HEATING OF A GAS BY A POWERFUL LIGHT PULSE

Yu. P. RAĬZER

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The absorption of a powerful light pulse in a gas is considered and the temperatures to which the gas is heated are estimated. Three possible mechanisms of transfer of the zone of light absorption (absorption and heating waves) are discussed; they are, respectively, a hydrodynamic mechanism, a "breakdown" mechanism, and a radiative mechanism. The corresponding wave velocities are calculated. Heating and absorption waves are considered in a general form and a "shock adiabat" of the substance absorbing the light flux is constructed. A general relation between the wave velocity, which can be measured experimentally, and the heating temperature is derived.

1. INTRODUCTION

As experiments show,[1-8] breakdown occurs in gases under the action of a light pulse of sufficient intensity. If the flux greatly exceeds the threshold value for breakdown, the gas is very strongly ionized and the resultant plasma absorbs the light completely, being heated to high temperatures. As the measurements carried out in[7] have shown, for an energy of the pulse of 2.5 Joules, duration 40 nanoseconds, and a ray focused in a circle of radius $r_0 \approx 10^{-2}$ cm, atmospheric air at the focal point is heated to temperatures $\sim 6 \times 10^5$ deg K (The threshold of breakdown in these experiments correspond to an energy of about 1 Joule.)

Theoretical researches devoted to this phenomenon have touched mainly on the problem of the appearance and threshold of breakdown.[9-13] Only in the paper of Ramsden and Savic[14] is the absorption of the ray after breakdown considered; there, one of the possible mechanisms of occurrence of this process is noted ("detonation"). Certain fundamental questions, also related to this mechanism, were not touched upon, and some incorrect expressions, which contradict the law of conservation of energy, are obtained for the temperature to which the gas is heated. There are also very tentative estimates of the power necessary for heating hydrogen to thermonuclear temperatures.[15,16]

In the present work, the process of absorption of the light flux after the first breakdown at the focus is considered; possible mechanisms of the phenomenon are discussed and the temperatures achievable in this process are calculated. We shall be interested only in comparatively heavy gases, such as air, in which complete ionization of the atoms to the nucleus does not take place.

2. "THE ABSORPTION WAVE" AND THE MECHANISM OF ITS PROPAGATION

At temperatures of several tens and thousands of degrees, small light quanta are absorbed in the gas as the result of free-free transitions of electrons in the ionic field. The absorption coefficient, corrected for the induced emission (with account of the fact that $h\nu \ll kT$), is equal to[17,18]

$$\varkappa_\nu = \frac{4}{3}\left(\frac{2\pi}{3}\right)^{1/2}\frac{e^6 Z^2 n_e n_+}{(mkT)^{3/2}c\nu^2}\,g \approx \frac{3{,}1\cdot 10^{-31}Z^3 n^2 g}{(T^\circ\mathrm{K})^{3/2}(h\nu_{\mathrm{eV}})^2}\ [\mathrm{cm}^{-1}],$$

$$g = \frac{\sqrt{3}}{\pi}\ln\left(\frac{4kT}{Ze^2 n_e^{1/3}}\right) \approx 0{.}55\ln\left(\frac{2{.}4\cdot 10^3\,T^\circ\mathrm{K}}{Z^{4/3}n^{1/3}}\right).$$

Here n_e, n_+ and n are the numbers of electrons, ions, and original atoms per cm^3. We introduce numerical values of the range $l_\nu = 1/\kappa_\nu$ for air of normal density and ruby quanta $h\nu = 1.8$ eV:

$10^{-5}\,T^\circ$K:	0.5	1	2.5	5	7	10
Z:	1.35	2.7	5.0	5.2	5.7	6.6
$10^3\,l_\nu$, cm:	6.0	2.3	1.9	4.7	5.9	7.0

The effective charge of the ions Z is taken under the assumption that the ionization is equilibrium.[17]

We assume that at the focus, in a very small region of the light channel (see Fig. 1), where the light flux is maximal, breakdown has taken place and a very high degree of ionization and temperature have been achieved. The light is absorbed in a very thin layer of the order of the range of the quanta l_ν and heats the gas. One of the most noteworthy peculiarities of this process, which is quite evident physically and observable experimentally,[5,7] consists in the shift of the absorp-

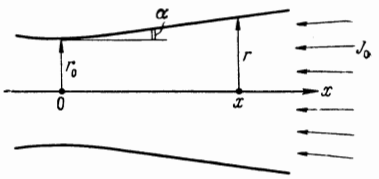

FIG. 1

tion zone against the light flux. In fact, the light quanta are strongly absorbed in the highly ionized medium. As only the degree of ionization in front of the layer which is absorbing at the given instant achieves for one reason or another a rather high value, the ion layer becomes nontransparent and is transformed into an absorbing layer. Thus, "a wave of absorption and heating" is propagated along the light channel against the ray. This effect inhibits the release of all the energy of the pulse into a very small volume of the focus, where the breakdown takes place first, and limits the achievement of very high temperatures in a continuous medium.

One can point to three completely different and independent mechanisms which lead to the generation of the absorption wave.

1. If the light flux at the focus appreciably exceeds the threshold or breakdown, then it expands within a certain region along a direction toward the light channel. Breakdown occurs also in these parts of the channel, but with a lag relative to the narrowest place. The larger the cross section of the channel and the smaller the current, the greater the lag. Thus the "breakdown wave" moves against the beam.

