RAMAN RADIATION OF A UNIFORMLY MOVING CHARGE

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It is shown that the interaction between the proper-field of a uniformly moving charge and excited atoms of matter leads to the emission of radiation of a frequency which differs from the excitation energy of the atom. The radiation intensity depends on the velocity of the particle, on the angle of emission of the quantum and on the excitation energy of the atoms in such a manner that if the material contains atoms with one excited state it is possible to measure the energy of the particle in terms of the angle of emission of the quantum.

1. INTRODUCTION

IN order that emission of radiation should occur when a charge is moving uniformly in a medium it is necessary that the proper field of the charge be transformable into transverse electromagnetic waves as a result of its interaction with the medium. We consider the case when Vavilov-Cerenkov radiation is absent. Then radiation can occur as a result of the scattering of the proper field of the charge by spatial inhomogeneities in the medium. The properties of such radiation have been investigated earlier by many authors[1]. We pointed out[2] the existence of another mechanism of radiation, associated with Raman scattering of the proper field of the charge in a medium. For the appearance of such radiation (to which it is natural to refer as Raman radiation) it is necessary that certain definite relations be satisfied.

It is essential to point out the fact that the most interesting possibility is that of Raman radiation in a strictly homogeneous medium (without transfer of momentum to the substance). This possibility is related to a certain peculiar feature of Raman scattering of the proper field of the charge. Indeed, in Raman scattering of transverse waves the change in the photon energy leads to a change in its momentum (the laws of conservation admit such a process only if momentum is transferred to the substance), so that Raman scattering of transverse waves is essentially related to the existence of inhomogeneities in the substance.

If the Fourier-component of the proper field is scattered

$$\mathbf{E}_0(\mathbf{r}, t) = \int d^3q \mathbf{E}_0(\mathbf{q}) \exp[i\mathbf{q}(\mathbf{r} - \mathbf{v}t)],$$

$$\mathbf{E}_0(\mathbf{q}) = \frac{ie}{2\pi^2\varepsilon_0} \frac{\mathbf{v}(\mathbf{qv})\varepsilon_0 - \mathbf{q}}{\mathbf{q}^2 - (\mathbf{qv})^2\varepsilon_0}, \qquad (1.1)$$

then the situation is altered, since the relation between the momentum of the Fourier-component $\mathbf{q}$ and the energy $\mathbf{q}\cdot\mathbf{v}$ is less restrictive than for transverse waves. In particular, one can find such an angle between $\mathbf{q}$ and $\mathbf{v}$ for which Raman scattering occurs without transferring any momentum at all to the substance. Such a version of the process is distinct from all the others due to the fact that it can occur in a strictly homogeneous substance, i.e., it is not associated with the presence of inhomogeneities in the medium. Therefore, Raman radiation of this type is more probable than radiation involving a transfer of momentum.

From an investigation of the Raman scattering of the proper field it follows that radiation without transfer of momentum to the medium is possible if the angle of emission of the quantum ϑ satisfies the following condition[2]

$$\cos\vartheta = \frac{1}{v\sqrt{\varepsilon_0}}\left(1 - \frac{\Delta\omega}{\omega}\right), \qquad (1.2)$$

where $\Delta\omega$ is the energy transferred to the medium by the quantum as a result of scattering. From the requirement $\cos^2\vartheta \leq 1$ we obtain the following condition for the existence of radiation

$$v^2\varepsilon_0 \geqslant \left(1 - \frac{\Delta\omega}{\omega}\right)^2. \qquad (1.3)$$

If Vavilov-Cerenkov radiation is absent, i.e., $v^2\epsilon_0 < 1$, then Raman radiation is possible only for $\Delta\omega > 0$. From (1.2) it also follows that the spectrum of the frequencies radiated has the form

$$\omega = \frac{\Delta\omega}{1 - v\sqrt{\varepsilon_0}\cos\vartheta}, \qquad (1.4)$$

so that high frequencies can also be emitted near the threshold of the Vavilov-Cerenkov radiation. It is evident that emission of frequencies lying

far from the characteristic frequencies of the medium is of interest since otherwise it will be impossible to distinguish the radiation from the charge against the background of spontaneous radiation from the medium. This investigation is restricted by the assumption that the radiation represents a process which is rapid in comparison to the interaction with neighboring atoms (molecules) so that thermal motion does not affect the emission of radiation. It is evident that this condition is fulfilled if the frequency of radiation ω and the time τ characteristic of relaxation processes satisfy the inequality $\omega\tau \gg 1$.

