THEORY OF TRANSVERSE DIFFUSION AND STATIC AND HIGH-FREQUENCY CONDUCTIVITY OF A PLASMA IN A STRONG MAGNETIC FIELD

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It is shown that in a completely ionized plasma with an electron temperature T_e which considerably exceeds the ion temperature T_i, the effective collision frequency of the particles is proportional to $T_e^{-1/2}T_i^{-1}$. The effect of Coulomb interaction between the particles on the time they spend in the region of action of the forces is taken into account. Weak nonlinear effects of removal of the particles from the interaction region as a result of electric drift are detected.

INTRODUCTION

THE theory of diffusion and conductivity of a fully ionized laminar plasma in a strong magnetic field was developed in several papers. In the present article we shall deal with a fully ionized non-isothermal plasma situated in such a magnetic field that the Larmor radii of the particles become comparable with or even smaller than the Debye-screening radius. The difference between the results obtained below for the transverse diffusion coefficient and the corresponding results of other papers[1-5] lies in the fact that along with the usual double-logarithmic additions to the Coulomb logarithm there arise in our analysis also additive terms, proportional to the ratio of the difference between the electron and ion temperatures to the ion temperature. Therefore, under the conditions when the electron temperature (T_e) is one order of magnitude larger than the ion temperature (T_i), the transverse frequency of collisions, which determines the coefficient of transverse diffusion and the transverse conductivity, can turn out to be, as shown below, proportional to $T_e^{-1/2}T_i^{-1}$.

Comparing this dependence with the $T_e^{-3/2}$ law, which follows from the theory of plasma transport phenomena in a weak magnetic field,[6] we can state that under the conditions of interest to us an appreciable increase in the coefficient of transverse diffusion takes place. Moreover, whereas in the isothermal case the order of magnitude of the coefficient of transverse diffusion remained in fact unchanged, owing to the increase in the time of interaction between the magnetized particles, in the non-isothermal case which we are considering ($T_e \gg T_i$), we must speak of both the occurrence of a new qualitative dependence on the ion temperature and of a change in the order of magnitude. The result obtained by us for diffusion and for the static transverse conductivity is in partial agreement with one of the results of Lovetskiĭ[7] for the high-frequency dielectric constant of a non-isothermal plasma in a range of frequencies exceeding the Langmuir frequency of the electrons. We present below also the results for the frequency dependent dielectric constant of a plasma. Compared with[7], first, we have greatly broadened the frequency range and, second, we took into account the influence of the Coulomb interaction of the particles on the time during which the colliding plasma particles interact, and third, we considered weakly nonlinear effects of the outward electric drift of the particles from the interaction region.

1. INITIAL EQUATIONS AND THE GENERALIZED OHM'S LAW

To construct the theory of conductivity and diffusion of plasma, we make use of a kinetic equation that takes into account the influence of strong fields on the motion of particles during the time of the collision. Such a kinetic equation is of the form[8,9]

$$\frac{\partial f_a}{\partial t} + \mathbf{v}_a \frac{\partial f_a}{\partial \mathbf{r}_a} + \frac{e_a}{m_a}\left(\mathbf{E} + \frac{1}{c}[\mathbf{v}_a\mathbf{B}]\right)\frac{\partial f_a}{\partial t}$$
$$= \sum_b \frac{\partial}{\partial p_a{}^i}\int d\mathbf{p}_b d\mathbf{r}_b \frac{\partial U_{ab}(|\mathbf{r}_a - \mathbf{r}_b|)}{\partial r_a{}^i}$$
$$\times \int_{-\infty}^{0} d\tau \frac{\partial U_{ab}(|\mathbf{R}_a - \mathbf{R}_b|)}{\partial r_a{}^j}\left\{\frac{\partial}{\partial P_a{}^j} - \frac{\partial}{\partial P_b{}^j}\right\} f_a$$

$$\times (\mathbf{P}_a, \mathbf{R}_a, t+\tau)\, f_b(\mathbf{P}_b, \mathbf{R}_b, t+\tau), \qquad (1.1)^*$$

$$\mathbf{P}_a \equiv \mathbf{P}_a(t+\tau, \mathbf{p}_a) = \mathbf{B}\frac{(\mathbf{B}\mathbf{p}_a)}{B^2} - \sin\Omega_a\tau\frac{[\mathbf{B}\mathbf{p}_a]}{B} - \cos\Omega_a\tau\frac{[\mathbf{B}[\mathbf{B}\mathbf{p}_a]]}{B^2} + e_a\int_t^{t+\tau} dt'\Big\{\mathbf{B}\frac{(\mathbf{B}\mathbf{E}(t'))}{B^2} - \frac{[\mathbf{B}\mathbf{E}(t')]}{B}\sin\Omega_a(t+\tau-t') - \frac{[\mathbf{B}[\mathbf{B}\mathbf{E}(t')]]}{B^2}\cos\Omega_a(t+\tau-t')\Big\}, \qquad (1.2)$$

$$\mathbf{R}_a \equiv \mathbf{R}_a(t+\tau, t, \mathbf{p}_a, \mathbf{r}_a) = \mathbf{r}_a + \mathbf{B}\frac{(\mathbf{B}\mathbf{v}_a)}{B^2}\tau - \frac{1-\cos\Omega_a\tau}{\Omega_a}\frac{[\mathbf{B}\mathbf{v}_a]}{B} - \frac{\sin\Omega_a\tau}{\Omega_a}\frac{[\mathbf{B}[\mathbf{B}\mathbf{v}_a]]}{B^2} + \frac{e_a}{m_a}\int_t^{t+\tau} dt'\int_t^{t'} dt''\Big\{\mathbf{B}\frac{(\mathbf{B}\mathbf{E}(t''))}{B^2} - \frac{[\mathbf{B}\mathbf{E}(t'')]}{B}\sin\Omega_a(t'-t'') - \frac{[\mathbf{B}[\mathbf{B}\mathbf{E}(t'')]]}{B^2}\cos\Omega_a(t'-t'')\Big\}, \qquad (1.3)$$

where e_a, m_a, $\mathbf{r}_a$, $\mathbf{v}_a$, and $\mathbf{P}_a$ are respectively the charge, mass, coordinate, velocity, and momentum of a particle of species a; $\Omega_a = e_aB/m_ac$—gyroscopic frequency. The distribution function f_a is normalized to N_a, which is the number of particles per unit volume. Finally, U_{ab} is the energy of the Coulomb interaction of a pair of particles. At large distances, the Coulomb interaction is assumed to be screened. At small distances, when perturbation theory breaks down or it becomes necessary to use a quantum-mechanical analysis, the Coulomb interaction, as is well known,[10] is effectively weakened. We shall therefore assume that at small values of the argument $U_{ab}(\mathbf{r})$ vanishes.

In order to obtain from (1.1) the generalized Ohm's law for spatially homogeneous distributions, we shall assume that the particle distribution functions entering into the collision integral of (1.1) are of the form[11]

$$f_a(\mathbf{p}_a, t) = f_{a0}\Big(\mathbf{p}_a - e_a\int_{-\infty}^{t} dt'\Big\{\frac{1}{B^2}\mathbf{B}(\mathbf{B}\mathbf{E}(t')) + \frac{[\mathbf{E}(t')\mathbf{B}]}{B}\sin\Omega_a(t-t') + \frac{[\mathbf{B}[\mathbf{E}(t')\mathbf{B}]]}{B^2}\cos\Omega_a(t-t')\Big\}\Big), \qquad (1.4)$$

where $f_{a0}(\mathbf{P}_a)$ is the Maxwellian distribution. In the high-frequency limit for relatively rare collisions, such an assumption corresponds to the assumption customarily used in the theory that obtains the results in the form of an expansion in powers of the collision frequency.

We shall be principally interested in the conductivity transverse to a strong magnetic field, for which, as is well known, the theory is analogous in many respects to the high-frequency limit. Therefore formula (1.4) will yield exact results if we can assume that the inequality $|\Omega_a^2 - \omega| \gg \nu^2$ is satisfied. Here ω is the frequency of the alternating electric field and ν is the collision frequency, an explicit expression for which will be obtained below.