2. The heated gas in the absorbing layer is propagated and sends a shock wave in all directions, including that along the light channel against the ray. In the shock wave, the gas is heated and ionized, so that the zone of light absorption and energy release in the gas is transported behind the shock front. This ("hydrodynamic") mechanism is quite similar to detonations of explosive materials.

3. The gas in front of the absorbing layer is ionized and acquires an ability to absorb light through absorption of thermal radiation emanating from the highly heated region of the gas (due to the front of the absorption wave).

Calculations show that the heating and ionization in front of the absorbing layer, which are connected with the electron conductivity and diffusion of electrons, play a small role. The effectiveness of each of the mechanisms is characterized by that velocity of displacement of the absorption wave which the mechanism gives to the wave, while the real wave naturally moves with the highest of possible velocities.

Below we shall consider all the mentioned mechanisms (and calculate the corresponding velocities). First of all, however, we shall investigate some laws which do not depend on the specific mechanism of propagation of the wave, and establish a general relation between the velocity of the wave and the temperature of the heated gas.

3. THE ENERGY BALANCE AND THE "SHOCK ADIABAT" OF THE ABSORPTION WAVE

In a time Δt during which the wave moves a distance of the order of its thickness Δx, the light flux and the velocity of the wave $D = \Delta x/\Delta t$ are not appreciably changed ($\Delta x \sim l_\nu \lesssim 10^{-2}$ cm, $D \gtrsim 100$ km/sec, $\Delta t \lesssim 1$ nanosec). Therefore, the process is quasistationary in the system of coordinates connected with the front of the wave. If we disregard the fact that the gas is set into motion when heated, the energy balance in the wave can be written down in elementary fashion. The light flux $J_0 dt$ incident in a time dt on a unit surface of the front is consumed in heating the mass of gas $\rho_0 D dt$, which is trapped by the wave during this time. Consequently,

$$\rho_0 D\varepsilon(T) = J_0, \tag{3.1}$$

where J_0 is the light flux (in erg/cm^2-sec), ρ_0 is the initial density of the gas, and $\epsilon(T)$ is the specific internal energy which the gas acquires as the result of absorption of the flux.

Let us consider in detail an idealized plane wave whose width is much less than the radius of the surface of the front (the radius of the light channel). In this case, one can neglect the lateral expansion of the gas inside the wave. We write down the general equations for the conservation of flow of mass, momentum and energy in the wave, as is usually done in the theory of hydrodynamic discontinuities:[17,19]

$$\rho u = \rho_0 D, \quad p + \rho u^2 = \rho_0 D^2,$$
$$\varepsilon + p/\rho + {}^1/_2 u^2 = {}^1/_2 D^2 + J_0/\rho_0 D. \tag{3.2}$$

Here p, ρ, u are the pressure, density, and velocity of the gas (relative to the front) behind the wave. The pressure and the energy in front of the wave are considered small.

We assume an equation of state in the form

$$\varepsilon = p/(\gamma - 1)\rho, \tag{3.3}$$

where γ is the effective exponent of the adiabat. Eliminating ϵ, u, and D from (3.2) and (3.3), we get the equation of the "shock adiabat" of the gas,

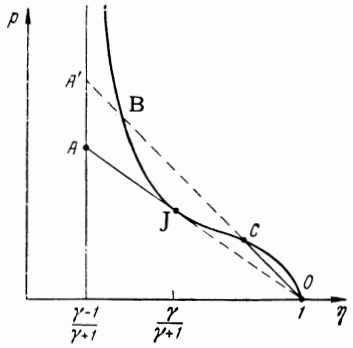

FIG. 2

in which the energy of the light flux is released: [1]

$$p=\left[2(1-\eta)^{1/2}\left(\frac{\gamma+1}{\gamma-1}\eta-1\right)^{-1}J_0\rho_0^{1/2}\right]^{2/3},\quad \eta=\frac{\rho_0}{\rho}. \quad (3.4)$$

The equation of energy balance, which generalizes (3.1), has the form

$$\rho_0 D\varepsilon(T,\eta)=J_0\beta,\qquad \beta=\left(1-\frac{\gamma-1}{2}\frac{1-\eta}{\eta}\right)^{-1}, \quad (3.5)$$

while the reciprocal of the compressibility η is connected with ϵ and D by another equation

$$(\gamma-1)\frac{\varepsilon}{D^2}=\eta(1-\eta), \quad (3.6)$$

which allows us to eliminate it from (3.5).

4. DETONATION AND SUPERDETONATION REGIMES

The first two equations of (3.2) give $p=\rho_0 D^2(1-\eta)$. The velocity of the wave is determined by the slope of the line drawn on the $p\eta$ diagram (Fig. 2) from the origin O to the final state of the gas on the shock adiabat. As is seen in Fig. 2, for a given light flux J_0, there is a minimum velocity of propagation of the absorption wave, which corresponds, as in the case of the detonation of an explosive to the Jouguet point J at which the velocity of the wave relative to the heated material behind it is identical with the local sound velocity $u^2=c^2=\gamma p/\rho$. In the absence of other mechanisms of ionization ("ignition"), in addition to ionization of the shock wave, or else if the other mechanisms are less effective, this hydrodynamic ("detonation") regime is realized. The gas is compressed and heated by the shock wave to the state A; then, obtaining additional energy as the result of absorption of the light, it expands along the line AJ, reaching the Jouguet point at the moment of conclusion of energy release.