2. POLARIZATION OF EXCITED MATTER BY AN ELECTROMAGNETIC FIELD

We investigate the singularities which appear when a substance containing excited atoms is polarized by an electromagnetic field

$$\mathbf{E}(\mathbf{r}, t) = \int d\omega\, \mathbf{E}(\mathbf{r}, \omega) e^{-i\omega t}.$$

In carrying out this investigation we shall assume that the dielectric permittivity of the unexcited substance $\epsilon_0(\omega)$ is known. Let the wave function of an individual atom (molecule) prior to the switching on of the field be a superposition of stationary states

$$\Psi(\mathbf{r}, t) = \sum_n a_n \Psi_n^0(\mathbf{r}, t),$$

where the expansion coefficients a_n can be assumed to be either independent of the time or sufficiently slow functions of the time. This assumes that all the relaxation processes are slow compared to the radiation process under discussion.

The switching on of the field leads to a change in the wave function by a small quantity

$$\Phi(\mathbf{r}, t) = \sum_n c_n(t) \Psi_n^0(\mathbf{r}, t),$$

in which the expansion coefficients change with time much more rapidly than a_n. Writing down the Hamiltonian for the interaction with the long wavelength field in the form $-\mathbf{D}\cdot\mathbf{E}$, where $\mathbf{D}$ is the dipole-moment operator, it is not difficult to obtain from the Schrödinger equation in the approximation linear in the field

$$c_m(\mathbf{R}, t) = \sum_s a_s \int \frac{d\omega}{\omega_{ms} - \omega - i0} \mathbf{D}_{ms}\mathbf{E}(\mathbf{R}, \omega) \times \exp[i(\omega_{ms} - \omega)t], \quad (2.1)$$

assuming that the field was switched on at $t \to -\infty$. Generally speaking, the expansion coefficients c_m are different for each atom, and this is taken into account by the dependence of c_m on the coordinate of the center of mass of the atom $\mathbf{R}$. The dipole moment induced by the field in an individual atom has the form

$$\sum_{n,m} a_n^* c_m(\mathbf{R}, t) D_{nm} \exp(i\omega_{nm}t) + \text{compl. conj.}$$

It is convenient to break up the density of the dipole moment induced by the field $\mathbf{P}(\mathbf{R}, t)$ into a part corresponding to the unexcited medium, $\mathbf{P}_0(\mathbf{R}, t)$ and a correction to this quantity $\mathbf{P}_1(\mathbf{R}, t)$ which is due to the presence of excited states:

$$\mathbf{P}(\mathbf{R}, t) = \mathbf{P}_0(\mathbf{R}, t) + \mathbf{P}_1(\mathbf{R}, t),$$

$$\mathbf{P}_1(\mathbf{R}, t) = n_0 \sum_{n,s}{}' a_n^* a_s \int d\omega \sum_m (\omega_{ms} - \omega)^{-1} \times \overline{\mathbf{D}_{nm}(\mathbf{D}_{ms}(\mathbf{R}, \omega))} \exp[i(\omega_{ns} - \omega)t] + \text{compl. conj.}$$

where the bar denotes averaging over the directions of the vector $\mathbf{D}$, while the prime in the summation over n, s denotes that in the sum we omit the term corresponding to the unexcited medium ($n = s = 0$).

The distinction in principle between $\mathbf{P}_1$ and $\mathbf{P}_0$ can be easily seen on the example of a periodic field. In this case $\mathbf{P}_0(\mathbf{R}, t)$ oscillates with the frequency of the field, while $\mathbf{P}_1(\mathbf{R}, t)$ represents a superposition of oscillations with the combination frequencies $\omega - \omega_{ns}$. Such a dependence of the polarization on the time is equivalent to a variation of the properties of the medium with time, i.e., to a variation with time of the dielectric permittivity. The appearance of this dependence even in an equilibrium substance is associated with the fact that the discussion of the scattering of electromagnetic waves must always be carried out with greater accuracy than the usual considerations of macroscopic theory. The values of the macroscopic fields are here taken to be the result of averaging only over a physically infinitesimal volume. As is well known averaging over the motion of the particles is not carried out in the scattering problem (it can be carried out only in the final result), since the carrying out of such averaging at the initial stage of the investigation would lead to the disappearance of the scattering process that is of interest to us. Therefore, the dependence of the properties of the medium on the time arising in the case of the equilibrium substance is related to the absence of averaging over the motion of the particles.