Let the electric field depend on the time like

$$\mathbf{E}(t) = \sum_r \mathbf{E}_r\cos(\omega t + \delta_r).$$

Then, substituting in the right side of (1.1) functions of the form (1.4), we obtain the generalized Ohm's law (see[11])

$$\frac{d\mathbf{j}_a}{dt} + [\mathbf{\Omega}_a\mathbf{j}_a] - \frac{e^2N_a}{m_a}\mathbf{E}(t) = \mathbf{J}_a = \frac{2}{\pi}\sum_b\frac{N_aN_b(e_ae_b)^2e_a}{\varkappa m_a} \times \int d\mathbf{k}\frac{\mathbf{k}}{k^4}\int d\tau\sin\Big\{(\mathbf{k}, \rho_a - \rho_b)\,2\sin\frac{\omega\tau}{2}\Big\} \times \exp(-X_a - X_b)\frac{\partial}{\partial\tau}\Big(\frac{X_a}{T_a} + \frac{X_b}{T_b}\Big); \qquad (1.5)$$

where

$$X_a = \frac{v_a^2}{2}\Big\{\frac{(\mathbf{k}\mathbf{B})^2}{B^2}\tau^2 + 4\frac{[\mathbf{k}\mathbf{B}]^2}{B^2\Omega_a^2}\sin^2\frac{\Omega_a\tau}{2}\Big\}, \qquad (1.6)$$

$$\rho_a(\tau) = \sum\Big\{\frac{e_a\Omega_a}{m_a(\omega^2-\Omega_a^2)}\frac{[\mathbf{E}_r\mathbf{B}]}{B\omega}\cos\Big(\omega t + \delta_r - \frac{\omega\tau}{2}\Big) - \frac{e_a}{m_a}\Big(\mathbf{B}\frac{(\mathbf{B}\mathbf{E}_r)}{B^2\omega^2} + \frac{[\mathbf{B}[\mathbf{E}_r\mathbf{B}]]}{B^2(\omega^2-\Omega_a^2)}\Big)\sin\Big(\omega t + \delta_r - \frac{\omega\tau}{2}\Big)\Big\}, \qquad (1.7)$$

$v_a = (\kappa T_a/m_a)^{1/2}$ is the thermal velocity of particles of species a, κ is Boltzmann's constant, and T_a is the temperature.

Finally,

$$\tau_{max}(k) = k^{-3/2}|e_ae_b|^{1/2}(m_am_b/m_a + m_b)^{1/2}.$$

This quantity results from allowance for the fact that the Coulomb interaction causes the particles to be displaced from their trajectories.[4,5] Unlike the paper by one of the authors[11], in which the influence of a strong field was considered, we are interested here in the case of weak fields. Namely, we shall assume below that the velocity of the relative motion of the charged particles, resulting from the action of the electric field, is small compared with the thermal velocity of the electrons.

*$[\mathbf{v}_a\mathbf{B}] = \mathbf{v}_a \times \mathbf{B}$.

It is easy to see that

$$J_a = -\sum_b \frac{4N_aN_b}{\varkappa m_a} e_a^3 e_b^2 \int \frac{dk}{k} \int_0^{\tau_{max}} d\tau \int_0^{\pi} \sin\theta \, d\theta \exp(-X_a - X_b)$$

$$\times \frac{\partial}{\partial\tau}\left[\frac{X_a}{T_a} + \frac{X_b}{T_b}\right]\left\{\frac{\mathbf{B}(\mathbf{kB})}{B^2 k}\sin\left(2k\rho_{\|}\cos\theta\sin\frac{\omega\tau}{2}\right)\right.$$

$$\times J_0\left(2k\rho_\perp \sin\theta\sin\frac{\omega\tau}{2}\right) + \frac{[\mathbf{B}[\boldsymbol{\varrho}\mathbf{B}]]}{B^2\rho_\perp}\cos\left(2k\rho_{\|}\cos\theta\sin\frac{\omega\tau}{2}\right)$$

$$\left.\times J_1\left(2k\rho_\perp\sin\theta\sin\frac{\omega\tau}{2}\right)\right\}, \qquad (1.8)$$

where $B\rho_{\|} = \mathbf{B}\cdot\boldsymbol{\rho}$, $\mathbf{B}_\perp = |\mathbf{B}\times\boldsymbol{\rho}|$, $\rho = \rho_a - \rho_b$, and θ is the angle between the direction of the magnetic field and the vector $\mathbf{k}$.

In the approximation linear in the electric field, formula (1.8) takes the form

$$J_a = -\sum \frac{4N_aN_b}{\varkappa m_a} e_a^3 e_b^2 \int dk \int_0^{\tau_{max}} d\tau \int_0^{\pi} \sin\theta\, d\theta \sin\frac{\omega\tau}{2}$$

$$\times \frac{\partial}{\partial\tau}\left[\frac{X_a}{T_a} + \frac{X_b}{T_b}\right]\exp(-X_a - X_b)\left\{2\cos^2\theta\frac{\mathbf{B}(\boldsymbol{\varrho}\mathbf{B})}{B^2}\right.$$

$$\left. + \sin^2\theta\frac{[\mathbf{B}[\boldsymbol{\varrho}\mathbf{B}]]}{B^2}\right\}. \qquad (1.9)$$

The presence in (1.9) of the factor $\sin(\omega\tau/2)$ makes it necessary to carry out the integration with respect to τ for the dissipative part of the expression for $\mathbf{J}_a$, which is the only one in which we shall be interested below, to values which do not exceed $1/\omega$. Then

$$J_{ai} = -\frac{B_i}{B^2}\sum_r (\mathbf{B}\mathbf{E}_r)\sin(\omega t + \delta_r) e_a N_a \sum_{b} \left(\frac{e_a}{m_a} - \frac{e_b}{m_b}\right)$$

$$\times \frac{\nu_{ab}^{\|}(\omega)}{\omega} - e_a N_a \sum_b \nu_{ab}^{\perp}(\omega)\left\{\left(\frac{e_a}{m_a}\frac{\omega}{\omega^2 - \Omega_a^2}\right.\right.$$

$$\left. - \frac{e_b}{m_b}\frac{\omega}{\omega^2 - \Omega_b^2}\right)\frac{B^2\delta_{ij} - B_iB_j}{B^2} \sum_r E_{rj}\sin(\omega t + \delta_r)$$

$$-\left(\frac{e_a}{m_a}\frac{\Omega_a}{\omega^2 - \Omega_a^2} - \frac{e_b}{m_b}\frac{\Omega_b}{\omega^2 - \Omega_b^2}\right)$$

$$\left.\times e_{ijl}\frac{B_l}{B}\sum_r E_{rj}\cos(\omega t + \delta_r)\right\}. \qquad (1.10)$$

Here

$$\left\{\begin{matrix}\nu_{ab}^{\|}(\omega)\\ \nu_{ab}^{\perp}(\omega)\end{matrix}\right\} = \frac{2e_a^2e_b^2}{m_a} N_b \int_{k_{min}}^{k_{max}} dk k^2 \int_0^{\pi} d\theta \sin\theta \left\{\begin{matrix}\cos^2\theta\\ \sin^2\theta\end{matrix}\right\} \int_0^{\min\{\tau_{max}(k),\, 1/\omega\}} d\tau\,\tau$$

$$\times\left\{\frac{1}{m_a}\left[\tau\cos^2\theta + \frac{\sin\Omega_a\tau}{\Omega_a}\sin^2\theta\right]\right.$$

$$\left. + \frac{1}{m_b}\left[\tau\cos^2\theta + \frac{\sin\Omega_b\tau}{\Omega_b}\sin^2\theta\right]\right\}$$

$$\times \exp\left\{-\frac{v_a^2k^2}{2}\left(\tau^2\cos^2\theta + \frac{4}{\Omega_a^2}\sin^2\theta\sin^2\frac{\Omega_a\tau}{2}\right)\right.$$

$$\left. - \frac{v_b^2k^2}{2}\left(\tau^2\cos^2\theta + 4\frac{\sin^2\theta}{\Omega_b^2}\sin^2\frac{\Omega_b\tau}{2}\right)\right\}. \qquad (1.11)$$

In the case of weak fields, in which we are interested, there is still another reason for cutting off the integration with respect to τ on the high side.[12] Namely, the colliding particles can leave the interaction region under the influence of the electric field. Therefore the particle drift in the electric field imposes an upper limit on the time τ during which the colliding charged plasma particles interact. The drift along the magnetic field corresponds to the maximum possible interaction time:

$$\tau_{\|} = [\omega k\rho_{\|}(0)\cos\theta]^{-1}.$$

Similarly, the drift transverse to the magnetic field leads to a maximum time

$$\tau_\perp = [\omega k\rho_\perp(0)\sin\theta]^{-1}.$$

The corresponding cutoff in (1.8) arises automatically because of oscillations of the integrand.

It must be noted that, strictly speaking, the concept of the dielectric tensor which is customarily used in the linear theory can be employed here only if the electric drift is negligible, for only then does the current depend linearly on the field. The dissipative (antihermitian) part of the dielectric tensor of a plasma consisting of electrons and a single species of ions is of the form

$$\delta\varepsilon_{lj}^{(a)}(\omega) = i\left\{\frac{B_lB_j}{B^2}\nu_{ei}^{\|}(\omega) + \nu_{ei}^{\perp}(\omega)\left(\frac{\omega^4}{\Omega_e^2}\frac{\delta_{lj}B^2 - B_lB_j}{B^2}\right.\right.$$

$$\times\left[\left(\frac{\Omega_e}{\omega^2 - \Omega_e^2} - \frac{\Omega_i}{\omega^2 - \Omega_i^2}\right)^2\right.$$

$$\left. + \omega^2\left(\frac{1}{\omega^2 - \Omega_e^2} - \frac{1}{\omega^2 - \Omega_i^2}\right)^2\right]$$

$$- \frac{2i\omega^5}{\Omega_e^2} e_{jlk}\frac{B_k}{B}\left(\left[\frac{1}{\omega^2 - \Omega_e^2} - \frac{1}{\omega^2 - \Omega_i^2}\right]\right.$$

$$\left.\left.\left.\times\left[\frac{\Omega_e}{\omega^2 - \Omega_e^2} - \frac{\Omega_i}{\omega^2 - \Omega_i^2}\right]\right)\right)\right\}\frac{\omega_{Le}^2}{\omega^3} \qquad (1.12)$$

Here $\omega_{Le} = (4\pi N_e e^2/m)^{1/2}$ is the electron Langmuir frequency.

