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EXCITATION OF ELECTROMAGNETIC WAVES IN PLASMAS SITUATED IN EXTERNAL ELECTRIC FIELDS

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We investigate the excitation of low-frequency electromagnetic waves in a weakly ionized electron-ion plasma and electron-hole solid-state plasma in the presence of an external electric field. By solving the kinetic equation with the collision integral obtained by Davydov[13], we derive an expression for the dielectric tensor of the plasma medium. A detailed analysis is presented of the dispersion equation for small oscillations. We show that longitudinal oscillations are excited in a plasma only at electron drift velocities exceeding the phase velocity of the wave along the drift. The buildup of oscillations is connected in this case with a change in the sign of the high-frequency conductivity of the plasma under the conditions of the anomalous Doppler effect. The transverse electromagnetic waves, on the other hand, are excited practically at arbitrarily small electron-drift velocities, i.e., for arbitrarily small currents in the plasma. In an electron-hole solid-state plasma, under the conditions considered below, only transverse oscillations can build up. The frequencies and the growth increments of the oscillations are obtained and the conditions under which their buildup takes place are indicated.

1. INTRODUCTION

THE recent literature includes many theoretical and experimental papers devoted to the buildup of low-frequency sound oscillations in a solid state plasma under the influence of an external electric field. The strongest buildup of ultrasound occurs in piezoelectric semiconductors[1]. In nonpiezoelectric crystals the effect of sound buildup is much less pronounced[2]. Therefore principal attention is paid in the theoretical papers (see[3-6]) to the buildup of sound in piezoelectric crystals.

In such crystals, in the absence of an electric field, the attenuation of the sound or lattice vibrations is due to the conduction electrons. In the presence of an external electric field, the high-frequency electron conductivity of the crystal can reverse sign and become negative if the electron-drift velocity exceeds the phase velocity of the wave, i.e., the velocity of sound. As a result, the sound builds up instead of attenuating. Such a buildup of sound oscillations in a crystal is analogous to the kinetic buildup of ion sound in a collisionless nonisothermal plasma with current if all the electrons move relative to the ions[7]. The only difference is that the conductivity of the crystal is due to the collisions between the electrons and the lattice, while the conductivity of the collisionless plasma is due to the Cerenkov mechanism of absorption of waves by the plasma electrons. An essentially similar mechanism of sound buildup in a nonpiezoelectric crystal is discussed also by Konstantinov et al.[2], who assume that the coupling between the lattice vibrations and the conduction electrons is via the contact potential difference produced on the p-n junction boundary.

In most of the cited theoretical papers[2-5], only longitudinal oscillations of the electromagnetic field are considered. Such a limitation, however, is rigorously valid only for isotropic media and for crystals with cubic symmetry. Pustovoĭt[6] dispenses with this limitation and does not assume that the electromagnetic field in the crystal is potential. In calculating the electric conductivity of the crystal, however, he neglects the magnetic field of the wave, and obtains quantitatively incorrect results. The point is that it is incorrect to neglect the magnetic field in the frequency region of the anomalous Doppler effect, which is of greatest interest and in which the electron-drift velocity exceeds the phase velocity of the wave along the drift and the conductivity-tensor components reverse sign. Thus Pustovoĭt's formulas[6] likewise apply only to potential field oscillations in a plasma.[1)]

We shall show below that allowance for the magnetic field of the wave does not change the deduction that oscillations can build up when the waves propagate at an acute angle to the drift direction if the drift velocity of the electrons exceeds the phase velocity of the wave. This is the result of the fact that such waves are practically longitudinal. Such a statement is no longer true for the waves propagating at a large angle to the drift direction. We shall show below that such waves can build up in practice at arbitrarily small directional velocities of the carrier drift in a solid-state electron-hole plasma even if the lattice vibrations are completely neglected. The buildup of the oscillations occurs in the region of frequencies of the normal Doppler effect (i.e., the carrier drift velocities are smaller than the phase velocity of the wave along the drift), and is not connected with the change in the sign of the plasma conductivity. Such a buildup is analogous to that occurring at normal Doppler-effect frequencies in a collisionless current-carrying plasma[8], and is due to the plasma anisotropy caused by the relative drift of the carriers in the electric field.

In Sec. 4 of this paper, unlike in earlier papers[2-6] in which principal attention was paid to the buildup of sound oscillations of the lattice, we consider arbitrary low-frequency oscillations of an electron-hole solid state plasma in an external electric field, neglecting the lattice vibrations completely. An analogous formulation of the problem is contained in the paper of Pines and Schrieffer[9], who investigated high-frequency longitudinal oscillations of an electron-hole plasma in an electric field (with oscillation frequencies considerably exceeding the frequencies of the collisions between the carriers and the lattice) neglecting the sound oscillations of the lattice. The theory developed in[9] does not differ at all from the theory of oscillations of a collisionless plasma with current (see[7]). Moreover, it seems to us that the buildup of such high-frequency oscillations in a real crystal, where the carrier collision frequencies are on the order of 10^{11}—10^{13} $\sec^{-1}$, it is quite doubtful. As to the work of Pustovoĭt[6], who, in particular, considered also low frequency oscillations of an electron-hole plasma in an external electric field, his main deduction, namely that oscillations cannot build up in such a plasma, holds true only for longitudinal oscillations, this being the consequence of the already indicated neglect of the magnetic field of the wave.

Finally, we note that low-frequency oscillations of an electron-hole plasma in an external electric field should be similar to oscillations of a weakly-ionized electron-ion plasma if the collisions between the charged particles can be neglected. The longitudinal oscillations of a field in such a plasma were investigated by several workers[10-12]. Akhiezer and Sitenko[10] and Stepanov and Tkalich[11] neglected the ion motion completely. As expected, the vibrations of a purely electronic plasma were damped in this case. Liperovskiĭ[12] took into account the ion motion and indicated that such plasma oscillations can build up. However, a numerical solution of the dispersion equation for the oscillations, undertaken in[12], could not disclose the physical mechanism whereby the oscillations build up and did not yield an instability criterion.