Setting $\mathbf{E} = \mathbf{e}|\mathbf{E}|$ as a result of the uniqueness

of the selected direction e we have

$$\overline{(\mathbf{D}_{nm}\mathbf{e})\mathbf{D}_{ms}} = \mathbf{e}\overline{(\mathbf{D}_{nm}\mathbf{e})(\mathbf{D}_{ms}\mathbf{e})},$$

so that

$$4\pi\mathbf{P}_1(\mathbf{R}, t) = \sum_{n,s}{}' \int d\omega Q_{ns}(\omega)\mathbf{E}(\mathbf{R}, \omega)\exp[-i(\omega-\omega_{ns})t],$$

$$Q_{ns}(\omega) = 4\pi n_0 a_n^* a_s \sum_m \overline{(\mathbf{D}_{nm}\mathbf{e})(\mathbf{D}_{ms}\mathbf{e})}$$

$$\times[(\omega_{ms}-\omega)^{-1}+(\omega_{mn}+\omega)^{-1}], \qquad (2.2)$$

from which follows the relation associating the induction with the field:

$$\mathbf{D}(\mathbf{R}, t) = \int d\omega\left[\varepsilon_0(\omega) + \sum_{n,s}{}' Q_{ns}(\omega)\exp(i\omega_{ns}t)\right]$$

$$\times \mathbf{E}(\mathbf{R}, \omega)e^{-i\omega t} \qquad (2.3)$$

or

$$\mathbf{D}(\mathrm{k}.\ \omega) = \varepsilon_0(\omega)\mathbf{E}(\mathbf{k}, \omega)$$

$$+ \sum_{n,s}{}' Q_{ns}(\omega+\omega_{ns})\mathbf{E}(\mathbf{k}, \omega+\omega_{ns}). \qquad (2.3')$$

These relations prove the equivalence of a substance with excited atoms with a substance having a dielectric permittivity variable with time. For example, in the simplest particular case when two levels are populated - the ground level and the first excited level, we have[1)]

$$\varepsilon(\omega, t) = \varepsilon_0(\omega) + Q_{10}(\omega)\exp(-i\omega_{10}t) + 4\pi Q_{01}(\omega)$$

$$\times \exp(i\omega_{10}t).$$

Thus, the system of Maxwell's equations in a medium with excited atoms assumes the form

$$\mathrm{rot}\,\mathbf{H}(\omega) = 4\pi\mathbf{j}(\omega) - i\omega\varepsilon_0\mathbf{E}(\omega) - i\omega$$

$$\times \sum_{n,s} Q_{ns}(\omega+\omega_{ns})\mathbf{E}(\omega+\omega_{ns}),$$

$$\varepsilon_0\,\mathrm{div}\,\mathbf{E}(\omega) = 4\pi\rho(\omega)$$

$$- \sum_{n,s} Q_{ns}(\omega+\omega_{ns})\,\mathrm{div}\,\mathbf{E}(\omega+\omega_{ns}),$$

$$\mathrm{div}\,\mathbf{H}(\omega) = 0, \qquad \mathrm{rot}\ \mathbf{E}(\omega) = i\omega\mathbf{H}(\omega). \qquad (2.4)^*$$

3. INTENSITY OF RAMAN RADIATION

The solution of the system of Maxwell's equations (2.4) for the field of a uniformly moving charge can be carried out by the method of successive approximations. For this we assume that Q_{ns} are small quantities and the solution of the system (2.4) is written in the form

$$E = E_0 + E_1 + E_2 + \ldots,$$

in which the k-th term is proportional to $(Q_{ns})^k$.