The minimum velocity of the absorption wave is equal to [2]

$$D=\left[2(\gamma^2-1)\frac{J_0}{\rho_0}\right]^{1/3}. \quad (4.1)$$

Heating in the detonation regime has the maximum possible value and is equal to

$$\varepsilon=\frac{\gamma}{(\gamma^2-1)(\gamma+1)}D^2=\frac{2^{2/3}\gamma}{(\gamma^2-1)^{1/3}(\gamma+1)}\left(\frac{J_0}{\rho_0}\right)^{2/3}. \quad (4.2)$$

The coefficient β in Eq. (3.5) is equal to $\beta=2\gamma/(\gamma+1)$. The compression behind the wave is $1/\eta=(\gamma+1)/\gamma$. If we substitute in Eqs. (4.1) and (4.2) the numerical values $J_0=2\times10^{18}$ erg/cm^2-sec, $\rho_0=1.3\times10^{-3}$g/cm^3, and $\gamma=1.33$, corresponding to experiment [7] [3] about which we spoke in the Introduction, we get D = 133 km/sec and $\epsilon=1.35\times10^{14}$ erg/g. Such an energy at equilibrium corresponds to a temperature $T\approx9.1\times10^5$°K. (The experimental values are D ≈ 110 km/sec, $T\approx6\times10^5$ °K.) [4]

Hydrodynamic regimes with shock-wave ionization but with a velocity exceeding the "detonation" value (to which corresponds a compression in the shock wave to the state A′ and subsequent expansion to the final state B) do not exist. Motion behind the absorption wave would in this case be subsonic, and the expansion of the heated gas behind the wave would immediately weaken the wave, transforming it to a regime of "normal detonation." If for a given light flux J_0 any of the ionization

[1] It differs from the equation of the shock adiabat of explosive material with energy release per gram q:

$$p=2q\rho_0/\{[(\gamma+1)/(\gamma-1)]\eta-1\}$$

for the reason that the energy release per gram $J_0/\rho_0 D$ in the case of the absorption wave depends on the velocity of the wave.

[2] One can also obtain Eq. (4.1) from the formula for the velocity of the detonation $D=[2(\gamma^2-1)q]^{1/2}$, if one substitutes $q=J_0/\rho_0 D$ in it. Ramsden and Savic proceeded in this fashion[14] but they used in their formula, only as the result of an error made in the determination of the connection of q and J_0, the density behind the front ρ in place of ρ_0.

[3] Such is the effective exponent of the air adiabat for T $T\sim5\times10^5-10^6$ °K and density close to normal.[17]

[4] In the work of Ramsden and Savic,[14] in which the results of Ramsden and Davis[5] are interpreted, the velocity of the wave is correctly estimated from a formula of the type (4.1), which gives D ≈ 100 km/sec, as also for the experiments of[7]. However, these authors did not consider the existence of Eq. (4.2) and ascribed to the absorbing gas a temperature T ≈ 40000°K, which follows from the measured line width of the scattered light. This value has in fact nothing in common with temperature of the gas in the absorption zone, which is more than an order of magnitude larger (for more details on this, see[7].

mechanisms, for example, the "breakdown" mechanism gives a velocity of propagation of the absorption wave exceeding the "detonation" velocity (4.1), then no shock wave is formed in the light channel. The gas, absorbing the light flux, transforms from the initial state O in Fig. 2 to the final state C by means of a continuous compression along the line OC. The wave in this case is propagated along the heated gas remaining behind it, with supersonic velocity, so that no hydrodynamic perturbations overtake it. The compression in the superdetonation regime is less than $(\gamma + 1)/\gamma$ and the coefficient β in (3.5) is less than $2\gamma/(\gamma + 1)$. In the limit as $D \to \infty$, the gas is in general not set into motion and $\eta \to 1$, $\beta \to 1$. The coefficient β is always very close to unity ($1 < \beta \leq 2\gamma/(\gamma + 1) \approx 1.14$; $\gamma \approx 1.33$), which evidences an insignificant effect of motion and compression on the energy balance. Therefore, within the framework of the assumption of the smallness of the width of the wave in comparison with the radius of its surface, the simple energy equation (3.1) is valid with high accuracy for any mechanism of wave propagation.

5. LATERAL EXPANSION OF THE GAS

As a consequence of the radial expansion of the heated gas, a shock wave moves out from the line of intersection of the surface of the front of the energy-release wave with the "surface" of the light channel, into the gas lying outside the channel. If the width Δx of the wave of energy release is not much less than the radius of the channel R, but is comparable with this value, as is most frequently the case under experimental conditions, then a significant fraction of the released energy is transferred to the layer of the gas surrounding the channel during the time of energy release. This leads to a lowering of the mean temperature over the cross section. (Moreover, the surface of the front in the detonation regime is bent, as in the detonation of a cylindrical explosive charge with a diameter close to critical; see Fig. 3.)

