THE POSSIBILITY OF ANOMALOUS FLUCTUATIONS IN FERROMAGNETS AND ANTIFERROMAGNETS

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Submitted to JETP editor February 15, 1965

J. Exptl. Theoret. Phys. (U.S.S.R.) **49**, 296-301 (July, 1965)

Fluctuations in ferromagnetic and antiferromagnetic substances in which a directed movement of electrons occurs are investigated. It is shown that when the mean velocity of the electrons approaches the critical value above which spin wave instability sets in, the amplitude of the random spin waves sharply increases. The existence of such anomalous fluctuations may result in a sharp increase of the differential cross section for scattering of slow neutrons in ferro- and antiferromagnets.

1. As is well known, oscillations of the magnetic moment in ferromagnets and antiferromagnets propagate at low temperatures in the form of weakly damped spin waves. Under ordinary conditions the amplitude of these waves is determined by the temperature of the body. It is possible, however, to produce conditions under which the amplitudes of the spin waves will appreciably exceed the thermal level. One of the possibilities of such amplification of spin waves was considered by A. Akhiezer, Bar'yakhtar, and Peletminskiĭ, who have shown that if directional motion of the electrons is realized in a ferromagnetic or antiferromagnetic sample, then, starting with a certain critical value of the electron velocity, the spin waves cease to attenuate and start to grow.[1]

In the present paper we investigate the spectral and angular distributions of the fluctuations in ferromagnets and antiferromagnets in which directional motion of the electrons exists. We shall show that when the average electron velocity approaches the critical value, beyond which instability of spin waves sets in, the amplitude of the random spin waves becomes anomalously large. This phenomenon is analogous to the anomalous level of fluctuations in a plasma whose state is close to instability.[2,3]

Anomalous fluctuations in ferro- and antiferromagnets can be observed by the same methods as ordinary fluctuations, for example, with the aid of slow-neutron scattering. If the directional velocity of the electrons is close to the critical value, then the scattered-neutron spectrum line connected with the possibility of spin-wave propagation in the sample should be appreciably more intense than in the equilibrium state (a phenomenon analogous to critical opalescence).

The possibility of an anomalous growth of the fluctuations on approaching the stability limits is connected with the vanishing at this limit of the damping decrement of the spin waves, and does not depend on the concrete nature of the electron current that leads to the instability of the spin waves. We shall therefore consider an idealized problem, involving a system of ordered spins, through which there moves a current of free electrons, avoiding thereby the complications due to the finite dimensions of the openings in the sample (in the case of currents from external sources) or to the interaction of the electrons with the lattice (in the case of a current produced by application of an external electric field to the sample).

2. To investigate the fluctuations in a system of oriented spins, through which a current of charged particles flows, we start from the linearized kinetic equation for the Fourier component of the electron distribution function $f\,(f \sim \exp[i\mathbf{k}\cdot\mathbf{r} - i\omega t])$ and Maxwell's equations for the Fourier components of the electric field $\mathbf{E}$ and of the magnetic induction $\mathbf{B}$. Introducing, in accordance with the general theory of fluctuations, random forces y into the equations describing the system, we get [1)]

$$-iGf + \frac{e}{m}\left(\mathbf{E} + \frac{1}{c}[\mathbf{vB}]\right)\frac{\partial}{\partial \mathbf{v}}F_0 = y(\mathbf{v}); \quad [\mathbf{kE}] = \frac{\omega}{c}\mathbf{B};$$

[1)] A kinetic equation with random forces was first used by Abrikosov and Khalatnikov in the investigation of fluctuations in an equilibrium Fermi liquid.[5]

$$[\mathbf{k}\cdot\hat{\mu}^{-1}\mathbf{B}] = -\frac{\omega\varepsilon}{c}\mathbf{E} - \frac{4\pi i}{c}\mathbf{j};\quad \mathbf{j} = e\int \mathbf{v}f d\mathbf{v};$$

$$G = \omega - \mathbf{kv} + \frac{ie}{mc}[\mathbf{vB}_0]\frac{\partial}{\partial\mathbf{v}} + \frac{i}{\tau};$$

$$F_0 \sim \exp\left\{\frac{-m(\mathbf{v}-\mathbf{u})^2}{2T_e}\right\}, \qquad (1)^*$$

where F_0 is the unperturbed electron distribution function, B_0 the unperturbed magnetic induction ($\mathbf{B}_0 \parallel \mathbf{u}$), $\hat{\mu}$ is the magnetic permeability tensor, ϵ is the dielectric constant of the ferro- (antiferro-) magnet, and τ^{-1} is the frequency of the electron collisions, which we shall let approach zero in the final results. (Being interested in the case when the fluctuations of the electromagnetic quantities greatly exceed the thermal level, we have introduced random forces only into the equation describing the electrons, without introducing additionally random fields in Maxwell's equations.)