In the zeroth approximation the system (2.4) coincides with Maxwell's equations for an unexcited medium. The field of a uniformly moving charge in such a medium $\mathbf{E}_0(\mathbf{r}, t)$ is known and is given by formula (1.1). In the first approximation with respect to Q the system (2.4) yields

$$\mathrm{rot}\,\mathbf{H}_1(\omega) = -i\omega$$

$$\times \sum_{n,s} Q_{n,s}(\omega+\omega_{ns})\mathbf{E}_0(\omega+\omega_{ns}),$$

$$\mathrm{div}\,\mathbf{E}_1(\omega) = -\frac{1}{\varepsilon_0}\sum_{n,s} Q_{ns}(\omega+\omega_{ns})\,\mathrm{div}\,\mathbf{E}_0(\omega+\omega_{ns}),$$

$$\mathrm{div}\,\mathbf{H}_1 = 0, \qquad \mathrm{rot}\,\mathbf{E}_1 = i\omega\mathbf{H}_1. \qquad (3.1)$$

From (3.1) it follows that $\mathbf{E}_1(\mathbf{R}, \omega)$ satisfies the equation

$$\Delta\mathbf{E}_1 + \omega^2\varepsilon_0\mathbf{E}_1 = -\sum_{n,s} Q_{ns}(\omega+\omega_{ns})\left[\omega^2\mathbf{E}_0(\omega+\omega_{ns})\right.$$

$$\left. + \frac{1}{\varepsilon_0}\,\mathrm{grad}\,\mathrm{div}\,\mathbf{E}_0(\omega+\omega_{ns})\right], \qquad (3.2)$$

which coincides in form with the equation for the retarded potentials. Therefore, the solution of (3.2) at large distances can be obtained by means of the well known approximate expression for retarded potentials at great distances from the source [3]:

$$\mathbf{E}_1(\mathbf{R}, \omega) = \frac{e^{i\mathbf{k}\mathbf{R}}}{R}\sum_{n,s} Q_{ns}(\omega+\omega_{ns})\int\frac{d^3r}{4\pi}e^{-i\mathbf{k}\mathbf{r}}$$

$$\times\left[\omega^2\mathbf{E}_0(\mathbf{r}, \omega+\omega_{ns}) + \frac{1}{\varepsilon_0}\nabla(\nabla\mathbf{E}_0(\mathbf{r}, \omega+\omega_{ns}))\right];$$

$$\mathbf{k} = \frac{\mathbf{R}}{R}\omega\sqrt{\varepsilon_0}.$$

Utilizing the expression (1.1) for $\mathbf{E}_0(\mathbf{r}, t)$ we can easily obtain

$$\mathbf{E}_1(\mathbf{R}, \omega) = \frac{e^{i\mathbf{k}\mathbf{R}}}{R}\sum_{n,s}(2\pi)^3\left\{Q_{ns}(\omega+\omega_{ns})\left[\omega^2\mathbf{E}_0(\mathbf{k})\right.\right.$$

$$\left.\left. - \mathbf{k}(\mathbf{k}\mathbf{E}_0(\mathbf{k}))\frac{1}{\varepsilon_0}\right]\delta(\omega-\mathbf{k}\mathbf{v}+\omega_{ns})\right\}. \qquad (3.3)$$

As is well known, the energy emitted in the frequency interval $d\omega$ into the solid angle $d\Omega$ during the whole transit time T in the medium is given by the expression

$$d\mathscr{E}(\omega, \mathbf{n}) = R^2 d\Omega\,|\mathbf{E}(\mathbf{R}, \omega)|^2\sqrt{\varepsilon_0(\omega)}. \qquad (3.4)$$

We now assume that in the unexcited medium there

[1)] The radiation of a charged particle in a medium with a phenomonologically given dependence $\epsilon(\omega, t) = \epsilon_0(\omega) + \epsilon_1(\omega)\cos(\mathbf{k}\cdot\mathbf{r} - \Omega t)$ has been investigated recently by Barsukov and Bolotovskiĭ [4].