In the energy equation (3.1), we introduce a correction which takes roughly into account the losses to lateral expansion.[5] The velocity of flow of the gas through the lateral "surface" of the channel is of the order of the sound velocity c ($c = [\gamma(\gamma - 1)\epsilon]^{1/2}$). Therefore, the energy balance in the wave zone can be written approximately

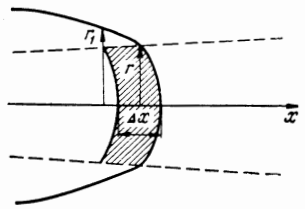

FIG. 3

in the form

$$\rho_0 D\varepsilon\pi r^2 + \rho_0 c\varepsilon \cdot 2\pi r\Delta x = J_0\pi r^2,$$

whence

$$\rho_0 D\varepsilon = J_0\delta, \qquad \delta = (1 + 2\Delta x c/rD)^{-1}. \qquad (5.1)$$

We can also reason somewhat differently. During the time of energy release $\Delta t \sim \Delta x/D$ the shock wave traverses in a radial direction a distance $\Delta r \sim c\Delta t \sim c\Delta x/D$, so that the released energy $J_0\pi r^2\Delta t$ goes into heating of the mass $\rho_0 D\pi\Delta t(r + \Delta r)^2$. For the mean energy of heating we have $\rho_0 D\epsilon = J_0[r/(r + \Delta r)]^2$, which reduces to (5.1) if $\Delta r < r$.

In the detonation regime $c \approx D/2$ [6] and $\delta \approx 1/(1 + \Delta x/r)$. This coefficient can be introduced directly in Eqs. (4.1) and (4.2) for estimate purposes, writing $J_0\delta$ in them in place of J_0. For example, for the experiments of [7] $\Delta x \approx l_\nu$, which is not much less than $r \approx 10^{-2}$ cm. If we set $\delta = 0.5$, then in place of the values estimated in Sec. 4 we get $D = 105$ km/sec, $\epsilon = 8.5 \times 10^{13}$ erg/g, and $T \approx 7.2 \times 10^5$°K, which are closer to the measured values.

Thanks to the radial expansion of the heated gas, within a certain time after the first breakdown the picture of the flow around the light channel becomes very similar to the picture of air flow in a supersonic motion of a body (the forward parts of the body correspond to the wave of energy release; see Fig. 3). The radial velocity of the shock wave at a given cross section x is somewhat smaller than the velocity of sound $c(\epsilon)$ even at the initial moment of separation of the wave from the surface of the light channel, that is, no more than half the velocity of propagation of the energy-release wave along the channel D. For large superdetonation velocities D it can be very much smaller than D. The radial velocity in the given section x decreases with passage of time, roughly speaking, like 1/R, where R is the radius of the shock

[5] We note that the energy loss to radiation is small. This is especially evident from an analysis of the transport of the wave by thermal radiation.

[6] For very large superdetonation velocities of wave propagation, $D \gg c$ and the losses on the lateral expansion are small even for $\Delta x \sim r$. The lateral discharge in this case encompasses only the peripheral part of the gas in correspondence with the Mach angle.

front. Numerical estimates of the law of radial propagation of the shock wave and the shape of the surface of the front can be made with the help of the well-known formulas for a cylindrical explosion, which must be applied to each cross section x, measuring the time from the moment of energy release in the cross section. In this case, the energy release per unit length of the channel is obviously equal to $J_0\pi r^2/D$ (erg/cm).

After a sufficient time following the end of the light pulse, when the shock wave moves off a distance large in comparison with the length of that part of the channel where the energy pulse is released, the motion of the gas acquires the character of motion in a point explosion. More detailed consideration of the general picture of the flow of the gas outside the zone of energy release and for the explosion phenomena indicated lie beyond the framework of the present article.

6. THE BREAKDOWN WAVE

We now proceed to a consideration of the nonhydrodynamic mechanisms of wave propagation.

Under the action of the light pulse an electron cascade is developed in the cold gas.[9] The electron density increases with passage of time according to the law

$$\frac{dn_e}{dt} = \frac{n_e}{\theta},$$

$$n_e = n_{e0} \exp\left[\int_0^t \frac{dt}{\theta}\right]. \qquad (6.1)$$

For large light fluxes, the only ones in which the mechanism of breakdown plays any role, the value θ is determined fundamentally as the time which is necessary for the electrons to acquire the energy needed for ionization (or excitation) of the molecules and atoms. In this case the rate of development of the cascade $1/\theta$ is simply proportional to the light flux: $1/\theta = AJ_0$. We shall not define the coefficient A more precisely here; see [9].

We assume that the breakdown sets in when the density of electrons reaches some critical value n_{ec} (for which the absorption of light quanta becomes sufficiently intense). This means that the moment t of onset of breakdown in section x of the light channel is determined by the equation

$$\int_0^t \frac{dt}{\theta} = A\int_0^t J_0(x,t)\,dt = \ln\frac{n_{ec}}{n_{e0}} = \mu. \qquad (6.2)$$

We shall assume the quantity μ, which depends only logarithmically on n_{ec} and n_{e0}, to be approximately constant.

