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SMALL ANGLE SCATTERING OF CHARGED PARTICLES

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A relativistic formula is derived for the small angle elastic scattering cross section of charged elementary particles, taking into account interference between nuclear and Coulomb interactions. The optical theorem is established for the nuclear amplitude taking this interference into account.

A formula for determining from experiment the real part of the nuclear elastic forward scattering amplitude of charged particles, taking account of the Coulomb phase of this amplitude, was first obtained by Bethe[1] using nonrelativistic quantum mechanics of an extended particle. From the point of view of experiments on the scattering of high energy elementary particles, it is of interest to obtain the corresponding formula starting from relativistic quantum field theory. We shall define the nuclear scattering amplitude for charged particles and establish the optical theorem for it.

In small angle elastic scattering of charged particles the main contributions to the amplitude come from diagrams 1 and 5 of the figure (the cross-hatched regions denote strong interactions), which correspond to nuclear and to Coulomb scattering. We shall also include the radiative corrections to these diagrams. With the presently attainable experimental accuracy, it is sufficient to consider only corrections of order $\alpha = 1/137$, although the inclusion of higher order corrections at small angles presents no difficulties in principle.[2]

The figure shows some typical diagrams 2, 3, 4 of radiative corrections to nuclear scattering. The contribution of diagrams like 4, corresponding to the exchange of a photon between real charged particles, contains infrared divergences. These diagrams correspond to the Coulomb contribution to the nuclear amplitude. Separating out explicitly the terms containing infrared divergences, we write the nuclear amplitude with the radiative corrections in the form

$$g_n(1 + F_\lambda), \qquad (1)$$

where the gauge- and Lorentz-invariant factor F_λ contains all the infrared divergences and takes account of the Coulomb contribution to the nuclear amplitude:[3]

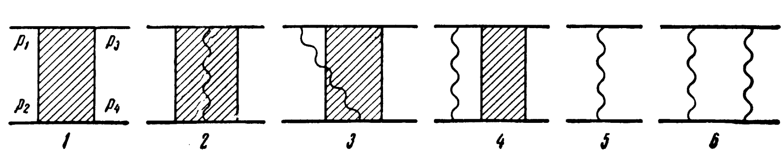
$$F_\lambda = -\sum_{i<i} z_i a_i z_j a_j \frac{i\alpha}{8\pi^3} \times \int \frac{d^4k}{k^2-\lambda^2}\left(\frac{2a_ip_i - k}{2a_ip_ik - k^2} - \frac{2a_jp_j + k}{2a_jp_jk + k^2}\right)^2. \qquad (2)$$

The summation goes over all charged particles at the start and end of the reaction; z_i and p_i are the sign of the charge and the 4-momentum of the particle; $a_i = 1$ for outgoing and $a_i = -1$ for incoming particles; λ is a fictitious small photon mass.

We shall call the quantity g_n in (1) the nuclear amplitude. It corresponds to diagrams of the type 1—3 and to the nondivergent parts of diagram 4, and differs from the purely nuclear amplitude (diagram 1) only by finite radiative corrections. It has the same general properties as the purely nuclear amplitude: it contains no divergences, is finite at zero scattering angle, is gauge invariant, has the same properties of crossing symmetry and (at least in low orders of perturbation theory) the same analytic properties.[2,4]

Let us consider the factor F_λ for zero scattering angle, i.e., for $t = (p_1 - p_3)^2 = 0$. It is not difficult to see that

$$F_\lambda = -i\eta \ln\frac{4p^2}{\lambda^2} + O\left(\alpha\frac{\ln p}{p^2}\right) \qquad \eta = \frac{z_1z_2\alpha}{v_L}, \qquad (3)$$



where v_L is the lab velocity of the incident particle and p is the modulus of the 3-dimensional momentum of the particle in the c.m.s. The quantity O contains no infrared divergences, goes to zero at high energies, and can be dropped.

Now consider diagrams for one- and two-photon exchange. Of their contributions it is sufficient to consider only the part which is most singular at small angles:

$$g_C\left(1 - i\eta \ln \frac{-t}{\lambda^2}\right), \tag{4}$$

where g_C is the Coulomb amplitude. At small angles

$$g_C = -2\eta / p\theta^2, \tag{5}$$

$$-t = p^2\theta^2, \tag{6}$$

where θ is the scattering angle in the c.m.s. In (4) we have neglected terms of the form $\alpha^2(-t)^{-1/2}$, $\alpha^2 \ln(-t)$, and in (5) terms of order $\alpha(-t)^{-1/2}$ and α, which come from the electromagnetic form factors of the particles in the one-photon diagram, but they can easily be included.