2. DIFFUSION TRANSVERSE TO THE MAGNETIC FIELD AND TRANSVERSE STATIC CONDUCTIVITY

The formula for the transverse static conductivity of the plasma, as follows from (1.12), can be written in the following form:

$$\sigma_\perp = \frac{e^2N_e}{m\Omega_e^2}\nu_{ei}^{\perp}(0), \qquad (2.1)$$

where

$$\nu_{ei}{}^{\perp}(0)=\frac{2N_i e^2 e_i^2}{\varkappa m}\int\limits_{k_{min}}^{k_{max}} dk\int\limits_0^{\pi} d\theta\sin^3\theta\int\limits_0^{\tau_{max}(k)} d\tau\,\tau\exp(-X_e-X_i)$$

$$\times\frac{\partial}{\partial\tau}\left[\frac{X_e}{T_e}+\frac{X_i}{T_i}\right]. \qquad (2.2)$$

According to Einstein's relation,[6] we have for the coefficient of transverse diffusion

$$D=\frac{\varkappa T_e}{m\Omega_e^2}\nu_{ei}{}^{\perp}(0). \qquad (2.3)$$

Therefore it is sufficient to consider in what follows the expression for the transverse collision frequency. In the limit of weak magnetic fields, when the Larmor radii of the particles greatly exceed the Debye screening radius r_D, formula (2.2) gives the well-known expression for the effective collision frequency:

$$\nu_{ei}{}^{\perp}(0)=\nu_{\rm eff}\equiv\frac{4}{3}\,\frac{\sqrt{2\pi}\,e^2e_i^2N_i}{m^2v_e^3}\ln\frac{r_D}{r_{min}}\equiv\nu_0\ln\frac{r_D}{r_{min}}. \qquad (2.4)$$

We assume here that $MT_i \gg mT_e$. We shall assume this inequality to be satisfied throughout.

In a strong magnetic field, when the Debye radius greatly exceeds the Larmor radius of the electrons ρ_e, formula (2.2) can be approximately represented by

$$\nu_{ei}{}^{\perp}(0)=\nu_0\left[\ln\frac{\rho_e}{r_{min}}+\frac{3}{4}\left(L_1+\frac{T_e-T_i}{T_i}L_2\right)\right]. \qquad (2.5)$$

Here

$$L_1=2\int\limits_1^{r_d^{3/2}/\rho_e r_{min}^{1/2}}\frac{d\xi}{\xi}\int\limits_{\rho_e/r_D}^{\rho_e^{1/3}/r_{min}^{1/3}\xi^{2/3}}\frac{d\varkappa}{\varkappa}\Phi(\varkappa\xi)\,e^{-\varkappa^2\psi(\xi)}, \qquad (2.6)$$

$$L_2=2\frac{mT_i}{MT_e}\frac{\Omega_e}{\Omega_i}\int\limits_1^{r_d^{3/2}/\rho_e r_{min}^{1/2}} d\xi\sin\frac{2\Omega_i}{\Omega_e}\xi$$

$$\times\int\limits_{\rho_e/r_D}^{\rho_e^{1/3}/r_{min}^{1/3}\xi^{2/3}} d\varkappa\,\varkappa\Phi(\varkappa\xi)\,e^{-\varkappa^2\psi(\xi)}. \qquad (2.7)$$

Here

$$\Phi(x)=\frac{2}{\sqrt{\pi}}\int\limits_0^x e^{-t^2}dt$$

is the probability integral and

$$\psi(\xi)=\sin^2\xi+\frac{\rho_i^2}{\rho_e^2}\sin^2\frac{\Omega_i\xi}{\Omega_e}. \qquad (2.8)$$

In view of the appearance of large logarithms, we can write, accurate to the principal terms, the following asymptotic expressions for the transverse collision frequency:

$$\nu_{ei}{}^{\perp}=\nu_0\left[\ln\frac{\rho_e}{r_{min}}+\frac{3}{2}\ln\frac{v_e}{v_i}\ln\frac{r_D}{\rho_e}+\frac{3}{2}\,\frac{T_e-T_i}{T_i}\ln\frac{r_D}{\rho_e}\right],$$

$$\rho_i\gg r_D>\rho_e>r_{min}\frac{v_e^2}{v_i^2}; \qquad (2.9)$$

$$\nu_{ei}{}^{\perp}=\nu_0\left[\ln\frac{\rho_e}{r_{min}}+\frac{3}{4}\ln\frac{r_D}{\rho_i}\ln\frac{\sqrt{r_D\rho_i}}{r_{min}}+\frac{3}{2}\,\frac{T_e-T_i}{T_i}\ln\frac{\rho_i}{\rho_e}\right.$$

$$\left.+\frac{3}{2}\ln\frac{v_e}{v_i}\ln\frac{\rho_i}{\rho_e}\right],\qquad r_D>\rho_i>\rho_e>r_{min}\frac{v_e^2}{v_i^2}; \qquad (2.10)$$

$$\nu_{ei}{}^{\perp}=\nu_0\left[\ln\frac{\rho_e}{r_{min}}+\frac{3}{2}\ln\frac{v_e}{v_i}\ln\frac{r_Dv_i^2}{r_{min}v_e^2}\right.$$

$$\left.+\frac{3}{8}\ln\frac{v_e^2r_{min}}{v_i^2\rho_e}\ln\frac{v_e^2\rho_e}{v_i^2r_{min}}+\frac{3}{2}\,\frac{T_e-T_i}{T_i}\ln\frac{r_Dv_i^2}{r_{min}v_e^2}\right],$$

$$\rho_i>r_D>r_{min}\frac{v_e^2}{v_i^2}>\rho_e; \qquad (2.11)$$

$$\nu_{ei}{}^{\perp}=\nu_0\left[\ln\frac{\rho_e}{r_{min}}+\frac{3}{4}\ln\frac{r_D}{\rho_i}\ln\frac{\sqrt{r_D\rho_i}}{r_{min}}+\frac{3}{2}\ln\frac{v_e}{v_i}\ln\frac{v_i^2\rho_i}{v_e^2r_{min}}\right.$$

$$\left.+\frac{3}{8}\ln\frac{v_e^2}{v_i^2}\frac{r_{min}}{\rho_e}\ln\frac{v_e^2\rho_e}{v_i^2\rho_i}+\frac{3}{2}\,\frac{T_e-T_i}{T_i}\ln\frac{\rho_iv_i^2}{r_{min}v_e^2}\right], \qquad (2.12)$$

$$r_d>\rho_i>r_{min}\frac{v_e^2}{v_i^2}>\rho_e;$$

$$\nu_{ei}{}^{\perp}(0)=\nu_0\left[\ln\frac{\rho_e}{r_{min}}+\frac{3}{4}\ln\frac{r_D}{\rho_e}\ln\frac{\sqrt{\rho_er_D}}{r_{min}}\right],$$

$$r_{min}\frac{v_e^2}{v_i^2}>\rho_i,\quad r_D>\rho_e. \qquad (2.13)$$

Here $\rho_i = v_i/\Omega_i$ is the ion Larmor radius.

In the limit of equal temperatures, formulas (2.9)—(2.13) correspond to formulas (3.5)—(3.10) of the paper by Aliev and one of the authors.[5] What is essentially new in our formulas is the occurrence of a term containing $(T_e - T_i)/T_i$. This term can become the largest if $T_e/T_i > 5$—10. Therefore in a strong magnetic field and for a sharply non-isothermal plasma with an electron temperature much larger than the ion temperature, the transverse effective collision frequency is inversely proportional to the ion temperature and to the square root of the electron temperature, and not to the temperature of the electrons raised to the 3/2 power, as in the case of a relatively weak magnetic field [formula (2.4)] or when the electron temperature does not exceed the ion temperature greatly. It must be emphasized that the ratio of the electron and ion temperatures must still not be larger than the ratio of the ion and electron masses, for otherwise the assumption which we made, namely that the Larmor radius of the electrons is smaller than that of the ions, will no longer be valid.