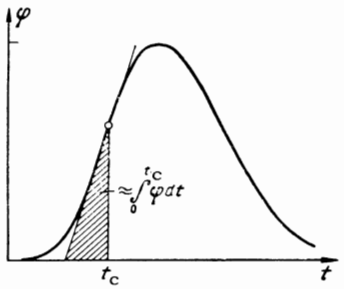

FIG. 4

We represent the light flux $J_0(x, t)$ in the form

$$J_0(x,t) = \frac{W}{\pi r^2}\varphi(t),$$

where W is the peak power of the generator, $\varphi(t)$ is a dimensionless function characterizing the shape of the pulse (a typical curve is shown in Fig. 4), and r is the radius of the channel at the section x. Substituting this expression in (6.2), we get

$$A\frac{W}{\pi r^2}\int_0^t \varphi(t)\,dt = \mu. \qquad (6.3)$$

If t_c is the instant of first breakdown in the focus, then

$$A\frac{W}{\pi r_0^2}\int_0^{t_c} \varphi(t)\,dt = \mu. \qquad (6.4)$$

Noting that approximately $r = r_0 + x\tan\alpha$ (see Fig. 1), we get from (6.3) and (6.4) an equation which determines the law of motion of the breakdown wave x(t):

$$\int_0^t \varphi(t)\,dt \Big/ \int_0^{t_c} \varphi(t)\,dt = \left(1 + \frac{x}{r_0}\operatorname{tg}\alpha\right)^2. \qquad (6.5)^*$$

For powers appreciably above threshold, the breakdown usually takes place even before the moment of onset of peak power. For convenience in estimating by means of (6.5), we approximate the curve $\varphi(t)$ in the region of power increase by a straight line as shown in Fig. 4 (we measure the time from the point of intersection of the straight line with the abscissa). Extrapolating the straight line to zero power, we get from (6.5)

$$x = D(t - t_c),$$

$$D = \frac{dx}{dt} = \frac{r_0}{t_c \operatorname{tg}\alpha}. \qquad (6.6)$$

In the region of power increase, the velocity of the wave is constant under the approximation made. Shortly before the instant of maximum power, it

*tg = tan.

begins to fall off. For the experiment of [7], about which we spoke in the Introduction, $r_0 \approx 10^{-2}$ cm, $t_c \approx 10$ nanosec, $\tan\alpha \approx 0.1$ and the velocity of the breakdown wave $D \approx 100$ km/sec. It is very close to the hydrodynamic value (see above). In the experiments of Ramsden and Davis,[5] $r_0 \approx 4 \times 10^{-3}$ cm, $t_c \approx 7$ nanosec, $\tan\alpha \approx 1$ (short focus lens); the breakdown takes place almost at maximum power. The velocity of the breakdown wave $D \approx 6$ km/sec is very small (the hydrodynamic velocity is $D \sim 100$ km/sec). We estimate the dependence of the initial velocity on the generator peak power W on the length of the pulse Δt_1 and on the geometry. For the estimate, we set $\varphi(t) = \text{const} \cdot t/\Delta t_1$, and obtain according to Eq. (6.4) $t_c \sim \Delta t_1^{1/2} W^{-1/2} r_0$; Eq. (6.6) in this case gives

$$D \sim (W/\Delta t_1)^{1/2}/\operatorname{tg}\alpha. \qquad (6.7)$$

In the case of short, powerful pulses and long-focus lenses (small α), the initial velocity of the breakdown wave can be very large and can appreciably exceed the hydrodynamic value. For example, in the geometry of the experiments of [7] ($r_0 = 10^{-2}$ cm, $\tan\alpha = 0.1$), but with a peak power one order of magnitude larger ($W \approx 1000$ MW) and a pulse length half as great ($\Delta t_1 \approx 20$ nanosec), the velocity of the breakdown wave [according to (6.6) and (6.7)] amounts to about 500 km/sec, while the corresponding velocity of the "detonation" is equal to about 200 km/sec.

7. RADIATIVE TRANSPORT OF THE WAVE

For temperatures of the order of several hundred thousand degrees, the range $l_\nu \sim 10^{-1} - 10$ cm of the thermal quanta with energies $h\nu \sim kT$, is much larger than the range of the light quanta ($l_\nu \sim 10^{-3} - 10^{-2}$ cm), the thickness of the waves, and also the characteristic dimensions of the heated region. The heated gas is transparent for thermal radiation and the radiation comes from the whole volume. This radiation is absorbed in the much colder layers where ionization is small; the corresponding range in atmospheric air $\lambda \approx 10^{-2} - 10^{-1}$ cm (for $h\nu \approx 20$–200 eV). As soon as the ionization brought about by absorption of the thermal radiation reaches a sufficient value (and this takes place when $T \sim (1.5 - 2) \times 10^{-4}$ °K), a new ionized layer begins to absorb the light flux intensely. Inasmuch as the focused light flux is much greater than the flux of thermal radiation, the layer quickly heats up and the boundary of the zone of high temperature moves toward it.

Let us consider a stationary regime in a system of coordinates in which the wave is at rest; see Fig. 5 (as will be seen from what follows, the stationary regime is possible). We assume that the degree of ionization and the path length of the light quanta l_ν are approximately determined by the energy of the gas ϵ. We call the front of the wave that point ($x = 0$) where the rates of energy release in the gas as a result of absorption of the thermal radiation Q (in erg/cm^3-sec) and of the light flux $dJ/dx = J/l_\nu$ are identical. In front of the wave front ($x > 0$) the gas heats up and is ionized principally through the absorption of thermal radiation. Therefore, the energy of the gas on the front, ϵ_0, is determined by the equation