As far as we know, general non-potential oscillations of a weakly-ionized plasma in an external electric field have not been investigated before. We therefore examine separately in Sec. 3 the spectrum of arbitrary low-frequency oscillations of a weakly-ionized electron-ion plasma. Unlike electron-hole plasma, in an electron-ion plasma the mass of the ions (positive carriers) is of the order of the mass of the neutral particles, and this leads the oscillations of such a plasma to differ appreciably from those of a solid-state plasma. The main difference is that low-frequency longitudinal oscillations can build up in a weakly-ionized electron-ion plasma, in the approximation considered, as will be shown below, whereas in a solid state plasma such oscillations are always damped.

2. ELECTRON CONDUCTIVITY AND DIELECTRIC CONSTANT OF A PLASMA IN AN EXTERNAL FIELD

To investigate small oscillations of a plasma medium, we start, as usual, from the dispersion equation

$$|k^2\delta_{ij} - k_ik_j - \omega^2c^{-2}\varepsilon_{ij}(\omega, \mathbf{k})| = 0, \tag{1}$$

where $\epsilon_{ij}(\omega, \mathbf{k})$ is the complex dielectric constant of the medium. In a weakly-ionized plasma, as in an electron-hole solid-state plasma, under conditions when the collisions of the charged particles with one another can be neglected, the dielectric constant is an additive function of the densities of the different species of particles. This means that the tensor $\epsilon_{ij}(\omega, \mathbf{k})$ can be represented in the form

$$\varepsilon_{ij}(\omega, \mathbf{k}) = \varepsilon_{ij}^{(0)}(\omega, \mathbf{k}) + 4\pi i\omega^{-1}\sigma_{ij}^{(e)}(\omega, \mathbf{k}), \tag{2}$$

where $\sigma_{ij}^{(e)}(\omega, \mathbf{k})$ is the electron conductivity of the

[1)]This means that in all formulas of this paper we must put $k_y = k_z = 0$.

plasma (i.e., the conductivity due to the negative carriers), and $\epsilon_{ij}^{(0)}(\omega,\mathbf{k})$ is the contribution made to the dielectric constant of the plasma by the remaining species of particles (ions, or positive carriers and neutral particles). Concrete expressions for $\epsilon_{ij}^{(0)}(\omega,\mathbf{k})$ are given in the next two sections. In the present section, we obtain only an expression for the electron conductivity of the plasma.

Following Davydov[13] (see also[14]), we expand the electron distribution function in a series of Legendre polynomials, confining ourselves to the first two terms of the expansion:

$$f(\mathbf{v}) = f_0(v) + \frac{\mathbf{v}}{v}\mathbf{f}_1(v). \tag{3}$$

(The conditions for the applicability of such an expansion will be indicated below.) As a result, the kinetic equation for the electrons reduces to a system of two equations[13,14]:

$$\frac{\partial f_0}{\partial t} + \frac{v}{3}\operatorname{div}\mathbf{f}_1 + \frac{e}{3mv^2}\frac{\partial}{\partial v}(v^2\mathbf{E}\mathbf{f}_1) + S_0(f_0) = 0,$$

$$\frac{\partial \mathbf{f}_1}{\partial t} + v\,\mathrm{grad}_r f_0 + \frac{e}{m}\mathbf{E}\frac{\partial f_0}{\partial v} + \frac{e}{mc}[\mathbf{B}\mathbf{f}_1] + \nu(v)\mathbf{f}_1 = 0, \tag{4*}$$

where $S_0(f_0)$ and $\nu(v)\mathbf{f}_1$ are the zeroth and first moments of the complete integral for the collisions between electrons and neutral particles:

$$S_0(f_0) = \frac{1}{4\pi}\int d\Omega\, S = -\frac{1}{2v^2}\frac{\partial}{\partial v}\left\{v^2\delta\nu\left(\frac{T}{m}\frac{\partial f_0}{\partial v} + v f_0\right)\right\},$$

$$\mathrm{S}_1 = \nu(v)\mathbf{f}_1 = \frac{3}{4\pi}\int d\Omega\, S\cos\vartheta \tag{5}$$

($\delta = 2m/M$—ratio of the electron mass to the neutral-particle mass [2]), $\nu(v)$ the frequency of the collisions between the electron and the neutral particle, and T the temperature of the neutral particles).

For a spatially homogeneous and stationary equilibrium state of the plasma, with equilibrium fields $\mathbf{E}_0$ and $\mathbf{B}_0$, we obtain from (4) the following solution[14]:

$$f_{00} = A\exp\left\{-\int_0^v mv\,dv\left[T + \frac{2e^2E_0^2}{3m\delta(\nu^2+\Omega^2)}\right]^{-1}\right\},$$

$$\mathbf{f}_{10} = -\mathbf{v}_0\partial f_{00}/\partial v, \tag{6}$$

where $\Omega = eB_0/mc$ is the Larmor frequency of the electrons,

$$\mathbf{v}_0 = \frac{e}{m\nu}\frac{1}{1+\Omega^2/\nu^2}\left\{\mathbf{E}_0 + \frac{\Omega}{\nu}\left[\frac{[\mathbf{E}_0\mathbf{B}_0]}{B_0} + \frac{\Omega}{\nu}\frac{\mathbf{B}_0(\mathbf{B}_0\mathbf{E}_0)}{B_0^2}\right]\right\} \tag{7}$$

is the drift velocity of the electrons in fields $\mathbf{E}_0$ and $\mathbf{B}_0$, and A is a normalizing multiplier determined from the normalization condition

$$\int d\mathbf{p}\, f_{00} = N_0 \tag{8}$$

(N_0 is the electron density).