3. DEPENDENCE OF THE TRANSVERSE COLLISION FREQUENCY ON THE ALTERNATING FIELD

As seen from (1.11), the transverse collision frequency of the electrons and the ions depends both on the frequency and on the magnitude of the alternating electric field. In the region of frequencies larger than the Langmuir frequencies of the electrons, it is well known that the effective collision frequency has a logarithmic dependence on ω.[13] In a strong magnetic field, the corresponding dependence on ω was considered earlier.[7,9] The results presented below greatly extend the values of the alternating-field frequencies for which the dependence of $\nu_{ei}^{\perp}$ on ω turns out to be appreciable. In addition, more accurate criteria are obtained for the applicability of the previously known formulas, and new expressions are obtained for the transverse collision frequency if these criteria are violated. Because of a consistent account of the relative drift of the charged particles of the plasma in the magnetic and alternating electric fields, we obtain below a weakly nonlinear (logarithmic and double logarithmic) dependence of the collision frequency on the electric field intensity. Compared with[12], in this respect the difference consists, first, in the disclosure of a qualitative increase in the transverse collision frequency due to the non-isothermal nature of the plasma, and, second, in the fact that the formulas obtained below for the weakly-nonlinear dependence on the field contain information on the dependence of the effective collision frequency on the time, which in fact determines the occurrence of new field harmonics in the plasma.

An appreciable dependence of the collision frequency on the magnetic field arises under conditions when the radius of gyroscopic rotation of the electrons is small compared with the radius of the Debye screening and compared with the ratio of the thermal velocity of the electron to the frequency of the alternating field. Being interested precisely in this case and bearing in mind the appearance of large logarithms, we can write an expression for the transverse collision frequency in the form (2.5), the only difference being that L_1 and L_2 are functions of the alternating-field frequency

$$L_1(\omega)=\frac{\Omega_e}{\omega}\int\limits_{\varkappa_{min}}^{1}\frac{d\varkappa}{\varkappa}\int\limits_{1}^{\xi_{max}(k)}\frac{d\xi}{\xi^2}\sin\left(2\frac{\omega}{\Omega_e}\xi\right)e^{-\varkappa^2\psi(\xi)}\Phi(\varkappa\xi), \tag{3.1}$$

$$L_2(\omega)=\frac{1}{2}\frac{v_i^2}{v_e^2}\frac{\Omega_e^2}{\Omega_i\omega}\int\limits_{\varkappa_{min}}^{\varkappa_{max}}d\varkappa\varkappa\int\limits_{1}^{\xi_{max}(k)}\frac{d\xi}{\xi}\sin\left(2\frac{\omega}{\Omega_e}\xi\right) \tag{3.2}$$

$$\times\sin\left(2\frac{\Omega_i}{\Omega_e}\xi\right)e^{-\varkappa^2\psi(\xi)}\Phi(\varkappa\xi).$$

Here $\Phi(x)$ is the probability integral, and the function $\psi(\xi)$ is determined by expression (2.8).

In (3.1) and (3.2) we introduce the following dimensionless variables:

$$\varkappa=\rho_e k,\quad \xi=\frac{\Omega_e\tau}{2},$$

$$\varkappa_{min}=\frac{\rho_e}{r_D},\qquad \varkappa_{max}=\frac{\rho_e}{r_{min}};$$

$\xi_{max}(\kappa)$ is defined as follows:

$$\xi_{max}(\varkappa)=\min\left[\varkappa_{max}^{1/2}\varkappa^{-3/2},\ \frac{v_e}{\varkappa v_{E\parallel}},\ \frac{v_e}{\varkappa v_{E\perp}}\right]. \tag{3.3}$$

Here the relative electric drift velocity is $v_{E\perp,\parallel} = \omega\rho_{\perp,\parallel}(0)$. The values of $\rho_{\perp,\parallel}(0)$ are determined in accordance with (1.9) for $\tau = 0$.

We first analyze the case when the nonlinear effects of the electric particle drift are insignificant, that is, the values of L_1 and L_2 do not depend on v_E. Here, as already stated in the first section, the usual concept of the dielectric tensor is meaningful. Therefore, bearing in mind formula (1.2) and giving below asymptotic formulas for the functions L_1 and L_2, which are connected with the transverse collision frequency by formula (2.5), we determine by the same token the ω-dependent transverse conductivity of the plasma.

As in Sec. 2, we shall determine L_1 with doubly logarithmic accuracy. The integration ranges with respect to κ and ξ are limited by the conditions

$$\varkappa_{min}<\varkappa<1,\quad 1<\xi<\xi_{max},\quad \varkappa\xi>1,$$
$$\varkappa^2\psi<1,\quad \xi<\Omega_e/\omega. \tag{3.4}$$

The entire region of possible impact parameters is subdivided into several intervals, the number of which coincides with the number of different mechanisms which limit the interaction of the particles in time. Let us list these mechanisms. First, limitation by the period of the external field ω^{-1}[13]; second, departure from the interaction region due to the electric drift; third, the displacement of the particles from their trajectories because of Coulomb interaction, and, finally, fourth, the departure of the free ion from the interaction region. The latter takes place for impact parameters smaller than the Larmor radius of the ion. The number of the doubly logarithmic terms in L_1 coincides with the number of the indicated intervals. In this connection, L_1 can be represented in the form

$$L_1=\frac{1}{2}\sum_k\ln\frac{r_{k+1}}{r_k}\ln\frac{\xi_{k+1}}{r_{k+1}}\frac{\xi_k}{r_k}. \tag{3.5}$$

Here the argument of the first logarithm represents the ratio of the final and initial values of the impact parameters, which limit one of the intervals into which the entire region of impact parameters is subdivided. In each such interval there appears one of the possible causes which limit the interaction of the particles in time. In the numerator of the argument of the second logarith there appears the product of the minimum and maximum interaction time for a given impact-parameter interval. The product of the impact parameters which limit the considered interval enters in the denominator of the argument of the second logarithm.

We present first the values of L_1 which do not depend on ω. This occurs when the period of the oscillations of the external field is much larger than the time during which the interaction of the particles will be limited either by the Coulomb acceleration mechanism, or by the aforementioned free emergence of the unmagnetized ion. In this case we have

$$L_1 = \ln\frac{r_D}{\rho_e}\ln\frac{v_e}{v_i}, \quad r_{min}\frac{v_e^2}{v_i^2} < \rho_e < r_D < \rho_i, \frac{v_i}{\omega}, \quad v_E < v_i; \tag{3.6}$$

$$L_1 = \ln\frac{v_e}{v_i}\left(\frac{r_{min}}{\rho_e}\right)^{1/2}\ln\frac{v_e}{v_i}\left(\frac{\rho_e}{r_{min}}\right)^{1/2} + \ln\frac{r_D}{r_{min}}\frac{v_i^2}{v_e^2}\ln\frac{v_e}{v_i}, \tag{3.7}$$

$$\rho_e < r_{min}\frac{v_e^2}{v_i^2} < r_D < \frac{v_i}{\omega}, \quad \rho_i, v_E < v_i;$$

$$L_1 = \ln\frac{\rho_i}{\rho_e}\ln\frac{v_e}{v_i} + \frac{1}{2}\ln\frac{r_D}{\rho_i}\ln\frac{\sqrt{\rho_i r_D}}{r_{min}}, \tag{3.8}$$

$$r_{min}\frac{v_e^2}{v_i^2} < \rho_e < \rho_i < r_D < r_{min}^{1/3}\left(\frac{v_e}{\omega}\right)^{2/3}, \ r_{min}\frac{v_e^2}{v_E^2};$$

$$L_1 = \ln\frac{v_e}{v_i}\left(\frac{r_{min}}{\rho_e}\right)^{1/2}\ln\frac{v_e}{v_i}\left(\frac{\rho_e}{r_{min}}\right)^{1/2}$$

$$+ \ln\frac{\rho_i v_i^2}{r_{min} v_e^2}\ln\frac{v_e}{v_i} + \frac{1}{2}\ln\frac{r_D}{\rho_i}\ln\frac{\sqrt{r_D\rho_i}}{r_{min}}, \tag{3.9}$$

$$\rho_e < r_{min}\frac{v_e^2}{v_i^2} < \rho_i < r_D < r_{min}^{1/3}\left(\frac{v_e}{\omega}\right)^{2/3}, \ r_{min}\frac{v_e^2}{v_E^2};$$

$$L_1 = \frac{1}{2}\ln\frac{r_D}{\rho_e}\ln\frac{\sqrt{\rho_e r_D}}{r_{min}},$$

$$\rho_e < r_D < r_{min}\frac{v_e^2}{v_i^2}, \ v_e^2\frac{r_{min}}{v_E^2}, \ r_{min}^{1/3}\left(\frac{v_e}{\omega}\right)^{2/3}. \tag{3.10}$$

In formulas (3.6)—(3.10) the speed of the electric drift is defined as follows: $v_E = \max\{v_{E\perp}, v_{E\parallel}\}$.

