CONSERVATION OF VECTOR CURRENT AND THE $\nu + N \to \mu + N + \pi$ PROCESS

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The relation between the $\nu + N \to \mu + N + \pi$ process and the electroproduction of π mesons is established phenomenologically on the basis of the hypothesis of the conservation of vector current. Numerical values are obtained by employing the experimental data on the electroproduction of π mesons.

ONE of the interesting results of the experiment performed at CERN [1] on the interaction of high energy neutrinos with matter is the approximate equality of the cross sections for the "elastic" process

$$\nu + N \to \mu + N \tag{1}$$

and the ineleastic process for the production of a single π meson

$$\nu + N \to \mu + N + \pi. \tag{2}$$

Theoretical predictions [2] with respect to process (1) based on the hypothesis of conserved vector current [3] have been confirmed in this experiment. With respect to process (2) there exist estimates for cross sections in the domain of small transferred momenta [4] which indicate that this domain of transferred momenta gives no essential contribution to the observed cross section.

In the work of Bell and Berman [5] the total cross section for the process (2) was obtained on the basis of a static model with the ($3/2$, $3/2$) resonance in the πN interaction. A more exact calculation taking into account the recoil of the nucleon and on the assumption that the process (2) proceeds through the intermediate ($3/2$, $3/2$) isobar is contained in the paper by Berman and Veltman [6]. In this paper we obtain an estimate of the cross section of process (2) based on a phenomenological approach utilizing the hypothesis of conserved vector current and the experimental data on the electroproduction of π-mesons.

1. PHENOMENOLOGICAL DISCUSSION OF THE PROCESS OF ELECTROPRODUCTION OF π MESONS

The matrix element for the process

$$e + N \to e + N + \pi \tag{3}$$

is represented in the form

$$M = ieJ_\mu A_\mu (2\pi)^4 \delta^4 (p_1 + s_1 - p_2 - s_2 - q), \tag{4}$$

where

$$A_\mu = \frac{ie\bar{u}(s_2)\gamma_\mu u(s_1)}{(s_1 - s_2)^2}, \qquad J_\mu = i\langle p_2, q | I_\mu | p_1\rangle. \tag{5}$$

Here u are spinors, s_1 and s_2 are the four-momenta of the electron before and after scattering and J_μ is the current of strongly interacting particles.

The requirement of relativistic and gauge invariance leads to the following expression for the matrix element J_μ [7]:

$$J_\mu = \frac{1}{\sqrt{2E_q}} \bar{u}(p_2) \{\gamma_5 \alpha_\mu f_1 + \gamma_5 \beta_\mu f_2 + \hat{N}\alpha_\mu f_3 + \hat{N}\beta_\mu f_4 + N_\mu f_5 + \gamma_5 \hat{N} N_\mu f_6\} u(p_1). \tag{6}$$

In this expression p_1 and p_2 are the four-momenta of the initial and final nucleons, E_q is the energy of the π meson, while the vectors α, β, and N are defined in the following manner. We introduce the notation

$$k = s_1 - s_2, \quad \lambda^2 = -k^2,$$

$$\Delta_\mu = \frac{(p_1 - p_2)_\mu}{2}, \qquad P_\mu = \frac{(p_1 + p_2)_\mu}{2}$$

$$S_\mu = \frac{2(ks_2)}{\lambda^2} s_{1\mu} - \frac{2(ks_1)}{\lambda^2} s_{2\mu}.$$

Then the vectors

$$N_\mu = \varepsilon_{\mu\nu\rho\sigma} P_\nu k_\rho \Delta_\sigma, \qquad \alpha_\mu = S_\mu - \frac{(NS')}{N^2} N_\mu,$$
$$\beta_\mu = -\frac{1}{\lambda^2}[\lambda^2 (S\Delta) + (\Delta k)(Sk)] P_\mu + \frac{1}{\lambda^2}[(\alpha P)(k\Delta) - (kP)(\alpha\Delta)] k_\mu - (\alpha P)\Delta_\mu \tag{7}$$

and k_μ constitute a complete orthogonal set.