It follows from (6) that the plasma electrons become heated in an electric field, and a stationary temperature is established, the order of magnitude of which is

$$T_e \approx T + 2e^2E_0^2/3m\delta(\bar{\nu}^2+\Omega^2), \tag{9}$$

where $\bar{\nu}$ is some effective electron collision frequency. As shown by Ginzburg and Gurevich[14], the electron temperature of a weakly-ionized plasma in a sufficiently strong electric field can greatly exceed the neutral-particle temperature. On the other hand, the ion temperature remains of the same order as the neutral-particle temperature, i.e., $T_i \approx T$ but $T_e \gg T$. For such a strongly heated plasma state, the solutions (6) hold true for all values of the field $\mathbf{E}_0$, since the main condition for the applicability of the approximate solution, viz., smallness of the directed electron drift velocity $\mathbf{v}_0$ compared with their thermal velocity $v_{Te} = [T_e/m]^{1/2}$, is always satisfied because of the small mass ratio $\delta \ll 1$.

We now consider a small deviation of the plasma from the equilibrium state, induced by alternating fields **E** and **B**. In accordance with (3), we represent the non-equilibrium addition to the distribution function (6) in the form

$$\delta f = \varphi_0(v) + \frac{\mathbf{v}}{v}\boldsymbol{\varphi}_1(v). \tag{10}$$

Linearizing the systems (4) and assuming that all the non-equilibrium quantities depend on the time and on the coordinates like exp(−iωt + i), we obtain

$$-i\omega\varphi_0 + i\frac{v}{3}\mathbf{k}\boldsymbol{\varphi}_1 + \frac{e}{3mv^2}\frac{\partial}{\partial v}(v^2\mathbf{E}_0\boldsymbol{\varphi}_1) + S_0(\varphi_0) = -\frac{e}{3mv^2}\frac{\partial}{\partial v}(v^2\mathbf{E}\mathbf{f}_{10}), \tag{11}$$

$$-i\omega\boldsymbol{\varphi}_1 + iv\mathbf{k}\varphi_0 + \frac{e}{m}\mathbf{E}_0\frac{\partial\varphi_0}{\partial v} + \frac{e}{mc}[\mathbf{B}_0\boldsymbol{\varphi}_1] + \nu\boldsymbol{\varphi}_1 = -\frac{e}{m}\mathbf{E}_1\frac{\partial f_{00}}{\partial v};$$

$$E_{1i} = \gamma_{ij}E_j = \left\{\left(1 - \frac{\mathbf{k}\mathbf{v}_0}{\omega}\right)\delta_{ij} + \frac{k_i v_{0j}}{\omega}\right\}E_j. \tag{12}$$

In the derivation of the system (11) we took Max-

*$[\mathbf{B}\mathbf{f}_1] \equiv \mathbf{B}\times\mathbf{f}_1$.

[2]The letter M will henceforth denote also the mass of the ions in a weakly-ionized plasma, assuming for simplicity that this mass coincides with the mass of the neutral particle.

well's equations into account.

Equations (11) go over into the system investigated by Pustovoĭt [6] if we substitute $\gamma_{ij} \rightarrow \delta_{ij}$, which is equivalent to neglecting the magnetic field of the wave and, strictly speaking, is valid only for potential oscillations of the field, when $\mathbf{E}_1 = \mathbf{E} = -\nabla\Phi$.

Determining the function $\varphi_1(\mathbf{v})$ from the second equation of (11) and substituting ∞ in the formula for the density of the current induced in the plasma by the electrons, we obtain after simple transformations

$$\mathbf{j} = e\int d\mathbf{p}\,\mathbf{v}\delta f = \frac{e}{3}\int d\mathbf{p}\,v\varphi_1$$

$$= \frac{ie^2(\omega+i\nu)}{m[(\omega+i\nu)^2-\Omega^2]}\left\{\mathbf{E}_1\int d\mathbf{p}\,f_{00} + \mathbf{E}_0\int d\mathbf{p}\,\varphi_0 - \frac{im}{3e}\mathbf{k}\int d\mathbf{p}\,v^2\varphi_0 + \frac{i\Omega}{\omega+i\nu}\left[\frac{[\mathbf{E}_1\mathbf{B}_0]}{B_0}\int d\mathbf{p}\,f_{00} + \frac{[\mathbf{E}_0\mathbf{B}_0]}{B_0}\int d\mathbf{p}\,\varphi_0 - \frac{im}{3e}\frac{[\mathbf{k}\mathbf{B}_0]}{B_0}\int d\mathbf{p}\,v^2\varphi_0\right] - \frac{\Omega^2}{(\omega+i\nu)^2}\frac{\mathbf{B}_0}{B_0}\left[\frac{\mathbf{E}_1\mathbf{B}_0}{B_0}\int d\mathbf{p}\,f_{00} + \frac{\mathbf{E}_0\mathbf{B}_0}{B_0}\int d\mathbf{p}\,\varphi_0 - \frac{im}{3e}\frac{\mathbf{k}\mathbf{B}_0}{B_0}\int d\mathbf{p}\,v^2\varphi_0\right]\right\}. \quad (13)$$

Here and throughout we neglect for simplicity the velocity dependence of the electron collision frequency. An account of this dependence greatly complicates the calculations but, as shown in [6], does not affect the results essentially. We therefore assume below that $\nu = \nu(T_e) = \text{const}$.

