental error (5) does not contradict the given data although a final decision may be made only by appreciably improving the measurements of the parameters of hadronic decays of hyperons (particularly the parameter γ of the decay $\Sigma^+ \to p\pi^0$).

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1)This result becomes clear if one thinks in terms of quarks: the strange quark Λ_0 cannot give rise to a π^+ meson regardless of whether the quark is in a symmetric state (baryons described by representation 56) or an antisymmetric state (baryons described by representation 20). The quark picture is inapplicable to the p-wave amplitude, which violates SU(6).

2)It is interesting to note that in[10, 11] in order to find this relationship it was necessary to assume that $(\Sigma^+ \to n\pi^+)_S = 0$. For $(\Sigma^+ \to n\pi^+)_P = 0$ there is a different relation between the S and P amplitudes of the triangle, disagreeing with experiment.

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TRANSFORMATION OF A METAL INTO A DIELECTRIC AND SINGULARITIES OF THE ELECTRICAL CHARACTERISTICS OF METALS IN STRONG FIELDS

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1. It is well known that metals with equal numbers of electrons n_e and holes n_h can exist only owing to the overlapping of bands. From Fig. 1., in which the shaded areas indicate filled states, it is clear that a shift of the band boundary by an amount $\delta\epsilon$ equal or greater than $\Delta\epsilon = \epsilon_2 - \epsilon_1$ would transform at absolute zero a metal (a) into a dielectric (b) 1).

If the metal has not only one type of carrier (not one band), but $n_e \neq n_h$, then a shift of the boundary might "deplete" one of the bands (see also (2)). At the moment of disappearance of carriers from even only one band, all the elec-

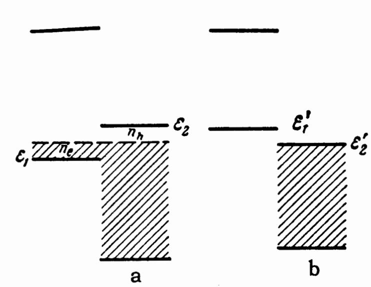

FIG. 1.

tronic properties of the metal would exhibit at $T = 0$ a singular behavior that vanishes gradually with increasing temperature. For metals of the type of bismuth and for a few anomalous empty bands in other metals, the required shift in energy is of the order of a few hundred degrees (for the ground state band of good metals it is of the order of 10^4-10^5°K).

2. To displace the boundary of a band it is possible to use a constant magnetic field H. We explain this effect with the example of electrons with a quadratic dispersion law $\epsilon_e = \epsilon_1 + p^2/2m_e^*$, where $m_e^* > 0$ is the cyclotron effective mass of the electron. In a magnetic field directed along the z axis we have

$$\varepsilon_e = \varepsilon_1 + (k + 1/2)\mu_e H \pm \mu_s^e H + p_z^2/2m_e^*, \quad (1)$$

$$\mu_e = |e|\hbar/m_e^* c, \qquad \mu_s^e = |e|\hbar/2m_s^e c; \quad (2)$$

$m_s^e > 0$ is the spin effective mass, which determines paramagnetic splitting of the Landau levels.

The magnetic field shifts the end point of the spectrum from $\epsilon_e = \epsilon_1$ (for $H = 0$) to

$$\varepsilon_e = \varepsilon_1'(H) = \varepsilon_1 + (1/2\mu_e - \mu_s^e)H,$$

and thus there are no states with energy less than $\epsilon_1'(H)$ at all. The continuity of the spectra for $\epsilon_e > \epsilon_1'(H)$ is preserved; the presence of the discrete k in the formula for ϵ_e leads only to singularities in the density of states $\nu_e(\epsilon) = dn_e(\epsilon)/d\epsilon$ at points equidistant in energy. (This can be shown by direct calculation, using the well known formulas for the number of states $n_e(\epsilon)$ with energy not greater than ϵ_e.) For holes with a quadratic dispersion law $\epsilon_h = \epsilon_2 - p^2/2m_h^*$ (where m_h^* is the absolute value of the negative effective mass of the hole) in a magnetic field, the upper boundary of the band is displaced from $\epsilon_h = \epsilon_2$ to

$$\varepsilon_h = \varepsilon_2'(H) = \varepsilon_2 - (1/2\mu_h - \mu_s^h)H.$$

The spectrum is continuous for $\epsilon_h < \epsilon_2'(H)$; μ_h and μ_s^h are given by formula (2) by changing the index e to h; $\mu_s^h > 0$. A necessary condition for the transformation of a metal to a dielectric in a magnetic field in the examined case is evidently the fulfillment of the inequality:

$$A = 1/m_e^* - 1/m_s^e + 1/m_h^* - 1/m_s^h > 0. \quad (3)$$

The resulting formulae allow an estimate of the magnetic field necessary for observation of the described singularities. For "convenient" (corresponding to a maximum $\delta\epsilon$) directions of the magnetic field and unusually small bands, and for metals of the type of bismuth, a field of strength $H \sim 10 - 10^6$ Oe is necessary.