The isotopic dependence of each of the coeffi-

Table I

Operator	1) $p \to p + \pi^0$	2) $n \to n + \pi^0$	3) $p \to n + \pi^+$	4) $n \to p + \pi^-$
$\frac{1}{2}(\tau_3\tau_\alpha + \tau_\alpha\tau_3)$	1	1	0	0
$\frac{1}{2}(\tau_3\tau_\alpha - \tau_\alpha\tau_3)$	0	0	$-\sqrt{2}$	$\sqrt{2}$
τ_α	1	-1	$\sqrt{2}$	$\sqrt{2}$

cients can be represented in the form

$$f = \frac{1}{2}(\tau_3\tau_\alpha + \tau_\alpha\tau_3) f^{(+)} + \frac{1}{2}(\tau_3\tau_\alpha - \tau_\alpha\tau_3) f^{(-)} + \tau_\alpha f^{(0)}, \quad (8)$$

where τ are the Pauli matrices operating in isotopic space, and $f^{(+)}$, $f^{(-)}$, and $f^{(0)}$ are scalar functions of the invariants (p_1k), (p_1p_2) and λ^2. Relation (8) means that the total amplitude M can be represented in the form

$$M = \frac{1}{2}\{\tau_3\tau_\alpha\} M^{(+)} + \frac{1}{2}[\tau_3\tau_\alpha] M^{(-)} + \tau_\alpha M^{(0)}. \quad (8')$$

The matrix elements of the isotopic operators chosen above for the different charge states of the meson-nucleon system are shown in Table I.

The amplitudes $M^{(+)}$, $M^{(-)}$ and $M^{(0)}$ can be expressed in terms of the amplitudes for the transition to the final state with total isotopic spin T equal to $3/2$, $1/2$. Indeed, the amplitudes of the processes 1)-4) (Table I) can be expressed in terms of amplitudes with definite isotopic spin in the following manner [8]:

$$A_1 = 2t_3 + t_1 + s, \quad A_2 = 2t_3 + t_1 - s,$$

$$A_3 = \sqrt{2}(-t_3 + t_1 + s), \quad A_4 = \sqrt{2}(t_3 - t_1 + s), \quad (9)$$

where t_3 and t_1 are the amplitudes for the transition into states T respectively equal to $3/2$ and $1/2$ determined by the isotopically vector part of the current J_μ; s is the amplitude for the transition into the state $T = 1/2$ determined by the isotopically scalar part of the current J_μ.

From a comparison of (9) with the representation (8′) and taking Table I into account we obtain

$$M^{(+)} = 2t_3 + t_1, \; M^{(-)} = t_3 - t_1, \; M^{(0)} = s. \quad (10)$$

2. PHENOMENOLOGICAL DISCUSSION OF PROCESS (2)

The matrix element of the process of production of the π meson in reaction (2) has the form

$$M_\nu = \frac{G}{\sqrt{2}} J_\mu^w j_\mu (2\pi)^4 \delta^4(p_1 + s_1 - p_2 - s_2 - q), \quad (11)$$

where

$$j_\mu = \bar{u}(s_2)\gamma_\mu(1 + \gamma_5) u(s_1), \quad J_\mu^w = i\langle p_2, q | I_\mu^w | p_1 \rangle. \quad (12)$$

The current J_μ^W can be represented [in analogy with (6)] in the form

$$J_\mu^w = \frac{2}{\sqrt{2E_q}} \bar{u}(p_2)\{\gamma_5\alpha_\mu(f_1' + \gamma_5 g_1) + \gamma_5\beta_\mu(f_2' + \gamma_5 g_2) + \hat{N}\alpha_\mu(f_3' + g_3\gamma_5) + \hat{N}\beta_\mu(f_4' + g_4\gamma_5) + N_\mu(f_5' + g_5\gamma_5) + \gamma_5\hat{N}N_\mu(f_6' + g_6\gamma_5) + \gamma_5 k_\mu(f_7' + g_7\gamma_5) + \hat{N}k_\mu(f_8' + g_8\gamma_5)\} u(p_1). \quad (13)$$

The isotopic structure of the coefficient f_i' has the form

$$f' = \frac{1}{2}(\tau_+\tau_\alpha + \tau_\alpha\tau_+) f'^{(+)} + \frac{1}{2}(\tau_+\tau_\alpha - \tau_\alpha\tau_+) f'^{(-)}, \quad (14)$$

the structure of g_i is analogous.