With the aid of (3.5) we shall analyze the foregoing formulas. In (3.6) the interaction of the particles at arbitrary impact parameters in the region from ρ_e to r_D is limited by the free emergence from the interaction sphere. In (3.7), at distances from ρ_e to $r_{min}v_e^2/v_i^2$ the interaction is limited by the effect of the Coulomb acceleration, and from $r_{min}v_e^2/v_i^2$ to r_D it is limited by the free emergence of the ion. Formulas (3.8)—(3.10) pertain to the case when both species of particles are magnetized. The first term in (3.8) corresponds to the region of impact parameters from the electronic to the ionic Larmor radii. In this region the interaction is limited by the free emergence of the ion. The second term of formula (3.8) and the third formulas of (3.9) constitute the contribution made to L_1 by the interaction of the particles at impact distances larger than the Larmor radius of the ion and smaller than the Debye radius. In this region the time of interaction is limited by the effect of the Coulomb acceleration. In Eq. (3.9) the first term corresponds to the region of impact parameters from ρ_e to $r_{min}v_e^2/v_i^2$, in which the interaction time is limited by the Coulomb acceleration effect, and the second corresponds to the region $r_{min}v_e^2/v_i^2$ to ρ_i, where free emergence of the ion takes place.

The presence in formula (3.10) of only one term is connected with the fact that here, in the entire region of impact parameters from the electronic Larmor radius to the Debye-screening radius, the interaction of the particles is limited by the mechanism of Coulomb acceleration. It must be pointed out that in the case of a non-isothermal plasma formulas (3.6)—(3.9) go over into formulas (3.5)—(3.9) from[5], and are also a generalization of the results obtained by Golant[4] to the case of a non-isothermal plasma.

We present below formulas which are close in their structure to formulas (3.6)—(3.10), but in this case L_1 depends explicitly on the oscillation frequency of the external field. We have:

$$L_1 = \ln\frac{v_i}{\omega\rho_e}\ln\frac{v_e}{v_i} + \frac{1}{2}\ln\frac{r_D\omega}{v_i}\ln\frac{v_e^2}{v_i\omega r_D}, \tag{3.11}$$

$$r_{min}\frac{v_e^2}{v_i^2} < \rho_e < \frac{v_i}{\omega} < r_D < \rho_i, \quad v_E < v_i;$$

$$L_1 = \ln\frac{v_e}{v_i}\left(\frac{r_{min}}{\rho_e}\right)^{1/2}\ln\frac{v_e}{v_i}\left(\frac{\rho_e}{r_{min}}\right)^{1/2} + \ln\frac{v_e}{v_i}\ln\frac{v_i^3}{v_e^2\omega r_{min}}$$

$$+ \frac{1}{2}\ln\frac{r_D\omega}{v_i}\ln\frac{v_e^2}{v_i\omega r_D}, \tag{3.12}$$

$$\rho_e < r_{min}\frac{v_e^2}{v_i^2} < \frac{v_i}{\omega} < r_D, \rho_i, \quad v_E < v_i, \quad r_D < \frac{v_e}{\omega};$$

$$L_1 = \ln\frac{\rho_i}{\rho_e}\ln\frac{v_e}{v_i} + \frac{1}{9}\ln\frac{v_e r_{min}^{1/2}}{\omega\rho_i^{3/2}}\ln\frac{v_e\rho_i^{3/2}}{\omega r_{min}^{5/2}}$$

$$+ \frac{1}{2}\ln\frac{r_D\omega^{2/3}}{r_{min}^{1/3}v_e^{2/3}}\ln\frac{v_e^{4/3}}{\omega^{4/3}r_D r_{min}^{1/3}}, \tag{3.13}$$

$$r_{min}\frac{v_e^2}{v_i^2} < \rho_e < \rho_i < r_{min}^{1/3}\left(\frac{v_e}{\omega}\right)^{2/3} < r_D, \ r_{min}\frac{v_e^2}{v_E^2},$$

$$r_D < v_i/\omega;$$

$$L_1 = \ln\frac{v_e}{v_i}\left(\frac{r_{min}}{\rho_e}\right)^{1/2}\ln\frac{v_e}{v_i}\left(\frac{\rho_e}{r_{min}}\right)^{1/2} + \ln\frac{v_i^2\rho_i}{v_e^2 r_{min}}\ln\frac{v_e}{v_i}$$

$$+\frac{1}{9}\ln\frac{v_e r_{min}^{1/3}}{\omega\rho_i^{3/2}}\ln\frac{v_e\rho_i^{3/2}}{\omega r_{min}^{5/2}} + \frac{1}{2}\ln\frac{r_D\omega^{2/3}}{r_{min}^{1/3}v_e^{2/3}}\ln\frac{v_e^{4/3}}{\omega^{4/3}r_D r_{min}^{1/3}} \quad (3.14)$$

$$\rho_e < r_{min}\frac{v_e^2}{v_i^2} < \rho_i < r_{min}^{1/3}\frac{v_e^{2/3}}{\omega^{2/3}} < r_D, \ r_{min}\frac{v_e^2}{v_E^2}, \ r_D < \frac{v_i}{\omega};$$

$$L_1 = \frac{1}{9}\ln\frac{\Omega_e}{\omega}\left(\frac{r_{min}}{\rho_e}\right)^{1/2}\ln\frac{v_e\rho_e^{3/2}}{\omega r_{min}^{5/2}} + \frac{1}{2}\ln\frac{r_D\omega^{2/3}}{r_{min}^{1/3}v_e^{2/3}}$$

$$\times\ln\frac{v_e^{4/3}}{\omega^{4/3}r_D r_{min}^{1/3}},$$

$$\rho^e < r_{min}^{1/3}\left(\frac{v_e}{\omega}\right)^{2/3} < r_D < r_{min}\left(\frac{v_e}{v_i}\right)^2, \qquad r_{min}\left(\frac{v_e}{v_E}\right)^2,$$

$$r_D < \frac{v_e}{\omega}. \quad (3.15)$$

The transition from formulas (3.6)—(3.10) to the expressions (3.11)—(3.15) can be explained in the following manner. If the maximum time of interaction becomes smaller than the period of the oscillations as the frequency of the oscillations of the external field increases, then the last term in (3.6)—(3.10), corresponding to the impact-parameter region bounded by the Debye radius, goes over into the two last terms of formulas (3.11)—(3.15). Thus, the last term in (3.11) and (3.12) corresponds to the region of impact parameters between the Debye radius and the average distance traversed by the ion during the period of field oscillation. In formulas (3.13)—(3.15) the last term is due to the interaction of the particles in the interval of values of the impact parameters from $r_{min}^{1/3}(v_e/\omega)^{2/3}$ to the radius of the Debye sphere. The last term of formulas (3.11)—(3.15) corresponds to the region of those values of the impact parameters, in which the time of interaction does not exceed the period of oscillation of the external field.

With further increase in frequency, we obtain in place of (3.13)

$$L_1 = \ln\frac{\rho_i}{\rho_e}\ln\frac{v_e}{v_i} + \ln\frac{r_D}{\rho_i}\ln\frac{v_e}{\omega\sqrt{\rho_i r_D}},$$

$$r_{min}\left(\frac{v_e}{v_i}\right)^2 < \rho_e < \rho_i < r_D < \frac{v_e}{\omega},$$

$$\frac{v_i}{\omega} > \rho_i > \frac{v_E}{\omega}, \ r_{min}^{1/3}\left(\frac{v_e}{\omega}\right)^{2/3}; \quad (3.16)$$

$$L_1 = \ln\frac{\rho_i}{\rho_e}\ln\frac{v_e}{v_i} + \frac{1}{2}\ln^2\frac{v_e}{\omega\rho_i},$$

$$r_{min}\frac{v_e^2}{v_i^2} < \rho_e < \rho_i < \frac{v_i}{\omega} < \frac{v_e}{\omega} < r_D,$$

$$\rho_i > \frac{v_E}{\omega}, \ r_{min}^{1/3}\left(\frac{v_e}{\omega}\right)^{2/3}; \quad (3.17)$$

$$L_1 = \ln\frac{\rho_i}{\rho_e}\ln\frac{v_e}{v_i} + \frac{1}{9}\ln\frac{v_e r_{min}^{1/2}}{\omega\rho_i^{3/2}}\ln\frac{v_e\rho_i^{3/2}}{\omega r_{min}^{5/2}} + \frac{1}{18}\ln^2\frac{v_e}{\omega r_{min}},$$

$$r_{min}\frac{v_e^2}{v_i^2} < \rho_e < \rho_i < r_{min}^{1/3}\left(\frac{v_e}{\omega}\right)^{2/3} < \frac{v_i}{\omega} < \frac{v_e}{\omega} < r_D,$$

$$r_{min}\left(\frac{v_e}{v_E}\right)^2 > \frac{v_E}{\omega}. \quad (3.18)$$

The first term in formulas (3.16)—(3.18), as in (3.13), corresponds to the region of impact parameters between the electronic and ionic Larmor radii. In this region the time of interaction is limited by the free emergence of the ion. The second term in (3.16) is due to the interaction of the particles at distances larger than the Larmor radius of the ion and smaller than the radius of the Debye sphere; the time of interaction does not exceed here the period of oscillation of the external field. The second term in (3.17) corresponds to the region between the ionic Larmor radius and the average distance which the electron traverses during one period of the external field. It is precisely this distance which plays here the role of the maximum impact parameter. The third term in (3.18) corresponds to the region of impact parameters from $r_{min}^{1/3}(v_e/\omega)^{2/3}$ to v_e/ω, and the second to the region from the ionic Larmor radius to $r_{min}^{1/3}(v_e/\omega)^{2/3}$.