Relation (13) connects the electron current density with the unknown function φ_0, which in turn is determined by the solution of the system (11). In our case of weak spatial dispersion, when expansion (10) is valid, there is no need for an exact solution of this system. The expansion (10) is valid if $|\omega + i\nu| \gg kv_{Te}$ or kv_0. If, in addition, the following inequality is satisfied [6]

$$|\omega - \mathbf{k}\mathbf{v}_0| \gg (kv_{Te})^2/\nu, \quad (14)$$

then the terms containing the integral

$$\int d\mathbf{p}\,v^2\varphi_0 \sim v_T^2\int d\mathbf{p}\,\varphi_0,$$

can be neglected in Eq. (13) [3]. Using further the continuity equation

$$e\omega\int d\mathbf{p}\,\varphi_0 = \mathbf{kj}, \quad (15)$$

we obtain from (13) the density of the electron current induced in the plasma, and an expression for the electron conductivity:

$$j_i = \sigma_{ij}^{(e)}E_j;$$

$$\sigma_{ij}^{(e)} = \frac{ie^2N_0(\omega+i\nu)}{m[(\omega+i\nu)^2-\Omega^2]}\left\{\beta_{i\mu} + \frac{ie(\omega+i\nu)}{m\omega[(\omega+i\nu)^2-\Omega^2]}\times\left(1-\frac{ie(\omega+i\nu)k_\mu\beta_{\mu\nu}E_{0\nu}}{m\omega[(\omega+i\nu)^2-\Omega^2]}\right)^{-1}\beta_{is}E_{0s}k_\nu\beta_{\nu\mu}\right\}\gamma_{\mu j}, \quad (16)$$

$$\beta_{ij}^{(\omega)} = \delta_{ij} + \frac{i\Omega}{\omega+i\nu}\left[e_{ij\nu}\frac{B_{0\nu}}{B_0} + \frac{i\Omega}{\omega+i\nu}\frac{B_{0i}B_{0j}}{B_0^2}\right]. \quad (17)$$

Here $e_{ij\nu}$ is a unit completely antisymmetrical tensor of third rank.

Expression (16) becomes much simpler in the low-frequency limit, when $\omega \ll \nu$. Indeed, according to (7)

$$v_{0i} = \frac{e\nu}{m(\nu^2+\Omega^2)}\beta_{ij}^{(0)}E_{0j}, \quad (17')$$

where $\beta_{ij}^{(0)}$ is the limit of the tensor β_{ij} when $\omega \ll \nu$. Taking this into account, we can rewrite (16) in the form

$$\sigma_{ij}^{(e)} = \frac{e^2N_0}{m(\nu^2+\Omega^2)}\alpha_{i\mu}\beta_{\mu\nu}^{(0)}\gamma_{\nu j}, \quad (18)$$

$$\alpha_{ij} = \delta_{ij} + \frac{v_{0i}k_j}{\omega - \mathbf{k}\mathbf{v}_0}. \quad (19)$$

To conclude this section we present for comparison an expression for the electron conductivity of a weakly ionized plasma, obtained from the equation of hydrodynamics for cold electrons

$$\frac{\partial\mathbf{v}}{\partial t} + (\mathbf{v}\nabla)\mathbf{v} = \frac{e}{m}\left\{\mathbf{E} + \frac{1}{c}[\mathbf{vB}]\right\} - \nu\mathbf{v},$$

$$\partial n/\partial t + \operatorname{div} n\mathbf{v} = 0. \quad (20)$$

By determining the stationary equilibrium state of the plasma in external fields $\mathbf{E}_0$ and $\mathbf{B}_0$, and then using the usual linearization procedure, we can obtain from (20) the following hydrodynamic expression for the electron conductivity of the plasma:

$$\sigma_{ij} = \frac{ie^2N_0(\omega - \mathbf{k}\mathbf{v}_0 + i\nu)}{m[(\omega-\mathbf{k}\mathbf{v}_0+i\nu)^2-\Omega^2]}\alpha_{i\mu}\beta_{\mu\nu}(\omega - \mathbf{k}\mathbf{v}_0)\gamma_{\nu j}, \quad (21)$$

where the quantities $\mathbf{v}_0$, α_{ij}, β_{ij}, and γ_{ij} were defined above.

It follows from a comparison of formulas (16), (18), and (21) that the results of the kinetic and hydrodynamic approaches coincide if: (a) we assume that $|\omega + i\nu| \gg \mathbf{k}\cdot\mathbf{v}_0$ in the hydrodynamic expression (21), and (b) if we put in the kinetic expression (16)

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[3]An analysis of the opposite limit, under conditions of weak spatial dispersion, seems unjustified to us. A published investigation of such a limit[6] contains an unfortunate arithmetic error that leads to incorrect deductions.

$\omega \ll \nu$ (in addition, naturally, we assume that inequalities (14) are satisfied). Taking this circumstance into account, we investigate below the plasma oscillation spectra when both conditions are satisfied, i.e., in the overlapping region of validity of both the kinetic and hydrodynamic approaches, when expression (18) is applicable.

3. SPECTRUM OF OSCILLATIONS OF AN ELECTRON-ION PLASMA IN AN EXTERNAL ELECTRIC FIELD IN THE ABSENCE OF A MAGNETIC FIELD

The electromagnetic oscillations of a spatially homogeneous electron-ion plasma in an external constant electric field, under conditions when the particle collisions can be neglected, were investigated in detail by several workers [7,15]. We consider here plasma oscillations in the opposite limiting case, when the particle collisions play the decisive role. Namely, we analyze with the aid of the formulas derived in the preceding section the oscillations of a weakly-ionized electron-ion plasma in the frequency region $\omega \ll \nu_e$ in the absence of an external magnetic field. The latter denotes that we assume that $\nu_e \gg \Omega_e$ (the subscripts denote the particles to which the particular quantities pertain).