The coefficients $f'^{(\pm)}$ and $g^{(\pm)}$ are, as before, scalar functions of the invariants (p_1k), (p_1p_2) and λ^2, where $f'^{(\pm)}$ in accordance with the hypothesis of conserved vector current are simply related to the coefficients describing the electroproduction of π mesons. Specifically,

$$f_i'^{(\pm)} = f_i^{(\pm)} \quad (i = 1, 2, \ldots, 6), \qquad f_7' = f_8' = 0. \quad (15)$$

The factor two in formula (13) is necessary for the following reasons. The Lagrangians of the electromagnetic and the weak interactions can be represented in the form

$$\mathcal{L}_{em} = \frac{e}{2}\bar{\psi}(1 + \tau_3)\psi, \quad \mathcal{L}_w = \frac{G}{\sqrt{2}}\bar{\psi}\tau_+\psi.$$

According to the hypothesis of conserved vector current the initial interactions $\bar{\psi}\tau_3\psi$ and $\bar{\psi}\tau_+\psi$ are renormalized in the same manner as a result of strong interaction. If in addition to that a π meson is emitted, then $\bar{\psi}\tau_3\psi$ goes over into

$$\bar{\psi}(a\tau_3\tau_\alpha + b\tau_\alpha\tau_3)\psi\varphi_\alpha,$$

while the interaction $\bar{\psi}\tau_+\psi$ goes over into

$$\bar{\psi}(a\tau_+\tau_\alpha + b\tau_\alpha\tau_+)\psi\varphi_\alpha.$$

Therefore, if the isotopic representation of the coefficient f′ is chosen in the form (14), then the relation (15) corresponds to the replacement

$$e \to 2G/\sqrt{2}.$$

The matrix elements of the operators $\frac{1}{2}\{\tau_+\tau_\alpha\}$ and $\frac{1}{2}[\tau_+\tau_\alpha]$ are shown in Table II. The expressions for the amplitudes of processes I-III (Table II) in terms of the amplitudes of the states of the meson-nucleon system with total

Table II

Operator	I) $\nu+n\to\mu^-+p+\pi^0$	II) $\nu+n\to\mu^-+n+\pi^+$	III) $\nu+p\to\mu^-+p+\pi^+$
$\frac{1}{2}(\tau_+\tau_\alpha+\tau_\alpha\tau_+)$	0	$1/\sqrt{2}$	$1/\sqrt{2}$
$\frac{1}{2}(\tau_+\tau_\alpha-\tau_\alpha\tau_+)$	−1	$-1/\sqrt{2}$	$1/\sqrt{2}$

isotopic spin T equal to $\frac{3}{2}$ and $\frac{1}{2}$ is given by the formulas

$$A_{\rm I}=(t_3'-t_1'),\qquad A_{\rm II}=\frac{1}{\sqrt{2}}(t_3'+2t_1'),\qquad A_{\rm III}=\frac{3}{\sqrt{2}}t_3'. \tag{16}$$

In the case of the vector interaction, as shall be seen later, the amplitudes t_3' and t_1' are obtained from the amplitudes t_3 and t_1 by multiplying them by $\sqrt{8}G/e^2$. Relations (16) enable us to establish different relations between the cross sections of processes I-III if the amplitudes t_3 and t_1 are known.