*rot ≡ curl.

is no radiation from a uniformly moving charge ($v^2\epsilon_0 < 1$). This means that the field $\mathbf{E}_0(\mathbf{R}, \omega)$ falls off at large distances faster than $1/R$. In this case at sufficiently large distances one can omit $\mathbf{E}_0$ in (3.4), retaining only $\mathbf{E}_1(\mathbf{R}, \omega)$. From (3.3) it can be seen that in this case in (3.4) there appear squares of δ-functions with the same argument, and this means that the radiation is proportional to the total transit time T. Indeed, transforming

$$[\delta(\omega \pm \omega_{ns} - \mathbf{kv})]^2 = T(2\pi)^{-1}\delta(\omega \pm \omega_{ns} - \mathbf{kv}),$$

one can obtain from (3.3) and (3.4) that the energy dJ emitted in the frequency interval $d\omega$ into the solid angle $d\Omega$ per unit path is determined by the expression

$$dJ = \frac{1}{vT} d\mathscr{E}(\mathbf{n}, \omega) = \omega^4 d\omega\, d\Omega\, v^{-1} 2\pi^3 \sqrt{\varepsilon_0}$$

$$\times \{[\mathbf{n}\mathbf{E}_0(\mathbf{k})]\}^2 \sum_{n,s} |Q_{ns}(\omega + \omega_{ns})|^2 \delta(\omega + \omega_{ns} - \mathbf{kv})$$

or, substituting $\mathbf{E}_0(\mathbf{k})$ by the expression

$$dJ(\mathbf{n}, \omega) = \frac{e^2}{2\pi v} \frac{\sqrt{\varepsilon_0(\omega)}}{\varepsilon_0(\mathbf{kv})} \omega^2 d\omega\, d\Omega \frac{(\mathbf{kv})^2[\omega^2\varepsilon_0(\mathbf{kv}) - (\mathbf{kv}^2)]}{[\mathbf{k}^2 - (\mathbf{kv})^2\varepsilon_0(\mathbf{kv})]^2}$$

$$\times \sum_{n,s} |Q_{ns}(\omega + \omega_{ns})|^2 \delta(\omega + \omega_{ns} - \mathbf{kv}). \qquad (3.5)$$

It follows from (3.5) that the angle ϑ between the direction of the photon momentum $\mathbf{k}$ and the direction of the particle velocity $\mathbf{v}$ is related to the photon frequency by the equations

$$\cos\vartheta = \frac{1}{v\sqrt{\varepsilon_0(\omega)}}\left(1 - \frac{|\omega_{ns}|}{\omega}\right),$$

and this agrees with formula (1.1) obtained from considering the conservation laws for emission.

Integrating over the angles with the aid of the δ-function we can easily find the energy emitted by a uniformly moving charge into the frequency interval $d\omega$ in all directions per unit path:

$$dJ(\omega) = \frac{e^2}{v^2}\omega\, d\omega \sum |Q_{ns}(\omega + \omega_{ns})|^2$$

$$\times \frac{(\omega + \omega_{ns})^2[\omega^2\varepsilon_0(\omega + \omega_{ns}) - (\omega + \omega_{ns})^2]}{\varepsilon_0(\omega + \omega_{ns})[\omega^2\varepsilon_0(\omega) - (\omega + \omega_{ns})^2\varepsilon(\omega + \omega_{ns})]^2},$$

$$|\omega_{ns}| > \omega(1 - v\sqrt{\varepsilon_0(\omega)}). \qquad (3.6)$$

As an example we consider the simplest case when prior to the switching on of the field only the ground and the first excited states of the atoms are populated. Then $Q_{ns} = 0$ for $s \geq 2$, $n \geq 2$; the diagonal elements yield no oscillations with sum or difference frequencies and do not lead to emission of radiation. Therefore, it remains to take into account only Q_{01} and Q_{10}, and only two terms will remain from the sums over n and s. Assuming that the condition $\omega_{10} > \omega(1 - v\sqrt{\epsilon_0(\omega)})$ is satisfied we obtain from (3.6)

$$dJ = \frac{e^2}{v^2} \frac{\omega\, d\omega}{\varepsilon_0(\omega + \omega_{10})}$$

$$\times \frac{(\omega + \omega_{10})^2[\omega^2\varepsilon_0(\omega + \omega_{10}) - (\omega + \omega_{10})^2]}{[\omega^2\varepsilon_0(\omega) - (\omega + \omega_{10})^2\varepsilon_0(\omega + \omega_{10})]^2}$$

$$\times \{|Q_{10}(\omega + \omega_{10})|^2 + |Q_{01}(-\omega - \omega_{10})|^2\}, \qquad (3.7)$$

where radiation of the given frequency ω is emitted only at a single well defined angle determined by the equation

$$\cos\vartheta = \frac{1}{v\sqrt{\varepsilon_0}}\left(1 - \frac{\omega_{10}}{\omega}\right).$$