For the ion contribution to the dielectric constant of the plasma, we make use of a hydrodynamic expression of the type (21), neglecting completely the ion drift in the electric field[4]. The dielectric constant of the plasma can then be written in the form

$$\varepsilon_{ij} = \left(1 - \frac{\omega_{Li}^2}{\omega(\omega + i\nu_i)}\right)\delta_{ij} + \frac{i\omega_{Le}^2}{\omega^2\nu_e}\left[(\omega - \mathbf{kv}_0)\delta_{ij} + k_i v_{0j} + k_j v_{0i} + \frac{k^2 v_{0i} v_{0j}}{\omega - \mathbf{kv}_0}\right], \quad (22)$$

where $\mathbf{v}_0 = e\mathbf{E}_0/m\nu_e$. Substituting this expression into the dispersion equation (1), we obtain two equations:

$$k^2 - \frac{\omega^2}{c^2}\left(1 - \frac{\omega_{Li}^2}{\omega(\omega + i\nu_i)} + i\frac{\omega_{Le}^2(\omega - \mathbf{kv}_0)}{\omega^2\nu_e}\right) = 0, \quad (23)$$

$$\left[k^2 - \frac{\omega^2}{c^2}\left(1 - \frac{\omega_{Li}^2}{\omega(\omega + i\nu_i)} + i\frac{\omega_{Le}^2(\omega - \mathbf{kv}_0)}{\omega^2\nu_e}\right)\right] \times \left(1 - \frac{\omega_{Li}^2}{\omega(\omega + i\nu_i)} + i\frac{\omega_{Le}^2}{\nu_e(\omega - \mathbf{kv}_0)}\right) + i\frac{k_\perp^2 v_0^2}{c^2}\frac{\omega_{Le}^2\omega_{Li}^2}{\omega\nu_e(\omega + i\nu_i)(\omega - \mathbf{kv}_0)} = 0,$$

the first of which describes purely transverse plasma oscillations, which attenuate in time. The second equation, on the other hand, describing the oscillations of the electric field in the $(\mathbf{E}_0, \mathbf{k})$ plane, admits also of unstable solutions. Thus, in the frequency region $\nu_i \ll \omega \ll \mathbf{k}\cdot\mathbf{v}_0$ we get from this equation

$$\omega^2 = \omega_{Li}^2\left\{k^2c^2 + \omega_{Li}^2 + i\frac{\omega_{Le}^2}{\nu_e\mathbf{kv}_0}[(\mathbf{kv}_0)^2 + k_\perp^2 v_0^2]\right\} \times \left(1 - i\frac{\omega_{Le}^2}{\nu_e\mathbf{kv}_0}\right)^{-1}\left(k^2c^2 + \omega_{Li}^2 + i\frac{\omega_{Le}^2\mathbf{kv}}{\nu_e}\right)^{-1}. \quad (24)$$

For waves propagating at an acute angle to the field $\mathbf{E}_0$, i.e., when $k_\parallel^2 \gg k_\perp^2$, this expression simplifies to

$$\omega^2 = \frac{\omega_{Li}^2}{1 + \omega_{Le}^4/\nu_e^2(\mathbf{kv}_0)^2}\left(1 + i\frac{\omega_{Le}^2}{\nu_e\mathbf{kv}_0}\right). \quad (25)$$

We see therefore that the oscillations in question always increase with increasing time, and in the long-wave region, $\omega_{Le}^2 \gg \nu_e\mathbf{k}\cdot\mathbf{v}_0$, the instability of the plasma is aperiodic:

$$\omega = \pm(1 + i)\left(\frac{m\nu_e\mathbf{kv}_0}{2M}\right)^{1/2}. \quad (26)$$

On the other hand, in the region of short waves, $\omega_{Le}^2 \ll \nu_e\mathbf{k}\cdot\mathbf{v}_0$, periodic oscillations build up with a small growth increment:

$$\omega = \pm\,\omega_{Li}, \qquad \gamma = \frac{\omega_{Le}^2}{2\nu_e\,\mathbf{kv}_0}\,\omega_{Li}. \quad (27)$$

The maximum increment corresponds to oscillations with wavelength $\nu_e\mathbf{k}\cdot\mathbf{v}_0 = \omega_{Le}^2$, and is equal to

$$\gamma_{max} = {}^1/_2\omega_{Li}(1 + \sqrt{2})^{1/2} \approx 0.8\omega_{Li}.$$

The buildup of these oscillations is due to the change of the sign of the imaginary part of the longitudinal dielectric constant of the plasma in the region of the anomalous Doppler-effect frequencies, $\omega < \mathbf{k}\cdot\mathbf{v}_0$, owing to the electron drift (when $k_\parallel^2 \gg k_\perp^2$ these oscillations become almost longitudinal).

Finally, starting from the conditions for the applicability of the formulas derived above, let us indicate when the buildup of such oscillations is possible in a plasma. From the conditions

$$\nu_i,\ k\nu_{Ti} \ll \omega \ll \nu_e,\ \mathbf{kv}_0$$

it follows that the oscillations in question are possible in a plasma in which $\omega_{Li} \gg \nu_i$ and $\nu_e \gg [M/m]^{1/2}\nu_i$, i.e., $T_e \gg T \approx T_i$, with the buildup of the oscillations occurring only in fields $\mathbf{E}_0$ in which $v_0 = eE_0/m\nu \gg v_{Ti}$. This condition, in turn, ensures heating of the electrons and the required non-isothermal behavior of the plasma [see formula (9)].

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[4]We note that such a neglect is valid for a weakly ionized plasma when $m\nu_e(T_e) \ll M\nu_i(T)$, i.e., $T_e/T \ll M/m$.