Representation (13) enables us to carry out easily the summation over the spins in $|M_\nu|^2$ and to obtain an expression for the total cross section of process (2). Specifically,

$$d\sigma_\nu=\frac{G^2}{(2\pi)^4\cdot 8M^2E_{s_1}{}^2}d\lambda^2\,dw^2[V'(w^2,\lambda^2)+A(w^2,\lambda^2)], \tag{17}$$

where

$$V'(w^2,\lambda^2)=\int\frac{d^3p_2}{E_{p_2}}\frac{d^3q}{E_q}\delta^4(p_2+q-p_1-k)$$
$$\times\{(|f_1'|^2-N^2|f_3'|^2)(p_1p_2-M^2)\alpha^2(\alpha^2-s_1^2-s_2^2-\lambda^2)$$
$$+(|f_2'|^2-N^2|f_4'|^2)(p_1p_2-M^2)\beta^2(-s_1^2-s_2^2-\lambda^2)$$
$$+(|f_5'|^2-N^2|f_6'|^2)(p_1p_2+M^2)[(NS)^2$$
$$-N^2(s_1^2+s_2^2+\lambda^2)]\}, \tag{18a}$$

$$A(w^2,\lambda^2)=\int\frac{d^3p_2}{E_{p_2}}\frac{d^3q}{E_q}\delta^4(p_2+q-p_1-k)$$
$$\times\{(|g_1|^2-N^2|g_3|^2)(p_1p_2+M^2)\alpha^2(\alpha^2-s_1^2-s_2^2-\lambda^2)$$
$$+(|g_2|^2-N^2|g_4|^2)(p_1p_2+M^2)\beta^2(-s_1^2-s_2^2-\lambda^2)$$
$$+(|g_5|^2-N^2|g_6|^2)(p_1p_2-M^2)[(NS)^2$$
$$-N^2(s_1^2+s_2^2+\lambda^2)]+(|g_7|^2-N^2|g_8|^2)\lambda^2$$
$$\times(s_1^2+s_2^2+\lambda^2)(p_1p_2+M^2)\}. \tag{18b}$$

We note that, generally speaking, $|M_\nu|^2$ contains the cross products $f_i^*f_k$ and $f_ig_k^*$, but as a result of integration over the variables p_2 and q they drop out. This circumstance facilitates the comparison of process (2) with the electroproduction of π mesons. The cross section for the latter process is equal to

$$\sigma_{ep}=\frac{e^4}{(2\pi)^4\cdot 64M^2E_{s_1}{}^2}\frac{d\lambda^2\,dw^2}{\lambda^4}V(w^2,\lambda^2,s_2^2=0), \tag{19}$$

where $V(w^2,\lambda^2)$ is expressed by formula (18a) with f_i' replaced by f_i.

Experiments carried out by Hand [9] on the electroproduction of π mesons on protons enable us to compare σ_ν and σ_{ep} in the energy range of the incident particle of the order of 1 GeV. Hand has studied the behavior of the ratio $d\sigma_{ep}/d\Omega dE_{s_2}$ as a function of the variables λ^2 and $K=E_{s_1}-E_{s_2}-\lambda^2/2M$. It can be easily seen that $K=(w^2-M^2)/2M$. If we use the notation

$$X(K,\lambda^2)\equiv\frac{d\sigma_{ep}}{d\Omega\,dE_{s_2}}\frac{1}{E_{s_2}}, \tag{20}$$

$$\varkappa(w^2,\lambda^2)\equiv\frac{V'(w^2,\lambda^2)}{V(w^2,\lambda^2)}, \tag{21}$$

then the part of the cross section of process (2) due to the interaction of the vector current is expressed in the form

$$\sigma_\nu{}^V=\frac{8G^2\langle\varkappa\rangle}{e^4}\frac{\pi}{E_{s_1}}\int dK\,d\lambda^2X(K,\lambda^2)\lambda^4, \tag{22}$$

where $\langle\kappa\rangle$ is the average value of κ over the range of variation of the variables w^2 and λ^2. The coefficient κ takes into account the fact that in the different charge modifications of processes (1) and (2) the matrix elements of the operators $\frac{1}{2}\{\tau_+\tau_\alpha\}$ and $\frac{1}{2}[\tau_+\tau_\alpha]$ appearing in f_i' and consequently in $V'(w^2,\lambda^2)$, differ from the matrix elements of the operators $\frac{1}{2}\{\tau_3\tau_\alpha\}$ and $\frac{1}{2}[\tau_3\tau_\alpha]$ contained in the f_i which determine $V(w^2,\lambda^2)$.