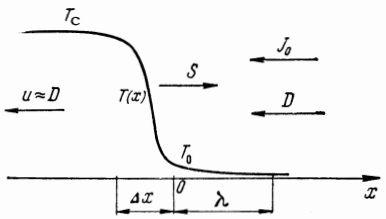

FIG. 5

$$\rho_0 D\varepsilon_0 = \int_0^\infty Q(x)\,dx = S, \qquad (7.1)$$

while the condition of the equality of energy release at the front has the form

$$J_0/l_{\nu_0} = Q_0, \quad l_{\nu_0} = l_\nu(\varepsilon_0), \quad Q_0 = Q(0), \quad J_0 = J(\infty). \qquad (7.2)$$

Behind the front ($x < 0$), the role of thermal radiation in the energy balance of the gas is small, and the variation of the temperature depends essentially on the absorption of the light flux.[7)] The final energy of the gas ϵ_f is given by Eq. (5.1):

$$\rho_0 D\varepsilon_f = J_0\delta. \qquad (7.3)$$

For an estimate of Q(x), we assume that the radiating volume is a cylinder with a constant temperature, equal to the final temperature (see Fig. 6). Let this cylinder be separated from the plane of the front by a non-radiating layer (in which the temperature increases to a value close

[7)] It is easy to find the energy distribution behind the front $\epsilon(x)$ if one does not take into account the motion of the gas. Then we have

$$\rho_0 D\frac{d\varepsilon}{dx} = -\frac{dJ}{dx} = -\frac{J}{l_\nu}, \quad \rho_0 D\varepsilon = J_0 - J, \quad J = \rho_0 D(\varepsilon_f' - \varepsilon),$$

where $\rho_0 D\epsilon_f' = J_0$. From these equations, it follows that

$$-\frac{d\varepsilon}{dx} = \frac{\varepsilon_f' - \varepsilon}{l_\nu}, \quad |x| = \int_{\varepsilon_0}^{\varepsilon} \frac{l_\nu(\varepsilon)\,d\varepsilon}{\varepsilon_f' - \varepsilon}.$$

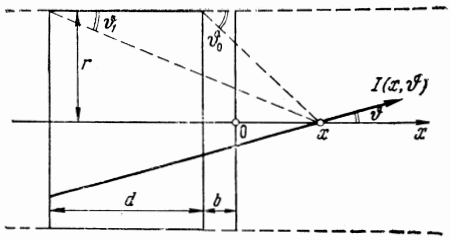

FIG. 6

to the final value). Attenuation of the flux of thermal radiation is small in this layer. Under these assumptions, the intensity of the radiation at the point x on the axis of the channel, $I(x, \vartheta)$ [erg/cm^2-sec-sr] is equal to (see Fig. 6)*

$$I(x,\vartheta)=\frac{j}{4\pi}\exp\left(-\frac{x}{\lambda\cos\vartheta}\right)\times\begin{cases}\dfrac{d}{\cos\vartheta}, & 0<\vartheta<\vartheta_1=\operatorname{arc\,tg}\dfrac{r}{d+b+x}\\ \dfrac{r}{\sin\vartheta}-\dfrac{b+x}{\cos\vartheta}, & \vartheta_1<\vartheta<\vartheta_0=\operatorname{arc\,tg}\dfrac{r}{b+x}.\end{cases}$$

Here $j = 4\sigma T_c^4/l_1$ [erg/cm^3-sec] is the emissivity of the gas; $l_1(T_c)$ is correspondingly the range averaged over the spectrum;[17] λ is the mean free path for absorption in the cold gas. The rate of energy release Q is

$$Q(x)=\int_0^{\vartheta_0}\frac{I}{\lambda}\cdot 2\pi\sin\vartheta\,d\vartheta.$$

On the wave front we have, at x = 0,

$$Q_0=\frac{jr}{2\lambda}f(\vartheta_0,\vartheta_1),$$

$$f=[\vartheta_0-\vartheta_1+\operatorname{ctg}\vartheta_0\ln\cos\vartheta_0-\operatorname{ctg}\vartheta_1\ln\cos\vartheta_1] \quad (7.4)\dagger$$

(f is at a maximum and is equal to $\pi/2$ for $d=\infty$, $b=0$).

The integral[8] $S=\int_0^{\infty}Q(x)\,dx$ can be computed (by reversal of the order of integration) for the simpler case of a semi-infinite cylinder closely adjoining the front ($b=0$, $d=\infty$). As a result, a relation is obtained between S and Q_0 which is approximately valid, according to the estimates, in the real range of values of b and d. Keeping this relation, we get

$$S=Q_0\lambda\psi\left(\frac{r}{\lambda}\right)=\frac{1}{2}jr\psi\left(\frac{r}{\lambda}\right)f(\vartheta_0,\vartheta_1),$$

$$\psi(\xi)=\pi^{-1}[\xi E_1(\xi)+2-\xi^{-1}+e^{-\xi}(\xi^{-1}-1)] \quad (7.5)$$

(ψ and f are slowly varying functions; $\psi(\infty)=0.64$, $\psi(1)=0.39$, $\psi(0.1)=0.14$; for $d/r\sim 1$ and $b/r\sim 1$, we have $f\sim 1$).

Equations (7.1)-(7.5) make it possible to find the unknown D, ϵ_f and ϵ_0. Actually, the situation reduces to the solution of the set

$$\frac{\varepsilon_f}{\varepsilon_0}=\frac{J_0\delta}{S(\varepsilon_f)}, \qquad \frac{\varepsilon_f}{\varepsilon_0}=\frac{l_{\nu_0}(\varepsilon_0)\delta}{\lambda\psi} \quad (7.6)$$

relative to ϵ_f and ϵ_0.