Let us investigate now the solution of the second equation of (23) in the frequency region

$$kc \gg \omega \gg \mathbf{k}\mathbf{v}_0, \nu_i$$

(the waves are almost transverse to the field $\mathbf{E}_0$). In this frequency region, Eq. (23) reduces to the cubic equation

$$A\omega^3 + B\omega^2 + C\omega + D = 0,$$
$$A = \nu_e(k^2c^2 + \omega_{Li}^2) + \omega_{Le}^4 / \nu_e,$$
$$B = i\omega_{Le}^2(k^2c^2 + 2\omega_{Li}^2), \qquad (28)$$
$$C = -\nu_e\omega_{Li}^2(k^2c^2 + \omega_{Li}^2),$$
$$D = ik_\perp^2 v_0^2\omega_{Le}^2\omega_{Li}^2. \qquad (29)$$

In the absence of an external electric field, i.e., when $v_0 = 0$, $D = 0$, and Eq. (28), as expected, has two damped solutions (since $C < 0$ and B is pure imaginary):

$$2A\omega_{1,2} = -B \pm (B^2 - 4AC)^{1/2} \qquad (30)$$

and one zero solution $\omega_3 = 0$. A small nonzero drift velocity $\mathbf{v}_0$ (i.e., a small field $\mathbf{E}_0$) has little effect on the large roots (30), but exerts a very strong influence on the zero root ω_3. Namely, the small root of (29) becomes equal to

$$\omega_3 = -\frac{D}{C} = i\frac{k_\perp^2 v_0^2\omega_{Le}^2}{\nu_e(k^2c^2 + \omega_{Li}^2)} \leqslant i\frac{v_0^2}{c^2}\frac{\omega_{Le}^2}{\nu_e}. \quad (31)$$

It is easy to see that this root corresponds to unstable aperiodically increasing plasma oscillations, with ω_3 representing the growth increment of the oscillations. From the conditions of applicability of formula (31) (we must take into account here also the inequalities $AD^2 \ll C^3$ and $BD \ll C^2$) it can be shown that such oscillations, like the ones considered above, are possible only in a non-isothermal plasma, in which $T_e \gg T$ but $\omega_{Li} \gg \nu_i$. It might seem that the instability takes place at arbitrarily small drift velocities $\mathbf{v}_0$, i.e., for arbitrarily small fields $\mathbf{E}_0$. Actually, however, the condition for non-isothermy of the plasma, in accordance with formula (9), imposes a limitation on $\mathbf{E}_0$, namely, for the plasma to build up it is necessary that $v_0 = eE_0/m\nu_e \gg v_{Ti}$. From a comparison of (26) and (31) we see that the growth increment of the transverse plasma oscillations in question is considerably smaller than the growth increment of the longitudinal oscillations. Therefore, under real conditions, such oscillations may likewise not be observed. The point is that before they have a chance to grow appreciably, the unstable longitudinal oscillations can greatly change the properties of the plasma.

Such an instability is analogous in its nature to the instability of a collisionless plasma with current at frequencies in the region of the normal Doppler effect [8] (the signs of the electronic conductivity tensor components do not change here), and is connected with the anisotropy of the plasma.

To conclude this section, let us consider the oscillations of a weakly ionized electron-ion plasma in the region of the lowest frequencies, when $\omega \ll \nu_i$. We note that in this frequency region the hydrodynamic expression for the ionic part of the dielectric constant is, generally speaking, no longer valid. Qualitatively, however, this expression reflects correctly the character of the oscillations of a weakly ionized electron-ion plasma even when $\omega \ll \nu_i$. We shall therefore use formula (22) in this frequency region, too. We note immediately that an investigation of such oscillations of an electron-ion plasma in an external electric field makes it possible to establish an analogy between such oscillations and low-frequency oscillations of an electron-hole solid state plasma, the analysis of which is valid also quantitatively.

The first expression of (23) (together with the second equation in the frequency region $\omega \ll \mathbf{k}\cdot\mathbf{v}_0$) has for $\omega \ll \nu_i$ only solutions that correspond to damped plasma oscillations. On the other hand, in the frequency region $\nu_i \gg \omega \gg \mathbf{k}\cdot\mathbf{v}_0$ the second equation of (23) reduces to the quadratic equation

$$\omega^2\left(k^2c^2 + \frac{\omega_{Le}^4}{\nu_e^2}\right) + i\omega k^2c^2\frac{\omega_{Le}^2}{\nu_e} + k_\perp^2 v_0^2\frac{\omega_{Le}^2\omega_{Li}^2}{\nu_e\nu_i} = 0. \quad (32)$$

One of the two roots $\omega_{1,2}$ of this equation, as can be readily seen, corresponds to plasma oscillations that increase in time. This instability, like the preceding one, is due to the anisotropy of the plasma in an external electric field. Indeed, when $\mathbf{v}_0 = 0$ we have

$$\omega_1 = -ik^2c^2\omega_{Le}^2\nu_e / (k^2c^2\nu_e^2 + \omega_{Le}^4), \qquad \omega_2 = 0, \quad (33)$$

i.e., the plasma oscillations attenuate in time.

A nonzero but small drift velocity $\mathbf{v}_0 \neq 0$ has little effect on the first large root ω_1, but greatly disturbs the second:

$$\omega_2 \approx i\frac{k_\perp^2 v_0^2}{k^2c^2}\frac{\omega_{Li}^2}{\nu_i} \lesssim i\frac{v_0^2}{c^2}\frac{\omega_{Li}^2}{\nu_i}. \qquad (34)$$

We note that the conditions for the applicability of formulas (32)—(34) impose in practice no lower limit on the drift velocity $\mathbf{v}_0$ at which the instability in question arises. The limitation imposed by the condition (14):

$$\frac{v_0}{c} > \frac{kv_{Te}}{\omega_{Li}}\sqrt{\frac{\nu_e}{\nu_i}},$$

is readily satisfied in a dense plasma at essentially arbitrarily small velocities $\mathbf{v}_0$. This denotes in turn that there is no need to heat the plasma electrons for such low frequency oscillations to build up, and the oscillations can build up also in an isothermal plasma, i.e., when $v_0 < v_{Ti}$. Thus, a weakly ionized dense electron-ion plasma is unstable in the absence of an external magnetic field at practically arbitrarily small fields E_0.