Adding expressions (1) and (4) and taking out a common phase factor, we get the following expression for the differential scattering cross section in the c.m.s.:

$$\frac{d\sigma}{d\Omega} = \left| g_C\left(1 - i\eta \ln \frac{-t}{4p^2}\right) + g_n\right|^2 = |g_C|^2 + |g_n|^2 - 2|g_C| \times\left(g_{nR} + 2g_{nI}\eta \ln \frac{2}{\theta}\right), \tag{7}$$

where g_{nR} and g_{nI} denote respectively the real and imaginary parts of the nuclear amplitude g_n without spin change. In Bethe's formula[1] the factor 2 under the logarithm was replaced by $\theta_0 = 1.06/pa$, where a is the range of the nuclear interaction.

We now establish the optical theorem for the nuclear amplitude g_n. If we introduce a small photon mass λ, the total forward scattering amplitude satisfies the usual unitarity condition

$$\operatorname{Im} g_\lambda = 4\pi W(2\pi)^4 \sum_n \frac{1}{m!} \prod_{i=1}^{n} \frac{d\mathbf{p}_{fi}}{(2\pi)^3 2p_{fi}^0} \times \sum_{\text{spin}} |\langle f|g_\lambda|p_1p_2\rangle|^2 \delta\left(\sum_j p_{fj} - p_1 - p_2\right), \tag{8}$$

where W is the total energy in the c.m.s. and m is the number of identical particles in the final state f.

We now separate out of both sides of (8) all terms which are infinite for $\lambda \to 0$. Since the infrared divergences caused by the virtual and real photons cancel one another in the total cross section, infinite terms on the right of (8) arise only from the integration of the Coulomb amplitude over the region of small (and also large, for identical particles) angles. Noting that the imaginary part of the amplitude for two-photon exchange on the left of (8) cancels against the Coulomb cross section on the right, we rewrite this equation in the form

$$\operatorname{Im}[g_n(1 + F_\lambda)] = \frac{p}{4\pi}[\sigma(>\tilde{\theta}_{min}) - \sigma_C(>\tilde{\theta}_{min})] + \frac{p}{2\pi} \int_{\theta' < \theta_{min}} d\Omega' \operatorname{Re}[g^*_{C\lambda}(\theta') g_n(\theta')], \tag{9}$$

where $\sigma(>\tilde{\theta}_{min})$ denotes the total cross section for the process in which the elastic scattering in the c.m.s. is taken into account at all angles greater than some minimum angle $\tilde{\theta}_{min}$ near the forward direction (and near the backward direction, in the case of identical particles), $\sigma_C(>\tilde{\theta}_{min})$ is the analogous cross section for Coulomb scattering, and

$$g_{C\lambda} = 2\eta p / (t - \lambda^2). \tag{10}$$

We see that the terms dependent on λ in (10) cancel one another, and we get the optical theorem for g_n:

$$g_{nI} = \frac{p}{4\pi}[\sigma(>\tilde{\theta}_{min}) - \sigma_C(>\tilde{\theta}_{min})] + 2g_{nR}\eta \ln \frac{2}{\tilde{\theta}_{min}}. \tag{11}$$

The Coulomb cross section $\sigma_C(>\tilde{\theta}_{min})$ is known. For small $\tilde{\theta}_{min}$

$$\sigma_C(>\tilde{\theta}_{min}) = \frac{4\pi\eta^2}{p^2\tilde{\theta}^2_{min}} + O\left(\frac{1}{\tilde{\theta}_{min}}\right). \tag{12}$$

Thus formula (11) contains only finite quantities and relates the real and imaginary parts of the nuclear forward scattering amplitude (without spin change) to the observed experimental cross section $\sigma(>\tilde{\theta}_{min})$.

The optical theorem (11) is given to terms of order α. It is not difficult to include terms of order α^2. To do this, in formulas (1) and (9) we must replace $1 + F_\lambda$ by $1 + F_\lambda + F^2_\lambda/2$, $g_n(\theta')$ by $g_n(\theta')(1 + F_\lambda)$, and instead of $g_{C\lambda}$, use

$$g_{C\lambda} = 2\eta p\left(\frac{1}{t - \lambda^2} - \frac{i\eta}{[t(t - 4\lambda^2)]^{1/2}} \ln \frac{(4\lambda^2 - t)^{1/2} - \sqrt{-t}}{(4\lambda^2 - t)^{1/2} + \sqrt{-t}}\right). \tag{13}$$

As a result we get

$$g_{nI} = \frac{p}{4\pi}[\sigma(>\tilde{\theta}_{min}) - \sigma_C(>\tilde{\theta}_{min})] + 2\left(g_{nR} + g_{nI}\eta \ln \frac{2}{\tilde{\theta}_{min}}\right)\eta \ln \frac{2}{\tilde{\theta}_{min}}. \tag{14}$$

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