As has been pointed out [2], this circumstance enables one to utilize Raman radiation for the detection of charged particles of high energy. The advantage of this method is the fact that by selecting the frequency of radiation ω and the excitation energy ω_{10}, one can always make the angle of emission of the quantum of the order of magnitude unity even for arbitrarily high energies of the particle.

4. IONIZATION LOSSES IN A MEDIUM WITH EXCITED ATOMS

A charged particle moving in a straight line in a medium loses by ionization and excitation of atoms during the whole time of flight T the energy

$$\Delta\mathscr{E} = ev \int_{-T/2}^{T/2} dt\, \mathbf{E}(\mathbf{v}t, t)$$

$$= e \int d\omega \int d^3k\, \mathbf{v}\mathbf{E}(\mathbf{k}, \omega) \int_{-T/2}^{T/2} dt \exp[it(\mathbf{kv} - \omega)].$$

For large values of T the integral over t behaves like a δ-function, and for the energy lost in the frequency interval $d\omega$ per unit time we can obtain

$$dJ = e\frac{2\pi}{T} d\omega \int d^3k\, \mathbf{v}\mathbf{E}(\mathbf{k}, \mathbf{kv})\{\delta(\omega - \mathbf{kv}) + \delta(\omega + \mathbf{kv})\}.$$

It is convenient to transform this expression in the following manner. We choose the gauge for the potentials in which $\varphi = 0$, and introduce the retarded Green's function for the electromagnetic field in a substance with excited atoms $D_{il}^R(\mathbf{k}, \omega, \omega')$ by means of the relation

$$A_i(\mathbf{k}, \omega) = \int d\omega' D_{il}^R(\mathbf{k}, \omega, \omega') j_l(\mathbf{k}, \omega'). \qquad (4.1)$$

The dependence of D_{il}^{R} on the two frequencies ω and ω' is related to the circumstance noted above that an excited substance is equivalent to a substance with a dielectric permittivity variable in time. In the final analysis the dependence of $D_{il}^{R}(\mathbf{k}, \omega, \omega')$ on the two frequencies, and the consequent nonequivalence of different instants of time for an equilibrium medium, is related to the previously mentioned absence in the preceeding investigation of averaging over the motion of the particles.

It is natural that in an equilibrium substance no singled-out instant of time remains after all the averages have been taken, and the Green's function must have the form $D_{il}^{R}(\mathbf{k}, \omega)\delta(\omega - \omega')$. Utilizing the relation

$$E_l(\mathbf{k}, \omega) = i\omega \int d\omega' D_{lj}^{R}(\mathbf{k}, \omega, \omega') j_j(\mathbf{k}, \omega')$$

and the expression for the current of a uniformly moving charge $j_i = ev_i(2\pi)^{-3}\delta(\omega - \mathbf{k}\cdot\mathbf{v})$, we can transform the spectral density of the ionization losses to the form

$$\frac{dJ}{d\omega} = -\frac{e^2\omega}{4\pi^2 T}\int d^3k \operatorname{Im} v_i v_l D_{il}^{R}(\mathbf{k}, \mathbf{kv}, \mathbf{kv}) \times \{\delta(\omega - \mathbf{kv}) + \delta(\omega + \mathbf{kv})\}. \tag{4.2}$$

From the Maxwell equations for the excited substance we can easily obtain the equation for $D_{il}^{R}(\mathbf{k}, \omega, \omega')$:

$$D_{il}^{R}(\mathbf{k}, \omega, \omega') = D_{il}^{0R}(\mathbf{k}, \omega)\delta(\omega - \omega') + D_{il'}^{0R}(\mathbf{k}, \omega) \times \sum_{ns} \Pi_{il'}(\omega + \omega_{ns}) D_{l'l}^{R}(\mathbf{k}, \omega + \omega_{ns}, \omega'),$$

where $D_{il}^{0R}(\mathbf{k}, \omega)\delta(\omega - \omega')$ is the Green's function of the electromagnetic field in the unexcited substance (in the gauge which we have chosen we have $D_{44}^{0R} = D_{4i}^{0R} = D_{i4}^{0R} = 0$).