In formula (22) the range of integration is determined by the relation

$$K_{max}=E_{s_1}\frac{\lambda^2}{\lambda^2+m_\mu^2}+\frac{m_\mu^2}{4M^2}-(\lambda^2+m_\mu^2)\frac{M+2E_{s_1}}{4ME_{s_1}}$$

$$K_{min}=m_\pi+\frac{m_\pi^2}{2M}. \tag{23}$$

In order to obtain the value of the integral (22) for the energy of the incident neutrino E = 1 GeV, the data of Hand have to be extrapolated a bit into the domain of larger values of K, but the possible errors involved in this are not great since the dependence X(K) is fairly smooth for all values of λ^2. Below we give the values of the integrand in formula (22) for various values of λ^2:[1)]

[1)]The value of $\int XdK$ is given in units of 10^{-32} cm²/Gev.

λ^2, GeV^2:	0.0776	0.194	0.310	0.465	0.620	0.776
$\int XdK$:	102	20.5	10	3.1	1.04	0.21
$\lambda^4\int XdK$:	0.612	0.77	0.96	0.67	0.4	0.126

As a result of integrating (22) we find that the part of the cross section σ_ν^V due to the vector current at an incident neutrino energy of 1 Gev amounts to

$$\sigma_\nu{}^V = 1.96\cdot 10^{-39}\langle\varkappa\rangle \text{ cm}^2 \qquad (24)$$

The numerical value can be obtained if we determine the value of $\langle\kappa\rangle$.

3. CONCLUSIONS

From relations (9) it follows that the cross section measured by Hand depends both on the isotopically vector and on the isotopically scalar current. Indeed, the cross section for a proton is the sum of cross sections of processes 1) and 3) (Table I):

$$|A_1|^2 + |A_3|^2 = 3(2|t_3|^2 + |t_1+s|^2). \qquad (25)$$

On the other hand, as follows from (16), the cross section for the production of π mesons by a neutrino on neutrons is

$$|A_{\rm I}|^2 + |A_{\rm II}|^2 \sim {}^3/_2(|t_3|^2 + 2|t_1|^2). \qquad (26)$$

The total cross section for the production of π mesons as a result of the interaction of a neutrino with matter containing an equal number of neutrons and of protons is

$$\sigma_\nu{}^{(n+p)} = |A_{\rm I}|^2 + |A_{\rm II}|^2 + |A_{\rm III}|^2 \sim 3(2|t_3|^2 + |t_1|^2). \qquad (27)$$

The coefficient of proportionality in relations (26) and (27) is $8G^2/e^4$. We have already taken it into account in formula (22). Therefore, if we are interested in $\sigma_\nu^{(n+p)}$, then in accordance with (25) and (27) we obtain

$$\varkappa = \frac{2|t_3|^2 + |t_1|^2}{2|t_3|^2 + |t_1+s|^2} \qquad (28)$$

The experimental data on the photoproduction of π mesons [10,11] enable us (with an accuracy $\sim 20\%$) to take the value of $\langle\kappa\rangle$ equal to unity (cf. the Appendix). Then the cross section $\sigma_\nu^{V(n+p)}$ of the neutrino process due to the vector interaction at an energy of 1 GeV turns out to be equal to 1.26×10^{-39} cm^2 or $\sim 1\times 10^{-39}$ per nucleon. The figures obtained above give a lower limit on the value of the cross section, since we do not know the contribution of the axial interaction. From the CERN experiment [1] it follows that the cross section per nucleon at an energy of 1 GeV apparently does not exceed 3×10^{-39} cm^2. If we accept this figure, then the part of the cross section due to the axial interaction exceeds the vector part by a factor two.