4. LOW FREQUENCY OSCILLATIONS OF AN ELECTRON-HOLE SOLID-STATE PLASMA IN AN EXTERNAL ELECTRIC FIELD

In an electron-hole solid-state plasma, unlike an electron-ion plasma, the mass of the neutral particles (lattice) is much greater than the mass of either the negative or the positive carriers. This causes all the relations derived in Sec. 2 to be valid for carriers of both signs. It follows hence, in particular, that both negative and positive carriers are heated in an electron-hole plasma situated in an external electric field, and that according to (9)

$$T_\mp \approx T + 2e_\mp^2 E_0^2 / 3m_\mp \delta_\mp (\nu_\mp^2 + \Omega_\mp^2), \qquad (35)$$

where T is the lattice temperature, $\delta_\pm = 2m_\pm/M$ is the ratio of the carrier mass to the lattice mass, and $\nu_\pm$ are the effective carrier collision frequencies[5)]. For simplicity we shall neglect their velocity dependence, assuming that $\nu_\pm = \nu_\pm(T_\pm) = \text{const}$.

We note that in real crystals $\nu_\pm \sim 10^{11}$—10^{13}. Therefore, up to magnetic fields $B_0 \lesssim 10^4$ Oe (taking into account the fact that in crystals the carrier masses are of the order of 0.1—1 of the mass of a real electron) we can neglect its influence on either the ground state or the perturbed state of the electron-hole plasma in the crystal. From (35) it follows in this case that the ratio of the carrier temperature varies with increasing electric field within the limits

$$1 \leqslant T_- / T_+ \leqslant (m_+ / m_-)^{1/2};$$

On the other hand, the ratio of the directed drift velocities $v_{0\pm} = e_\pm E_0 / m_\pm \nu_\pm$ varies within the limits

$$(m_+ / m_-)^{1/2} \geqslant v_{0-} / v_{0+} \geqslant (m_+ / m_-)^{1/4}.$$

In real crystals $m_+/m_- \sim 10$—30. Therefore the drift velocities of the positive and negative carriers turn out to be of the same order in any electric field. We shall show below that this circumstance limits the spectrum of the possible growing oscillations of an electron-hole plasma in an external field.

Using (16), we can write for the dielectric constant of the electron-ion plasma in a crystal, in the absence of a magnetic field,

$$\varepsilon_{ij} = \varepsilon_{ij}^{(0)} - \sum \frac{\omega_L^2}{\omega(\omega + i\nu)} \times \left\{ \delta_{i\mu} + \frac{i\nu}{\omega + i\nu} \frac{v_{0i} k_\mu}{\omega - i\nu k v_0 / (\omega + i\nu)} \right\} \gamma_{\mu j}. \qquad (36)$$

Here $\epsilon_{ij}^{(0)}$ is the dielectric constant of the crystal lattice itself, and the summation in the second term extends over the positive and negative carriers in the plasma. For simplicity we confine ourselves to a crystal with cubic symmetry, where $\epsilon_{ij}^{(0)} = \epsilon_0 \delta_{ij}$, and we assume that $\epsilon_0 \sim 1$.

In the region of high frequencies $\omega \gg \nu_\pm$, expression (36) reduces to the relation (we recall that $|\omega + i\nu| \gg kv_0$)

$$\varepsilon_{ij} = \left\{ \varepsilon_0 - \sum \frac{\omega_L^2}{\omega^2} \left(1 - i\frac{\nu}{\omega} \right) \right\} \delta_{ij}, \qquad (37)$$

which does not contain the electric drift at all, and when substituted in the dispersion equation (1) leads only to damped plasma oscillations.

An analogous situation takes place also in the region of intermediate frequencies, when $\nu_- \gg \omega \gg \nu_+$. Indeed, it follows from the latter inequality that $\omega \gg kv_{0+}$, and since $v_{0+} \sim v_{0-}$, we get $\omega \gg \mathbf{k} \cdot \mathbf{v}_{0-}$; taking this into account, we obtain from (37)

$$\varepsilon_{ij} = \left(\varepsilon_0 - \frac{\omega_{L+}^2}{\omega^2} + i\frac{\omega_{L-}^2}{\omega\nu_-} \right) \delta_{ij}. \qquad (38)$$

This expression, like (37), does not contain the electric drift and, naturally, does not lead to any buildup of oscillations.

To the contrary, in the region of low frequencies, in which $\omega \ll \nu_\pm$, it is possible for oscillations to build up in an electron-hole solid-state plasma, as well as in an electron-ion plasma, in analogy with the buildup considered above at the end of the preceding section. Expression (36) is written in this frequency region in the form [cf. (22)]

$$\varepsilon_{ij} = \varepsilon_0 \delta_{ij} + i \sum \frac{\omega_L^2}{\omega^2 \nu} \times \left[(\omega - \mathbf{k}\mathbf{v}_0) \delta_{ij} + k_i v_{0j} + v_{0i} k_j + \frac{k^2 v_{0i} v_{0j}}{\omega - \mathbf{k}\mathbf{v}_0} \right]. \qquad (39)$$

Substituting this expression into the dispersion equation (1) and neglecting terms of order $\omega^2/\epsilon_0 k^2 c^2$, we find that in the region of frequencies of the normal Doppler effect this equation has the following solutions:

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5)We note that actually we are dealing with the scattering of electrons by phonons and the presence of the small parameter $\delta_\pm$ is due to the small momentum transferred in such a scattering.