Since D_{il}^{R} and D_{il}^{0R} depend only on the single vector $\mathbf{k}$ we can without loss of generality represent them in the form

$$D_{il}^{R(0R)}(\mathbf{k}, \omega, \omega') = \left(\delta_{il} - \frac{k_i k_l}{k^2}\right) D^{t(0t)}(k, \omega, \omega') + \frac{k_i k_l}{k^2} D^{l(0l)}(k, \omega, \omega'),$$

where the quantities D^{0t} and D^{0l} are related to the dielectric permittivity of the unexcited medium ϵ_0:

$$D^{0t} = 4\pi(k^2 - \varepsilon_0\omega^2)^{-1}, \qquad D^{0l} = -4\pi/\varepsilon_0\omega^2.$$

The equation for D_{il}^{R} now assumes the form

$$D^{t(l)}(k, \omega, \omega') = D^{0t(0l)}(k, \omega)\delta(\omega - \omega') + D^{0t(0t)}(k, \omega) \times \sum_{n,s} \Pi^{t(l)}(\omega + \omega_{ns}) D^{t(l)}(k, \omega + \omega_{ns}, \omega'). \tag{4.3}$$

The solution of this equation behaves for $\omega \to \omega'$ as $D^{t(l)}(\mathbf{k}, \omega)\delta(\omega - \omega')$, where the function $D^{t(l)}(\mathbf{k}, \omega)$ can be obtained in the following manner. We replace $D(\mathbf{k}, \omega + \omega_{ns}, \omega')$ on the right hand side of (4.3) again by (4.3) and in the equation so obtained we let $\omega \to \omega'$ retaining only the principal terms with the δ-function singularity. In the four-fold sum over n, n', s, and s' the quantity $\delta(\omega - \omega')$ appears only in terms for which $\omega_{ns} + \omega_{n's'} = 0$, i.e., for $n = s'$ and $s = n'$. Therefore, only a twofold sum remains, and for the coefficients of $\delta(\omega - \omega')$ we obtain the relation

$$D^{t(l)}(k, \omega) = D^{0t(0l)}(k, \omega)(1 + \Delta^{t(l)}(k, \omega))D^{t(l)}(k, \omega),$$

where

$$\Delta^{t(l)} = \sum_{n,s} \Pi^{t(l)}(\omega + \omega_{ns}) D^{0t(0l)}(k, \omega + \omega_{ns}) \Pi^{t(l)}(\omega + \omega_{ns}),$$

and, consequently,

$$D^{t(l)}(k, \omega) = \frac{D^{0t(0l)}(k, \omega)}{1 - D^{0t(0l)}(k, \omega)\Delta^{t(l)}(k, \omega)}.$$

Now representing $\delta(\omega - \mathbf{k}\cdot\mathbf{v})$ in (4.2) again in the form of an integral over t and cancelling T in the numerator and the denominator we can easily obtain the spectral density of the losses in the form

$$\frac{dJ}{d\omega} = -\frac{e^2\omega}{2\pi^2}\int \frac{d^3k}{k^2}\delta(\omega - \mathbf{kv}) \times \operatorname{im}\left\{\frac{[\mathbf{k}\times\mathbf{v}]^2}{k^2 - \varepsilon_0\omega^2 - \Delta^t} - \frac{(\mathbf{kv})^2}{\varepsilon_0\omega^2 + \Delta^t}\right\}. \tag{4.4}$$