Relations (16) provide an interrelation between the different charge modifications of process (2). For example, the ratio of the cross sections for the production of charged π mesons compared to neutral ones in matter with the same number of neutrons and protons is

$$\frac{\sigma(\pi^+)}{\sigma(\pi^0)} = \frac{\langle 5|t_3|^2 + 2\,{\rm Re}(t_3{}^*t_1) + 2|t_1|^2\rangle}{\langle |t_3|^2 - 2\,{\rm Re}(t_3{}^*t_1) + |t_1|^2\rangle} \qquad (29)$$

In the Appendix it is shown that in the effective range of K, λ^2 for the vector part of the interaction we have

$$\int dK(2|t_3|^2 + |T_1|^2) \approx 1.4\int dK(|T_1|^2 - 4\,{\rm Re}(t_3{}^*T_1)).$$

Assuming that the isoscalar part of the amplitude s, appearing in $T_1 = t_1 + s$, is not great compared to the isovector part t_1, and utilizing the preceding relationship at an energy of 1 GeV we obtain

$$\sigma(\pi^+)\,/\,\sigma(\pi^0) \approx 2.5.$$

The experimental value of this ratio including all cases of energy up to 9 GeV, is [1]

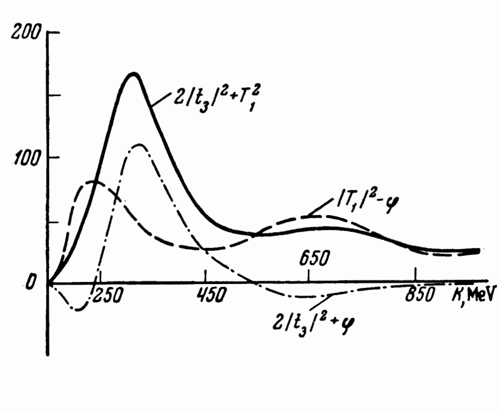
$$(\sigma_{\pi^+}/\sigma_{\pi^0})_{\rm exp} = 1.9 \pm 0.4.$$

This could mean that at large energies the state with $T = 1/2$ plays a dominant role.

The approach developed in the present paper enables us to obtain values of σ_ν^V over the whole energy region for which there exist experimental data on the electroproduction of π mesons. In particular, for the cross section σ_ν^V at an energy E_{s_1} equal to 0.5, 0.75, and 1.0 GeV, we obtain

$$\sigma_\nu{}^V(E = 0.75\text{ GeV}) \approx 0.57\,\sigma_\nu{}^V(E = 1\text{ GeV}),$$

$$\sigma_\nu{}^V(E = 0.5\text{ GeV}) \approx 0.16\sigma_\nu{}^V(E = 1\text{ GeV}).$$



A further extension of the experimentally investigated energy range for the electroproduction of π mesons would give us the energy dependence of the cross section of process (2) in the domain of high energies, and this is very important for comparison with experimental data [1].

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APPENDIX

In order to determine the value of the quantity κ one must know the contributions of the isotopically vector and isotopically scalar amplitudes to the electroproduction of a π meson. The data on the production of π^+ and π^- mesons on deuterium [11] do not contradict the assumption that the isoscalar amplitude makes a contribution to the state of isotopic spin T = ½ smaller than the isovector one. However, it is not possible to carry out a rigorous analysis, and, therefore, we estimate the value of κ from other considerations.