In analogy with the transition from (3.13) to formulas (3.16)—(3.18), there occurs also a transition from (3.14) to the following formulas:

$$L_1 = \ln\frac{v_e}{v_i}\left(\frac{r_{min}}{\rho_e}\right)^{1/2}\ln\frac{v_e}{v_i}\left(\frac{\rho_e}{r_{min}}\right)^{1/2} + \ln\frac{\rho_i v_i^2}{r_{min}v_e^2}\ln\frac{v_e}{v_i}$$

$$+\ln\frac{r_D}{\rho_i}\ln\frac{v_e}{\omega\sqrt{\rho_i r_D}},$$

$$\rho_e < r_{min}\frac{v_e^2}{v_i^2} < \rho_i < r_D < \frac{v_e}{\omega},$$

$$\frac{v_i}{\omega} > \rho_i > \frac{v_E}{\omega}, \quad r_{min}^{1/3}\left(\frac{v_e}{\omega}\right)^{2/3}; \quad (3.19)$$

$$L_1 = \ln\frac{v_e}{v_i}\left(\frac{r_{min}}{\rho_e}\right)^{1/2}\ln\frac{v_e}{v_i}\left(\frac{\rho_e}{r_{min}}\right)^{1/2} + \ln\frac{\rho_i v_i^2}{r_{min}v_e^2}\ln\frac{v_e}{v_i}$$

$$+\frac{1}{2}\ln^2\frac{v_e}{\omega\rho_i},$$

$$\rho_e < r_{min}\frac{v_e^2}{v_i^2} < \rho_i < \frac{v_i}{\omega} < \frac{v_e}{\omega} < r_D, \quad \rho_i > \frac{v_E}{\omega} r_{min}^{1/3}\left(\frac{v_e}{\omega}\right)^{2/3}; \quad (3.20)$$

$$L_1 = \ln\frac{v_e}{v_i}\left(\frac{r_{min}}{\rho_e}\right)^{1/2}\ln\frac{v_e}{v_i}\left(\frac{\rho_e}{r_{min}}\right)^{1/2} + \ln\frac{\rho_i v_i^2}{r_{min} v_e^2}\ln\frac{v_e}{v_i}$$

$$+\frac{1}{9}\ln\frac{v_e r_{min}^{1/2}}{\omega\rho_i^{3/2}}\ln\frac{v_e\rho_i^{3/2}}{\omega r_{min}^{5/2}} + \frac{1}{2}\ln^2\left(\frac{v_e}{\omega r_{min}}\right)^{1/3},$$

$$\rho_e < r_{min}\frac{v_e^2}{v_i^2} < \rho_i < r_{min}^{1/3}\left(\frac{v_e}{\omega}\right)^{2/3} < \frac{v_i}{\omega} < \frac{v_e}{\omega} < r_D,$$

$$r_{min}^{1/3}\left(\frac{v_e}{\omega}\right)^{2/3} > \frac{v_E}{\omega}. \qquad (3.21)$$

In the case when at all impact parameters the time of interaction is bounded by the period of oscillation of the external field, we have

$$L_1 = \frac{1}{2}\ln^2\frac{\Omega_e}{\omega}, \qquad \rho_e < \frac{v_e}{\omega} < r_D,\ \rho_e\frac{v_e}{v_i},\ \rho_e\frac{v_e}{v_E},\ \rho_e^{3/2} r_{min}^{-1/2}. \qquad (3.22)$$

A contribution is made to (3.22) by the region of impact parameters between the Larmor radius of the electron and a distance on the order of the average path of the electron over the period of the external field. If this path exceeds the radius of the Debye screening, and the time of interaction for all values of r is limited by the quantity ω^{-1} as before [in (3.22)], we have

$$L_1 = \ln\frac{r_D}{\rho_e}\ln\frac{v_e}{\omega\sqrt{r_D\rho_e}}, \qquad \rho_e < r_D < \frac{v_e}{\omega} < \frac{v_e}{v_i}\rho_e,\ \frac{v_e}{v_E}\rho_i,\ \frac{\rho_e^{3/2}}{r_{min}^{1/2}}. \qquad (3.23)$$

In the case when the average path of the electrons over the period of the external field is smaller than the Debye radius, and the average path traversed during the same time by the ion is larger than ρ_e, L_1 can be represented in the form of one term [formula (3.24)], in which account is taken of both the contribution from the region ρ_e to v_i/ω (where free emergence of the ion takes place), and the contribution from the interaction in the region from v_i/ω to v_e/ω:

$$L_1 = \ln\frac{v_e}{v_i}\ln\frac{\sqrt{v_e v_i}}{\omega\rho_e},$$

$$r_{min}\frac{v_e^2}{v_i^2} < \rho_e < \frac{v_i}{\omega} < \frac{v_e}{\omega} < r_D, \qquad v_E < v_i. \qquad (3.24)$$

If the path covered by the electron during the period of field oscillation is smaller than the radius of the Debye sphere, the following values of L_1 are also possible:

$$L_1 = \frac{1}{9}\ln\frac{\Omega_e}{\omega}\left(\frac{r_{min}}{\rho_e}\right)^{1/2}\ln\frac{v_e\rho_e^{3/2}}{\omega r_{min}^{5/2}} + \frac{1}{18}\ln^2\frac{v_e}{\omega r_{min}},$$

$$\rho_e < r_{min}^{1/3}\left(\frac{v_e}{\omega}\right)^{2/3} < r_D,\ r_{min}\frac{v_e^2}{v_i^2},\ r_{min}\frac{v_e^2}{v_E^2}, \qquad r_D > \frac{v_e}{\omega}; \qquad (3.25)$$

$$L_1 = \ln\frac{v_e}{v_i}\left(\frac{r_{min}}{\rho_e}\right)^{1/2}\ln\frac{v_e}{v_i}\left(\frac{\rho_e}{r_{min}}\right)^{1/2} + \ln\frac{v_e}{v_i}\ln\frac{v_i^3}{\omega v_e^2 r_{min}}$$

$$+\frac{1}{2}\ln^2\frac{v_e}{v_i},$$

$$\rho_e < r_{min}\frac{v_e^2}{v_i^2} < \frac{v_i}{\omega} < \frac{v_e}{\omega} < r_D, \qquad v_E > v_i. \qquad (3.26)$$

The second term in (3.25) differs from the second term of (3.15) in the fact that the region of the impact parameters is limited not to the value of the Debye radius, but to the average distance which is traversed by the electron during one period of the external field. Analogously, formula (3.26) differs from formula (3.12) in that in the last term r_D is replaced by the quantity v_e/ω. The first term in (3.25) coincides with the first term in (3.15).

It must be noted that in the case of an isothermal plasma formulas (3.6)—(3.26) go over into the results of the work of one of the authors.[14] The values of L_1 in (3.17), (3.22), and (3.24) coincide, respectively, with the values of the doubly-logarithmic terms in (10b), (10), and (10a) of[7], where the case $\omega > \omega_{Le}$ is considered. However, the limits of applicability of these formulas differ, since, in particular, it was assumed in[7] that the radius of the Debye screening of the non-isothermal plasma coincides with the Debye radius of the isothermal plasma.

Carrying out, with logarithmic accuracy, the integration in formula (3.8), we can readily verify that a nonvanishing result is obtained only when $v_E < v_i$. We ultimately obtain

$$L_2 = \ln\frac{r_D}{\rho_e}, \qquad r_{min}\frac{v_e^2}{v_i^2} < \rho_e < r_D < \rho_i\frac{v_i}{\omega}, \qquad (3.27)$$

$$L_2 = \ln\frac{r_D v_i^2}{r_{min} v_e^2}, \qquad \rho_e < r_{min}\frac{v_e^2}{v_i^2} < r_D < \rho_i,\ \frac{v_i}{\omega}, \qquad (3.28)$$

$$L_2 = \ln\frac{\rho_i}{\rho_e}, \qquad r_{min}\frac{v_e^2}{v_i^2} < \rho_e < \rho_i < \frac{v_i}{\omega},\ r_D, \qquad (3.29)$$

$$L_2 = \ln\frac{\rho_i v_i^2}{r_{min} v_e^2}, \qquad \rho_e < r_{min}\frac{v_e^2}{v_i^2} < \rho_i < \frac{v_i}{\omega},\ r_D, \qquad (3.30)$$

$$L_2 = \ln\frac{v_i}{\omega\rho_e}, \qquad r_{min}\frac{v_e^2}{v_i^2} < \rho_e < \frac{v_i}{\omega} < \rho_i,\ r_D, \qquad (3.31)$$

$$L_2 = \ln\frac{v_i^3}{\omega r_{min} v_e^2}, \qquad \rho_e < r_{min}\frac{v_e^2}{v_i^2} < \frac{v_i}{\omega} < \rho_i,\ r_D. \qquad (3.32)$$

We note that L_2 does not contain logarithmic expressions if

$$\max\left\{\rho_e,\ r_{min}\frac{v_e^2}{v_i^2}\right\} > \min\left\{r_D,\ \rho_i,\ \frac{v_i}{\omega}\right\}.$$

We point out that formula (10a) in[7] contains a logarithmic term proportional to T_e/T_i, but the argument of the logarithm, unlike in formula (3.34), contains $v_e/\omega\rho_e$. The difference from[7] is due also to the limits of applicability. This is connected both with the established new effects in our work, and with the use of the correct expression for the Debye screening radius.