We stress that a change of the number of free charges is impossible in the presence of only one band. If a given band "contracts" in a magnetic field (the lower boundary of the band rises, and the upper boundary drops), the number of states contained in it does not change (see also [3]) owing to the increase of the density of states, just like the number of particles in a liquid does not change upon contraction[2)]. Since the number of electrons also does not change, the number of unoccupied states in the band does not change. A change in the number of carriers in a band is possible only if they "flow" from band to band, the way a liquid flows into a vessel when a neighboring connected vessel is raised or lowered. The "holes" are analogous here to voids over the liquid.

3. We now determine the dependence of different electronic characteristics on the magnetic field.

It is clear, that so long as $H \ll H_k$ (H_k is the field for which the edges of the bands touch), the magnetic field has little effect on the number of carriers. When $H_k - H \sim H_k$ the number of carriers changes smoothly[3)]. When $H_k - H \ll H_k$ a relatively small change of the field must lead to an abrupt decrease of $n_{e,h}$, as a result of which the initially degenerate gas of electrons and holes becomes non-degenerate and $n_{e,h}$ are determined only by the temperature. When

$$B = (|e|\hbar A/ckT)\ (H - H_k) > 1$$

the number of carriers decreases exponentially as e^{-B} as the magnetic field is increased or the temperature is decreased. For $T = 0$°K, naturally, there is a singularity at the point $H = H_k$.

For simplicity we assume that $T = 0$°K[4)]. Let $H = H_k + H'$ and $|H'| \ll H_k$. For $H' < 0$ the chemical potential is

$$\xi(H_k + H') = \xi(H_k) + \beta|H'|, \quad n_e = n_h \sim [\xi - \xi(H_k)]^{1/2}$$

(this form of dependence of $n_{e,h}$ on ξ is due to the singularity of $\nu_{e,h}(\epsilon)$ near the border of the band), and thus $n_e = n_h = \eta|H'|^{1/2}$.

The thermodynamic potentials are proportional to

$$n_{e,h}[\xi - \xi(H_k)] \sim |H'|^{3/2}$$

(for the energy this is obvious, and the small additional terms related in this case to $n_{e,h}$ coincide for all potentials [9]).

When $H' > 0$ the current carriers vanish

$$n_{e,h}(H_k + H') = 0,$$

$$\xi(H_k + H') = \varepsilon_2' = \xi(H_k) - \delta H',$$

and the additional terms in the thermodynamic

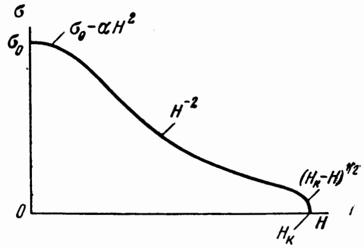

FIG. 2.

potentials are evidently equal to zero.

Since the conductivity is proportional to the number of carriers, for $H = H_k$ it becomes zero, having discontinuities with infinite derivative (Fig. 2.)[5)].

It is clear that when $n_e \neq n_n$ the conductivity suffers a discontinuity if the carriers vanish from one of the bands, but it does not go to zero.

The thermodynamic potentials and their derivatives (except derivatives with respect to the magnetic field) are continuous at $H = H_k$. The magnetic moment has the same singularity and the same dependence on H′ as does σ. Thus the magnetic susceptibility $\chi = -\partial F/\partial H$ (F is the free energy) experiences at T = 0°K an infinite jump when $H = H_k$. A similar type of singularity results also when the chemical potential ξ(H) goes through values of the energy that are singular for the given band (for which there appears a new equal energy surface, for which a change takes place from an open surface to a closed one and vice versa, etc.).

These singularities resemble singularities in the electronic characteristics of metals under high pressures, predicted by I. Lifshitz.[11]

If A < 0, then for $H \gg c\Delta\epsilon/|e|\hbar A$ the number of carriers must increase in a magnetic field (in proportion to $H^{3/2}$ for a quadratic dispersion law). A calculation of the resulting effects will be the subject of a future paper.

4. Thus, the change in the chemical potential in a magnetic field can be used to investigate the dispersion law and its singularities in a large energy interval (of the order of μH).

1)A decrease in the number of electrons and holes might, as shown by Arkhipov[1], change a metal into a dielectric abruptly when $\delta\epsilon < \Delta\epsilon$ even at finite temperature.

2)Conservation of the number of states and of the total charge makes the charge of any quasi-particle (electron or hole) equal to the charge of a free electron (with the accuracy with which the charge commutes with the quasi-momentum).

3)A change in the number of carriers in a relatively weak magnetic field was observed in bismuth[4-7].

4)For a quadratic anisotropic dispersion law one may write exact equations for arbitrary H. However, a strong spin-orbit coupling leads to a more complicated form of ϵ(p even near the edge of a band (see[8]). A non-quadratic ϵ(p results also from degeneracy; in this case it is necessary to take into account the "interaction" between bands and it is impossible to quantize them independently in each band. As far as finite temperature is concerned, its effect may be taken into account in the general case in a manner analogous to that described in[2].

5)The possibility of bismuth transforming in principle into a dielectric in very strong magnetic fields was first pointed out by Davydov and Pomeranchuk[10]. They did not, however, examine the singularities of the electronic characteristics in such a transition.

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