We consider the data on the photoproduction of charged and neutral π mesons on protons [10]. The cross sections for processes 1) and 3) (Table I) is expressed in the form

$$\sigma_1 = 4|t_3|^2 + \varphi + |t_1 + s|^2, \quad \sigma_3 = 2|t_3|^2 - \varphi + 2|t_1 + s|^2.$$

The quantity φ represents the interference of states with T equal to 3/2 and ½. We shall show that in the energy range of interest to us φ is negative. We consider the difference

$$^1/_3(2\sigma_1 - \sigma_3) = 2|t_3|^2 + \varphi.$$

From the data on the total cross sections for the photoproduction of π^0 and π^+ mesons on protons [10] it follows that this difference is negative for $K < 240$ MeV and for $K > 550$ MeV (cf. the Figure). Consequently, in these regions φ is negative and in absolute value is greater than twice the value of the square of the modulus of the amplitude with T = 3/2.

In the range $240 \leq K \leq 450$ MeV it is sufficient to take into account only the S- and P-waves in the meson-nucleon system. Then we have

$$t_3 = t_3(^3/_2, 1)e^{i\delta_{33}} + t_3(^1/_2, 1)e^{i\delta_{31}} + t_3(^1/_2, 0)e^{i\delta_3},$$

where $t(j, l)$ are real matrix elements for the transition with total angular momentum j and π-meson angular momentum l; δ are the phases for the scattering of π mesons by a nucleon [12]. Similarly we have

$$t_1 + s \equiv T_1 = T_1(^3/_2, 1)e^{i\delta_{13}} + T_1(^1/_2, 1)e^{i\delta_{11}} + T_1(^1/_2, 0)e^{i\delta_1}.$$

In the energy range under consideration the phases δ_{31}, δ_{13}, and δ_{11} do not exceed 5° [13] and can be neglected. If we take into account the fact that the angular distributions in the process $\langle p\gamma \mid \pi^0_p \rangle$ in the energy range $K < 650$ MeV are well described by the term $1 - (3/5)\cos^2\theta$ [10], it follows that in the given process the transitions $M_1P_{3/2}$ and $E_1D_{3/2}$ [14] are dominant. In the energy range $K \leq 450$ MeV the second transition is small, and therefore from (9) we obtain

$$2t_3(^1/_2, 0) = -T_1(^1/_2, 0), \quad 2t_3(^1/_2, 1) = -T_1(^1/_2, 1).$$

These relations immediately lead to the following expression for the contribution of φ to the total cross section:

$$\varphi = 4t_3(^3/_2, 1)T_1(^3/_2, 1)\cos\delta_{33} - 2[T_1^2(^1/_2, 1) + T_1^2(^1/_2, 0)\cos(\delta_3 - \delta_1)].$$

Near the resonance the phase δ_{33} goes through $\pi/2$ and

$$\varphi_{res} = -2[T_1^2(^1/_2, 1) + T_1^2(^1/_2, 0)\cos(\delta_3 - \delta_1)],$$

i.e., φ is negative. It is evident that φ is also negative in the energy range from resonance to K = 550 MeV, since otherwise the total cross sections would exhibit an irregularity in place of a continuous falling off in this region.

From the Figure it can be seen that

$$\int dK(2|t_3|^2 + |T_1|^2) \approx 1.4 \int dK(|T_1|^2 - \varphi),$$

and from this, if one sets φ equal to zero, it follows immediately that

$$2|t_3|^2 / (2|t_3|^2 + |T_1|^2) > 0.28.$$

If we take into account that φ is actually a negative quantity and assume that the contribution of the state T = ½, j = 3/2 is small, we obtain

$$2|t_3|^2 / (2|t_3|^2 + |T_1|^2) > 0.6.$$

Since κ actually contains another additional positive term, one can assume (with an accuracy 20%) a value of κ equal to unity.

We have obtained the value of κ at $\lambda^2 = 0$. The data of Hand [9] indicate a certain diminution in the dominance of the 3/2, 3/2 resonance at λ^2 different from zero. However, as λ^2 increases the range over K decreases, and with a decrease in the range with respect to K the ratio $2|t_3|^2/(2|t_3|^2 + |T_1|^2)$ increases (cf. the Figure). Therefore, for the average value of $\langle \kappa \rangle$ one can also take the value unity with good accuracy.

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