For high temperatures, ionization and excitation of the ions do not differ appreciably from equilibrium. The range l_1 can then be estimated by the method described in [17]. For air of normal density, in the interval $T\approx 2\times10^5-10^6$ °K, we have approximately, $l_1\approx 2\times(T°/5\times10^5)^3$ cm and $j\approx 7\times10^{18}(T°/5\times10^5)$ erg/cm^3-sec. The energy can be approximated by the formulas $\epsilon\approx 4.6\times10^{13}(T/5\times10^5)^{2/3}$ erg/g for $T\approx 2\times10^5-5\times10^5$ °K and $\epsilon\approx 4.5\times10^{13}(T/5\times10^5)^{7/4}$ erg/g for $T\approx 5\times10^5-10^6$ °K.

The ranges l_{ν_0} in front of the wave front can also be estimated by starting out from the assumption of thermodynamic equilibrium.[9] According to the Kramers-Unsold formula we have, for air in the vicinity of the first ionization and $h\nu = 1.8$ [17],

$$\varkappa_\nu=l_\nu^{-1}=0.57\,Te^{-165000/T}(e^{21000/T}-1)\,[\mathrm{cm}^{-1},\ T°]. \quad (7.7)$$

For a numerical example, we again consider the experiment cited above:[7] $J_0=2\times10^{18}$ erg/cm^2-sec, $r=10^{-2}$ cm. We set $\lambda=3\times10^{-2}$ cm, and here $\psi(1/3)=0.2$; let $d=r$ and $b=r/4$, then $f=0.6$; we get $\delta=0.5$. Equations (7.6) and (7.3) give:

*arc tg = tan^{-1}.

†ctg = cot.

[8] The quantity S is approximately that part of the flux of thermal radiation escaping from the surface of the wave front which is absorbed in the limits of the light channel.

[9] Here one can advance the following considerations. For air, one obtains $\epsilon_0\approx 16$ eV per molecule, to which correspond the equilibrium $T\approx 15000$°K and degree of ionization 0.1 electron per molecule. The energies of the thermal quanta ($h\nu\sim 50$ - 100 eV) are initially transferred to photoelectrons, the primary number of which $\sim\epsilon_0/h\nu$ is comparable with the equilibrium value. The electronic processes (excitation and deactivation of molecules, ionization) take place very quickly (the characteristic time of heating is $\Delta t\sim\lambda/D\sim 3\times10^{-2}/10^7=3\times10^{-9}$ sec). Recombination requires times comparable with Δt. The transfer of energy from the electrons to the heavy particles through elastic collisions takes place slowly ($\tau\sim 10^{-8}$ sec), but in a molecular gas there is another fast transfer mechanism: electronic excitation of molecules is simultaneously accompanied by excitation of vibrations (in correspondence with the Franck-Condon principle), and sometimes dissociation of the molecules also takes place.

$\epsilon_0 \approx 16$ eV per molecule or 5.3×11^{11} erg/g; $T_0 = 15300$°K, $l_{\nu 0} = 1.9$ cm, $T_c = 7.5 \times 10^5$ °K, $\epsilon_f = 8.3 \times 10^{13}$ erg/g, D = 95 km/sec. The fact that the path length $l_{\nu 0}(T_0)$ is very great on the front does not in any way give evidence as to the width of the wave. One can show that, thanks to the sharp exponential dependence of $l_\nu(T)$ for low temperatures, the width of the wave will always be of the order of $l_\nu(T_c)$.

For an estimate of the dependence of the velocity D on the flux J_0 and other parameters, we note that as a consequence of the extraordinarily strong dependence of $l_{\nu 0}$ on T_0 by Eq. (7.7), the energy on the front ϵ_0 is extremely stable; that is, one can assume approximately that $\epsilon_0 \approx 16$ eV per molecule always. Then the first of Eqs. (7.6) gives the direct connection of ϵ_f with J_0. Making use of (7.5) and the interpolation formulas for $\epsilon(T)$ (for $5 \times 10^5 < T < 10^6$ °K) and $j(T)$, we obtain the following estimated formulas for air:

$$T_c = 1.9 \cdot 10^{-2} (J_0 \delta / r \psi f)^{0.364} \text{ [deg]},$$
$$\varepsilon_f = 4.15 (J_0 \delta / r \psi f)^{0.636} \text{ [erg/g]},$$
$$D = 1.9 \cdot 10^2 (J_0 \delta)^{0.364} (r \psi f)^{0.636} \text{ [cm/sec]}. \quad (7.8)$$

As is seen, the wave velocity and its dependence on the light flux are shown to be approximately the same as for the detonation regime. (Of course, this coincidence must not be taken literally but only as an identity in order of magnitude; we note that the functions δ, ψ, f also depend on J_0 in the final analysis but not strongly.)