Let us turn to consider those values of L_1, for the determination of which it is essential to take into account the electric drift of the particles. In the case when in the entire interaction region, from the electron Larmor radius to the radius of the Debye sphere, the time of interaction of the particles does not exceed the period of oscillations of the external field, we have

$$L_1 = \ln\frac{r_D}{\rho_e}\ln\frac{v_e}{v_E}, \quad r_{min}\frac{v_e^2}{v_E^2} < \rho_e < r_D < \frac{v_E}{\omega}, \quad v_E > v_i; \tag{3.33}$$

$$L_1 = \ln\frac{v_e}{v_E}\left(\frac{r_{min}}{\rho_e}\right)^{1/2}\ln\frac{v_e}{v_E}\left(\frac{\rho_e}{r_{min}}\right)^{1/2} + \ln\frac{v_e}{v_E}\ln\frac{r_D v_E^2}{r_{min}v_e^2}$$

$$\rho_e < r_{min}\frac{v_e^2}{v_E^2} < r_D < \frac{v_E}{\omega}, \quad v_E > v_i; \tag{3.34}$$

$$L_1 = \ln\frac{\rho_i}{\rho_e}\ln\frac{v_e}{v_i} + \ln\frac{r_D}{\rho_i}\ln\frac{v_e}{v_E},$$

$$r_{min}\frac{v_e^2}{v_i^2} < r_{min}\frac{v_e^2}{v_E^2}, \ \rho_e < \rho_i < r_D < \frac{v_E}{\omega}; \tag{3.35}$$

$$L_1 = \ln\frac{\rho_i}{\rho_e}\ln\frac{v_i}{v_i} + \ln\frac{v_e}{v_E}\left(\frac{r_{min}}{\rho_i}\right)^{1/2}\ln\frac{v_e}{v_E}\left(\frac{\rho_i}{r_{min}}\right)^{1/2}$$

$$+ \ln\frac{r_D v_E^2}{r_{min}v_e^2}\ln\frac{v_e}{v_E},$$

$$r_{min}\frac{v_e^2}{v_i^2} < \rho_e < \rho_i < r_{min}\frac{v_e^2}{v_E^2} < r_D < \frac{v_E}{\omega}; \tag{3.36}$$

$$L_1 = \ln\frac{v_e}{v_i}\left(\frac{r_{min}}{\rho_e}\right)^{1/2}\ln\frac{v_e}{v_i}\left(\frac{\rho_e}{r_{min}}\right)^{1/2} + \ln\frac{\rho_i v_i^2}{r_{min}v_e^2}\ln\frac{v_e}{v_i}$$

$$+ \ln\frac{r_D}{\rho_i}\ln\frac{v_e}{v_E},$$

$$\rho_e < r_{min}\frac{v_e^2}{v_i^2} < r_{min}\frac{v_e^2}{v_E^2} < \rho_i < r_D < \frac{v_E}{\omega}; \tag{3.37}$$

$$L_1 = \ln\frac{v_e}{v_i}\left(\frac{r_{min}}{\rho_e}\right)^{1/2}\ln\frac{v_e}{v_i}\left(\frac{\rho_e}{r_{min}}\right)^{1/2} + \ln\frac{\rho_i v_i^2}{r_{min}v_e^2}\ln\frac{v_e}{v_i}$$

$$+ \ln\frac{v_e}{v_E}\left(\frac{r_{min}}{\rho_i}\right)^{1/2}\ln\frac{v_e}{v_E}\left(\frac{\rho_i}{r_{min}}\right)^{1/2} + \ln\frac{r_D v_E^2}{r_{min}v_e^2}\ln\frac{v_e}{v_E},$$

$$\rho_e < r_{min}\frac{v_e^2}{v_i^2} < \rho_i < r_{min}\frac{v_e^2}{v_E^2} < r_D < \frac{v_E}{\omega}. \tag{3.38}$$

Formula (3.33) differs from (3.6) in that v_i is replaced by v_E, for at any value of the impact parameter the drift terminates the interaction of the particles earlier than the mechanism of free emergence of the ion. The difference between (3.34) and (3.7) is connected with the replacement of v_i by v_E. The time of particle interaction at distances ρ_e to $r_{min}v_e^2/v_E^2$ is limited by the Coulomb acceleration effect, and at distances $r_{min}v_e^2/v_E^2$ to r_D it is limited by the particle drift in the electrical field. Formulas (3.35)—(3.38) correspond to the case when the particles of both species are magnetized. The first terms of (3.35) and (3.36) coincide with the first term of formula (3.8), while the first and second terms in the right side of (3.37) and (3.38) coincide with the first and second terms of (3.9). The corresponding discussion was presented above. The second term in (3.35) and the third term in the right side of (3.37) correspond to the interaction of the particles at impact parameters from the Larmor radius of the ion to the radius of the Debye sphere. The time of interaction for these values of the parameters is limited because of the particle drift. The second term of (3.36) and the third term of (3.38) correspond to the region from ρ_i to $r_{min}v_e^2/v_E^2$, where the time of interaction is limited by the effect of the Coulomb acceleration. Finally, the third term in (3.36) and the fourth in (3.38) pertain to the region $r_{min}v_e^2/v_E^2$ to r_D, where the interaction is limited by the particle drift in the electric field.

In analogy with the transition from (3.6)—(3.10) to the formulas (3.11)—(3.15), a transition is also effected from (3.33)—(3.38) to formulas (3.39). In this transition the last term in (3.33)—(3.38) is replaced by two terms. The first of these terms corresponds to the region of impact parameters smaller than v_E/ω, in which the time of interaction is limited by the same mechanism as in formulas (3.33)—(3.38). The second term corresponds to the region of impact parameters extending from a distance through which the particles separate with drift velocity during one period of the external field to the radius of the Debye screening. At these impact parameters, the time of interaction of the particles is limited by the period of the external field.

As a result we have:

$$L_1 = \ln\frac{v_E}{\omega\rho_e}\ln\frac{v_e}{v_E} + \frac{1}{2}\ln\frac{r_D\omega}{v_E}\ln\frac{v_e^2}{v_E r_D\omega},$$

$$r_{min}\frac{v_e^2}{v_E^2} < \rho_e < \frac{v_E}{\omega} < r_D, \quad v_E > v_i;$$

$$L_1 = \ln\frac{v_e}{v_E}\left(\frac{r_{min}}{\rho_e}\right)^{1/2}\ln\frac{v_e}{v_E}\left(\frac{\rho_e}{r_{min}}\right)^{1/2} + \ln\frac{v_E^3}{\omega v_e^2 r_{min}}\ln\frac{v_e}{v_E}$$

$$+ \frac{1}{2}\ln\frac{r_D\omega}{v_E}\ln\frac{v_e^2}{v_E r_D\omega},$$

$$\rho_e < r_{min}\frac{v_e^2}{v_E^2} < \frac{v_E}{\omega} < r_D, \quad v_E > v_i;$$

$$L_1 = \ln\frac{\rho_i}{\rho_e}\ln\frac{v_e}{v_i} + \ln\frac{v_E}{\omega\rho_i}\ln\frac{v_e}{v_E} + \frac{1}{2}\ln\frac{r_D\omega}{v_E}\ln\frac{v_e^2}{v_E r_D\omega},$$

$$\frac{r_{min}v_e^2}{v_i^2} < \frac{r_{min}v_e^2}{v_E^2}, \quad \rho_e < \rho_i < \frac{v_E}{\omega} < r_D;$$

$$L_1 = \ln\frac{\rho_i}{\rho_e}\ln\frac{v_e}{v_i} + \ln\frac{v_e}{v_E}\left(\frac{r_{min}}{\rho_i}\right)^{1/2}\ln\frac{v_e}{v_E}\left(\frac{\rho_i}{r_{min}}\right)^{1/2} + \ln\frac{v_E^3}{\omega v_e^2 r_{min}}\ln\frac{v_e}{v_E} + \frac{1}{2}\ln\frac{r_D\omega}{v_E}\ln\frac{v_e^2}{v_E r_D\omega},$$