We now discuss the physical meaning of the quantity $D_{il}^{R}(\mathbf{k}, \omega)\delta(\omega - \omega')$. We average the quantity $D_{il}^{R}(\mathbf{k}, \omega, \omega')$ over the motion of the particles. From general considerations it is evident that the answer must have the form $D_{il}^{R}(\mathbf{k}, \omega)\delta(\omega - \omega')$. In the approximation quadratic in Q nothing other than the solution obtained above can be obtained since all the terms proportional to $\delta(\omega - \omega')$ have been taken into account already. Therefore, the quantity obtained above $D_{il}^{R}(\mathbf{k}, \omega)\delta(\omega - \omega')$ does represent the completely averaged Green's function of the electromagnetic field in the Q^2 approximation. From this, in particular, it follows that the relation between the frequency and the propagation vector of the photon is determined by the roots of the equation

$$k^2 = \varepsilon_0(\omega)\omega^2 + \Delta^t(k, \omega),$$

which due to the smallness of Δ^t can be written in the form

$$k^2 \approx \varepsilon_0\omega^2 + \Delta^t(\omega\sqrt{\varepsilon_0}, \omega) \approx \varepsilon_0(\omega + \Delta^t/2\omega\varepsilon_0)^2.$$

Thus, ω represents the energy lost by the particle and because of Raman scattering does not agree with the frequency of the emitted quantum ω_k.

We now consider the spectral density of energy losses in that frequency region where the medium is transparent ($\mathrm{Im}\ \epsilon_0 \to 0$), the function $\epsilon_0(\omega)$ does not pass through zero and in the unexcited medium the Vavilov-Cerenkov radiation is absent ($v^2\epsilon_0 < 1$). In this case the spectral density of the losses in the unexcited medium reduces to zero. In the excited material the losses will no longer reduce to zero due to the presence of Raman radiation.

Utilizing the well known relation $\mathrm{Im}\,(x - i\delta)^{-1} = \pi\delta(x)$ one can in the frequency range under consideration represent the spectral density of the ionization losses in the form

$$\frac{dJ}{d\omega} = \frac{e^2\omega}{2\pi}\int \frac{d^3k}{k^2}(k^2v^2 - \omega^2)\,\delta(\omega - \mathbf{kv}) \times \delta\left(k^2 - \varepsilon_0\left(\omega + \frac{\Delta^t}{2\omega\varepsilon_0}\right)^2\right),$$

so that the spectral density of the Raman radiation assumes the form

$$\frac{dJ}{d\omega_k} = \frac{e^2\omega}{2\varepsilon_0\omega_k^2 v}\left(\frac{\partial\omega}{\partial\omega_k}\right)(v^2\omega_k^2\varepsilon_0 - \omega^2) \times \int_0^\pi \sin\vartheta\, d\vartheta\,\delta\left(\cos\vartheta - \frac{\omega}{v\sqrt{\varepsilon_0}\,\omega_k}\right). \quad (4.5)$$

Thus, the angle of emission of radiation is determined by the formula

$$\cos\vartheta = \frac{1}{v\sqrt{\varepsilon_0}}\frac{\omega}{\omega_k} = \frac{1}{v\sqrt{\varepsilon_0}}\left(1 - \frac{\Delta E}{\omega_k}\right)$$

where ΔE is the quantity $\Delta^t(\omega\sqrt{\epsilon_0}, \omega)/2\omega\epsilon_0$ expressed in terms of ω_k. The formula for the angle of emission of radiation agrees with formula (1.2) derived on the basis of kinematic considerations. The fact that ΔE does not coincide with any of the characteristic frequencies of the atom is not surprising since the discussion of this section takes into account the transfer of energy resulting from an arbitrary number of Raman scattering events.

The results of the method of successive approximations are obtained from (4.5) if we assume $k^2 - \omega^2\epsilon_0 \gg \Delta^t$ and expand (4.4) in terms of Δ^t. Consequently, the domain of applicability of (3.7) is restricted by the condition $k^2 - \omega^2\epsilon_0 \gg \Delta^t$ or $1 - v^2\epsilon_0 \gg \Delta^t/k^2$, i.e., by the remoteness from the threshold for the Vavilov-Cerenkov radiation for the unexcited substance. We note that at relativistic energies of the particle $E \gg M$ and for emission of frequencies higher than the atomic ones ($\epsilon_0 = 1 - (\omega_L/\omega)^2$, $\omega_L^2 = 4\pi ne^2Z/m$) the condition for the applicability of the method of successive approximations appears in the following form $(M/E)^2 + (\omega_L/\omega)^2 \gg \Delta^t/k^2$. This means, that as the particle energy and the frequency increase there can always be found such a region in which the method of successive approximations is not applicable and one should utilize the more exact formula (4.5).

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