If the regime is a detonation regime, the air in front of the shock wave is always a little heated by the thermal radiation; this heating is determined by the same Eq. (7.1). Together with this, a very thin layer in front of the shock wave is heated by the mechanism of electron conductivity, while in this layer the temperature is of the order of the final T_c. The width of the layer Δx_e is determined from the approximate consideration:

$$S_e \approx \rho_0 D \varepsilon_f \approx J_0 \delta,$$

where S_e is the thermal conduction flux from the front and

$$S_e \approx \eta_e(T_c) T_c / \Delta x_e,$$

where $\eta_e = aT^{5/2}$ is the coefficient of electron thermal conductivity. In air, for five fold ionization (Z = 5), we have $a \approx 5 \times 10^{-7}$ erg/sec-cm^2-deg$^{7/2}$. These equations give $\Delta x_e \approx 5 \times 10^{-7} T_c^{7/2} / J_0 \delta$. For example, for $J_0 \delta = 10^{18}$ erg/cm^2-sec and $T_c = 7 \times 10^5$ °K, we have $\Delta x_e \approx 1.4 \times 10^{-4}$ cm and $\Delta x_e \ll l_\nu(T_c) \approx 6 \times 10^{-3}$ cm, that is, the layer is transparent for light flux, which also demonstrates the insignificant role of the electron conductivity. We note that diffusion of the electrons from the front also plays an equally small role.

8. DISCUSSION OF THE RESULTS

As calculations have shown, the dynamic and radiation mechanisms give approximately the same wave velocities. The velocities are not "additive," and under conditions in which the mechanism or breakdown plays a small role, the real wave moves with a velocity which is given by either of the first two mechanisms. There is some basis for assuming that the effectiveness of the radiation mechanism in the calculations is somewhat underestimated. In particular, this is associated with the assumption that equilibrium is established in front of the wave-front, which raises misgivings, for essentially there is no spare time for this purpose. The lack of equilibrium always is such as to produce a larger degree of ionization and excitation of the atoms, since initially all the energy of thermal radiation is transferred by the fast electrons. Therefore, the gas acquires the necessary ability to absorb the light flux at lower fluxes of heat radiation and at lower final temperatures, i.e., the radiation wave propagates more rapidly. However, the phenomena are so complicated and the data on the ranges of the radiation are so incomplete that an increase in the accuracy of the calculations (which without doubt is possible) would hardly allow us to give any preference with assurance to the hydrodynamic or radiation mechanisms.

Obviously this problem must be solved by experiment. Even for similar velocities, the internal structure of the wave depends strongly on the transport method. In the hydrodynamic mechanism, there is a shock wave, that is, a steep front of compression and heating, where in the first place the ions acquire energy and the ion temperature is extremely high, even higher than the final temperatures.[17] In the other (superdetonation) mechanisms, the heating takes place by degrees, the ion temperature is not higher and most likely is lower than the electron temperature, and there is apparently no compression at all because of the lateral divergence.

For high powers and long focus lenses, a breakdown wave arises, but after some time, as the power in the pulse falls off, the breakdown wave slows down and the detonation wave or the radiation wave must evidently overtake it. Even if the nature of the mechanism in any case remains unclear, one can always determine the temperature

of the heating by measuring the velocity of the wave, making use of the law of conservation of energy (3.1) and (5.1).

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[1] R. G. Meyer and A. F. Haught, Phys. Rev. Lett. **11**, 401 (1963).

[2] E. K. Damon and R. G. Tomlinson, J. Appl. Opt. **2**, 546 (1963).

[3] R. W. Minck, J. Appl. Phys. **35**, 252 (1964).

[4] R. G. Meyer and A. F. Haught, Phys. Rev. Lett. **13**, 7 (1964).

[5] S. A. Ramsden and W. E. Davies, Phys. Rev. Lett. **13**, 227 (1964).

[6] Mandel'shtam, Pashinin, Prokhindeev, Prokhorov, and Sukhodrev, JETP **47**, 2003 (1964), Soviet Phys. JETP **20**, 1344 (1965).

[7] Mandel'shtam, Pashinin, Prokhorov, Raĭzer, and Sukhodrev, JETP (in press).

[8] Ambartsumyan, Basov, Boĭko, Zuev, Krokhin, Kryukov, Senatskiĭ, and Stoilov, JETP **48**, 1583 (1965), Soviet Phys. JETP **21**, in press.

[9] Ya. B. Zel'dovich and Yu. P. Raĭzer, JETP **47**, 1150 (1964), Soviet Phys. JETP **20**, 772 (1965).

[10] J. K. Wright, Proc. Phys. Soc. (London) **84**, 41 (1964).

[11] L. V. Keldysh, JETP **47**, 1945 (1964), Soviet Phys. JETP **20**, 1307 (1965).

[12] G. A. Askar'yan and M. S. Rabinovich, JETP **48**, 290 (1965), Soviet Phys. JETP **21**,

[13] D. D. Ryutov, JETP **47**, 2194 (1964), Soviet Phys. JETP **20**, 1472 (1965).

[14] S. A. Ramden and P. Savic, Nature, No. 4951, 1217 (1964).

[15] N. G. Basov and O. N. Krokhin, JETP **46**, 171 (1964), Soviet Phys. JETP **19**, 123 (1964).

[16] J. M. Dawson, Phys. Fluids **7**, 981 (1964).

[17] Ya. B. Zel'dovich and Yu. P. Raĭzer, Fizika udarnykh voln i vysokotemperaturnykh gidrodinamicheskikh yavleniĭ (Physics of Shock Waves and High Temperature Hydrodynamic Phenomena) (Fizmatgiz, 1963).

[18] K. W. Allen, Astrophysical Quantities (Russian translation, IIL, 1960).

[19] L. D. Landau and E. M. Lifshitz, Mekhanika sploshnykh sred (Mechanics of Continuous Media) (Gostekhizdat, 1954).

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