$$\omega_{1,2} = \frac{1}{2}\left[-ik^2c^2 \sum \frac{\omega_L^2}{\nu} \mp i \left\{ k^4c^4 \left(\sum \frac{\omega_L^2}{\nu} \right)^2 + 4k_\perp^2 v_0^2 \frac{\omega_{L+}^2 \omega_{L-}^2}{\nu_+ \nu_-} \times \left[k^2c^2\varepsilon_0 + \left(\sum \frac{\omega_L^2}{\nu} \right)^2 \right] \right\}^{1/2} \right] \times \left[k^2c^2\varepsilon_0 + \left(\sum \frac{\omega_L^2}{\nu} \right)^2 \right]^{-1}, \quad (40)$$

where $\mathbf{v}_0 = \mathbf{v}_{0-} - \mathbf{v}_{0+}$ is the relative velocity of the electric carrier drift. Exactly as in the case of an electron-ion plasma, one of the roots, ω_2, tends to zero in the absence of an electric field, while the second root, ω_1, corresponds to damped plasma oscillations.

A nonzero but small carrier drift leads to a strong change in the zero root ω_2, without essentially influencing the large root ω_1. Namely,

$$\omega_2 \approx i \frac{v_0^2}{c^2} \frac{\omega_{L+}^2 \omega_{L-}^2}{\omega_{L+}^2 \nu_- + \omega_{L-}^2 \nu_+}. \quad (41)$$

Thus, low-frequency ($\omega \ll \nu_\pm$) electromagnetic oscillations build up in an electron-hole solid-state plasma situated in an external electric field at practically arbitrarily small fields $\mathbf{E}_0$. The only limitation imposed by condition (14), $\omega \gg k(v_{T\pm})^2/\nu_\pm$ (where $v_{T\pm}$ are the thermal velocities of the carriers in the plasma), is more likely to limit from below the wavelength of the excited oscillations (or the transverse dimensions of the crystal) rather than the magnitude of the field $\mathbf{E}_0$. The fact that Pustovoĭt [6] has made just the opposite statement, is a consequence of the fact that the result of his investigations, as noted above, is valid only for longitudinal field oscillations.

For longitudinal field oscillations, the dispersion equation, taking (39) into account, is written in the form

$$\frac{k_i k_j}{k^2} \varepsilon_{ij} = \varepsilon_0 + i \sum \frac{\omega_L^2}{\nu(\omega - \mathbf{k}\mathbf{v}_0)} = 0. \quad (42)$$

It is easy to show that this equation has only solutions that correspond to damped longitudinal plasma oscillations. This result indicates, in particular, that growing oscillations, described by relation (41), are not longitudinal.

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[1] Hutson, McFee, and White, Phys. Rev. Letters 7, 237 (1961). Z. Esaki, ibid. 8, 4 (1962).

[2] Konstantinov, Grinberg, Kostal'skiĭ, and Ryvkin, DAN SSSR 159, 49 (1964), Soviet Phys. Doklady 9, 969 (1965).

[3] R. F. Kazarinov and V. G. Skobov, JETP 42, 910 (1962) and 43, 1496 (1962), Soviet Phys. JETP 15, 628 (1962), and 16, 1057 (1963).

[4] A. A. Vedenov and E. P. Velikhov, JETP 43, 1110 (1962), Soviet Phys. JETP 16, 784 (1963).

[5] M. E. Gertsenshteĭn and V. I. Pustovoĭt, JETP 43, 536 (1962), Soviet Phys. JETP 16, 383 (1963); PTÉ No. 7, 1009 (1962).

[6] V. I. Pustovoĭt, JETP 43, 2281 (1962), Soviet Phys. JETP 16, 1612 (1963); FTT 5, 2490 (1963), Soviet Phys. Solid State 5, 1818 (1964); Dissertation, Physics Institute, Academy of Sciences, 1964.

[7] E. Jackson, Physics of Fluids 3, 786 (1960). Bodgankevich, Rukhadze, and Silin, Izv. Vuzov Radiofizika 5, 1093 (1962). A. A. Rukhadze, Dissertation, Physics Institute Academy of Sciences, 1964.

[8] A. A. Rukhadze, Izv. Vuzov Radiofizika 6, 401 (1963). V. G. Makhan'kov and A. A. Rukhadze, Yadernyĭ sintez (Nuclear Fusion) 2, 177 (1962). N. V. Sholokhov, Izv. Vuzov Radiofizika 7, 452 (1964).

[9] D. Pines and J. R. Schrieffer, Phys. Rev. 124, 1387 (1961).

[10] A. I. Akhiezer and A. G. Sitenko, JETP 30, 216 (1956) Soviet Phys. JETP 3, 140 (1956).

[11] K. N. Stepanov and V. S. Tkalich, ZhTF 28, 1789 (1958), Soviet Phys. Tech. Phys. 3, 1649 (1959).

[12] V. A. Liperovskiĭ, JETP 39, 1363 (1960), Soviet Phys. JETP 12, 951 (1961).

[13] B. I. Davydov, JETP 7, 1069 (1937).

[14] V. L. Ginzburg and A. V. Gurevich, UFN 70, 201 and 393 (1960). Soviet Phys. Uspekhi 3, 115 (1960). V. L. Ginzburg, Rasprostranenie elektromagnitnykh voln v plazme (Propagation of Electromagnetic Waves in a Plasma), Fizmatgiz, 1960.

[15] O. Buneman, Phys. Rev. 115, 503 (1959). L. M. Kovrizhnykh and A. A. Rukhadze, JETP 38, 850 (1960), Soviet Phys. JETP 11, 615 (1960). L. M. Kovrizhnykh, ZhTF 31, 888 (1961), Soviet Phys. Techn. Phys. 6, 643 (1961). E. E. Lovetskiĭ and A. A. Rukhadze, JETP 48, 514 (1965), Soviet Phys. JETP 21, 526 (1965).

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