$$r_{min}\frac{v_e^2}{v_i^2} < \rho_e < \rho_i < r_{min}\frac{v_e^2}{v_E^2} < \frac{v_E}{\omega} < r_D;$$

$$L_1 = \ln\frac{v_e}{v_i}\left(\frac{r_{min}}{\rho_e}\right)^{1/2}\ln\frac{v_e}{v_i}\left(\frac{\rho_e}{r_{min}}\right)^{1/2} + \ln\frac{\rho_i v_i^2}{r_{min}v_e^2}\ln\frac{v_e}{v_i} + \ln\frac{v_E}{\omega\rho_i}\ln\frac{v_e}{v_E} + \frac{1}{2}\ln\frac{r_D\omega}{v_E}\ln\frac{v_e^2}{v_E r_D\omega},$$

$$\rho_e < r_{min}\frac{v_e^2}{v_i^2} < r_{min}\frac{v_e^2}{v_E^2} < \rho_i < \frac{v_E}{\omega} < r_D;$$

$$L_1 = \ln\frac{v_e}{v_i}\left(\frac{r_{min}}{\rho_e}\right)^{1/2}\ln\frac{v_e}{v_i}\left(\frac{\rho_e}{r_{min}}\right)^{1/2} + \ln\frac{\rho_i v_i^2}{r_{min}v_e^2}\ln\frac{v_e}{v_i} + \ln\frac{v_e}{v_E}\left(\frac{r_{min}}{\rho_i}\right)^{1/2}\ln\frac{v_e}{v_E}\left(\frac{\rho_i}{r_{min}}\right)^{1/2} + \ln\frac{v_E^3}{\omega v_e^2 r_{min}}\ln\frac{v_e}{v_E} + \frac{1}{2}\ln\frac{r_D\omega}{v_E}\ln\frac{v_e^2}{v_E r_D\omega},$$

$$\rho_e < r_{min}\frac{v_e^2}{v_i^2} < \rho_i < r_{min}\frac{v_e^2}{v_E^2} < \frac{v_E}{\omega} < r_D < \frac{v_e}{\omega}. \quad (3.39)$$

If the average distance traversed by the electrons during the period of the oscillations of the external field is smaller than the radius of the Debye screening, we have

$$L_1 = \ln\frac{v_e}{v_E}\ln\frac{\sqrt{v_E v_e}}{\omega\rho_e},$$

$$r_{min}\frac{v_e^2}{v_E^2} < r_{min}\frac{v_e^2}{v_i^2} < \rho_e < \frac{v_E}{\omega} < \frac{v_e}{\omega} < r_D;$$

$$L_1 = \ln\frac{v_e}{v_E}\left(\frac{r_{min}}{\rho_e}\right)^{1/2}\ln\frac{v_e}{v_E}\left(\frac{\rho_e}{r_{min}}\right)^{1/2} + \ln\frac{v_E^3}{\omega v_e^2 r_{min}}\ln\frac{v_e}{v_E} + \frac{1}{2}\ln^2\frac{v_e}{v_E},$$

$$\rho_e < r_{min}\frac{v_e^2}{v_E^2} < \frac{v_E}{\omega} < \frac{v_e}{\omega} < r_D, \qquad v_E > v_i. \quad (3.40)$$

In the case when the Larmor radii of the particles of both species are smaller than the Debye radius, the following formulas also hold true:

$$L_1 = \ln\frac{\rho_i}{\rho_e}\ln\frac{v_e}{v_i} + \ln\frac{v_E}{\omega\rho_i}\ln\frac{v_e}{v_E} + \frac{1}{2}\ln^2\frac{v_e}{v_E},$$

$$r_{min}\frac{v_e^2}{v_i^2} < r_{min}\frac{v_e^2}{v_E^2}, \ \rho_e < \rho_i < \frac{v_E}{\omega} < \frac{v_e}{\omega} < r_D;$$

$$L_1 = \ln\frac{\rho_i}{\rho_e}\ln\frac{v_e}{v_i} + \ln\frac{v_e}{v_E}\left(\frac{r_{min}}{\rho_i}\right)^{1/2}\ln\frac{v_e}{v_E}\left(\frac{\rho_i}{r_{min}}\right)^{1/2} + \ln\frac{v_E^3}{\omega v_e^2 r_{min}}\ln\frac{v_e}{v_E} + \frac{1}{2}\ln^2\frac{v_e}{v_E},$$

$$r_{min}\frac{v_e^2}{v_i^2} < \rho_e < \rho_i < r_{min}\frac{v_e^2}{v_E^2} < \frac{v_E}{\omega} < \frac{v_e}{\omega} < r_D;$$

$$L_1 = \ln\frac{v_e}{v_i}\left(\frac{r_{min}}{\rho_e}\right)^{1/2}\ln\frac{v_e}{v_i}\left(\frac{\rho_e}{r_{min}}\right)^{1/2} + \ln\frac{\rho_i v_i^2}{r_{min}v_e^2}\ln\frac{v_e}{v_i} + \ln\frac{v_E}{\omega\rho_i}\ln\frac{v_e}{v_E} + \frac{1}{2}\ln^2\frac{v_e}{v_E}$$

$$\rho_e < r_{min}\frac{v_e^2}{v_i^2} < \rho_i < \frac{v_E}{\omega} < \frac{v_e}{\omega} < r_D;$$

$$L_1 = \ln\frac{v_e}{v_i}\left(\frac{r_{min}}{\rho_e}\right)^{1/2}\ln\frac{v_e}{v_i}\left(\frac{\rho_e}{r_{min}}\right)^{1/2} + \ln\frac{\rho_i v_i^2}{r_{min}v_e^2}\ln\frac{v_e}{v_i} + \ln\frac{v_e}{v_E}\left(\frac{r_{min}}{\rho_i}\right)^{1/2}\ln\frac{v_e}{v_E}\left(\frac{\rho_i}{r_{min}}\right)^{1/2} + \ln\frac{v_E^3}{\omega v_e^2 r_{min}}\ln\frac{v_e}{v_E} + \frac{1}{2}\ln^2\frac{v_e}{v_E},$$

$$\rho_e < r_{min}\left(\frac{v_e}{v_i}\right)^2 < \rho_i < r_{min}\left(\frac{v_e}{v_E}\right)^2 < \frac{v_E}{\omega} < \frac{v_i}{\omega} < \frac{v_e}{\omega} < r_D. \quad (3.41)$$

As can be seen from (3.33)—(3.41), L_1, and therefore also $\nu_\perp$, depends explicitly on the external field. Therefore, if the conditions defined by the inequalities in (3.33)—(3.41) are satisfied, we are actually dealing with a nonlinear dependence of the current on the electric field. Therefore, generally speaking, the frequency of the collisions turns out to be a function of the time. We note that when a wave of circular polarization propagates in a plasma along the magnetic field, the velocity of the electric drift, that is also the transverse frequency of the collisions, does not depend on the time.

[1] S. T. Belyaev, Fizika plazmy i problema upravlyaemykh termoyadernykh reaktsiĭ (Plasma Physics and the Problem of Controlled Thermonuclear Reactions) **3**, AN SSSR, 1958, p. 66.

[2] Kihara, Midzuno, and Kaneko, J. Phys. Soc. Japan **15**, 1101, 1960.

[3] V. L. Gurevich and Yu. A. Firsov, JETP **41**, 1151 (1961), Soviet Phys. JETP **14**, 822 (1962).

[4] V. E. Golant, ZhTF **33**, 3 (1963), Soviet Phys. Tech. Phys. **8**, 1 (1963).

[5] Yu. M. Aliev and A. R. Shister, JETP **45**, 1499 (1963), Soviet Phys. JETP **18**, 1035 (1964).

[6] S. I. Braginskiĭ, Voprosy teorii plazmy (Problems of Plasma Theory) **1**, 183 (1963).

[7] E. E. Lovetskiĭ, Izv. Vuzov, Radiofizika **5**, 813 (1962).

[8] V. P. Silin, JETP **38**, 1771 (1960), Soviet Phys. JETP **11**, 1277 (1960).

[9] V. P. Silin, JETP **41**, 861 (1961), Soviet Phys. JETP **14**, 617 (1962).

[10] L. D. Landau, JETP **7**, 103 (1937).

[11] V. P. Silin, JETP **47**, 2254 (1964), Soviet Phys. JETP **20**, 1510 (1965).

[12] V. P. Silin, DAN SSSR **161**, 328 (1965), Soviet Phys. Doklady **10**, 230 (1965).

[13] V. L. Ginzburg, Rasprostranenie elektromagnitnykh voln v plazme (Propagation of Electromagnetic Waves in a Plasma), Fizmatgiz, 1960, p. 495.

[14] A. R. Shister, Teplofizika vysokikh temperatur (High-temperature Physics), No. 3, 1965.

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