Introducing the tensor of the electric susceptibility of the electrons

$$\varkappa_{ij}^e = \frac{e^2}{m\omega}\int v_i G^{-1}\frac{\partial}{\partial v_j}F_0 d\mathbf{v}, \qquad (2)$$

we rewrite (1) in the form

$$\frac{c}{\omega}[\mathbf{k}\cdot\hat{\mu}^{-1}\mathbf{B}] + \varepsilon\mathbf{E} + 4\pi\hat{\varkappa}^e\left(\mathbf{E} + \frac{1}{c}[\mathbf{u}\ \mathbf{B}]\right) = \frac{4\pi}{\omega}\mathbf{Y},$$

$$[\mathbf{k}\ \mathbf{E}] = \frac{\omega}{c}\mathbf{B}, \qquad \mathbf{Y} = e\int \mathbf{v}G^{-1}y\, d\mathbf{v}. \qquad (3)$$

In order to find the average values of the products of the random forces, we differentiate with respect to time the expression for the entropy S of the system. We confine ourselves to an examination of fluctuations with $\mathbf{k} \neq 0$, and represent $\dot{S}$ in the form

$$\dot{S} = -\int d\mathbf{r}d\mathbf{v}\dot{x}X;\quad \dot{x} = -\tau^{-1}f + y;\quad X = F_0^{-1}f.$$

Using further the method developed by Landau and Lifshitz [4] and Abrikosov and Khalatnikov [5], we normalize the random forces:

$$\langle y(\mathbf{v})y^*(\mathbf{v}')\rangle_{\mathbf{k}\omega} = 2\tau^{-1}F_0(\mathbf{v})\delta(\mathbf{v}-\mathbf{v}'). \qquad (4)$$

We can now determine the correlators of the quantities characterizing the system. To this end, expressing these quantities in terms of random forces, it is necessary to construct their bilinear combinations and then average over the random forces with the aid of (4). According to (3), the correlators of all the electromagnetic quantities are expressed in this case in terms of the averaged products of the components of the vector **Y**.

*[vB] = v × B.

Taking (2) into account and confining ourselves for simplicity to the case of not too cold electrons ($T_e \gg mu^2$), we have

$$\langle Y_i Y_j^*\rangle_{\mathbf{k}\omega} = -i\omega T_e(\varkappa_{ij}^e - \varkappa_{ji}^{e*}). \qquad (5)$$

We note that according to this relation we get $\mathbf{k}\cdot\mathbf{Y} \ll kY$ for low-frequency fluctuations ($\omega^2/k^2 \ll T_e/m$).

3. We consider first the case of an antiferromagnet with sublattice magnetic moments oriented along a chosen axis (axis 3). We assume in this case that the external magnetic field is zero. Noting that in this case the tensor $\hat{\mu}$ is diagonal with $\mu_{33} = 1$ and $\mu_{11} = \mu_{22} \equiv \mu_1$, we rewrite (3) in the form

$$B_1 = 4\pi(ck)^{-1}\cos\theta\cdot D_1^{-1}Y_2;\qquad B_2 = 4\pi(ck\sin\theta)^{-1}D_2^{-1}Y_3;$$

$$B_3 = -\mathrm{tg}\,\theta\cdot B_1;$$

$$D_1 = \sin^2\theta + \mu_1^{-1}\cos^2\theta - (\omega/ck)^2[\varepsilon + (1-\mathbf{ku}/\omega)4\pi\varkappa_t^e];$$

$$D_2 = \mu_1^{-1} - (\omega/ck)^2[\varepsilon + (1-\mathbf{ku}/\omega)4\pi\varkappa_t^e], \qquad (6)^*$$

where κ_t^e is the component of the electron electric susceptibility which is transverse to the wave vector, and θ is the angle between **k** and the axis 3,

$$\mu_1 = (\Omega_2^2 - \omega^2)/(\Omega_1^2 - \omega^2);$$

$$\mathrm{Re}\,\Omega_1^2 = (gM_0)^2 2\delta[\beta + (\alpha - \alpha_{12})k^2],$$

$$\mathrm{Re}\,(\Omega_2^2 - \Omega_1^2) = 8\pi(gM_0)^2[\beta + (\alpha - \alpha_{12})k^2], \qquad (7)$$

g is the gyromagnetic ratio, M_0 —the magnetic moment of the sublattice, β —the magnetic anisotropy constant, and α, α_{12} and δ are constants characterizing the exchange interaction in the antiferromagnet (see, for example, [6]), while the 2-axis is chosen perpendicular to the vector **k**.

The equations $D_{1,2} = 0$ determine the frequencies $\omega_{1,2}$ and the damping decrements $\gamma_{1,2}$ of two branches of the natural modes of the antiferromagnet with directional electron motion. We note that the interaction with the electron current affects little the frequencies of the spin waves, but can greatly change their damping; recognizing that

$$4\pi\,\mathrm{Im}\,\varkappa_t^e = \Omega_B^2(\omega k)^{-1}(\pi m/2T_e)^{1/2},$$

we obtain [2)]

$$\omega_1(\mathbf{k}) = \{\Omega_1^2 + (\Omega_2^2 - \Omega_1^2)\sin^2\theta\}^{1/2};\qquad \omega_2(\mathbf{k}) = \Omega_1;$$

$$\gamma_{1,2}(\mathbf{k}) = \overset{0}{\gamma}_{1,2}(\mathbf{k})\{(\mathbf{ku})_{1,2} - \mathbf{ku}\}\{(\mathbf{ku})_{1,2} - \omega_{1,2}\}^{-1}, \qquad (8)$$

*tg = tan.

2)Hereafter $\Omega_{1,2}$ will stand for the real parts of the corresponding quantities. Analogously, in relations (15) – (18) we shall omit the symbol for the real part in the case of Re Ω.

where $\gamma^0_{1,2}$ are the damping decrements of two types of spin waves in the free antiferromagnet

$$(\mathbf{ku})_{1,2} = \omega_{1,2}\{1 + 2\gamma^0_{1,2}k(2T_e/\pi m)^{1/2}(ck/\Omega_B)^2 \times (\Omega_2^2 - \omega^2_{1,2})^{-1}\} \quad (9)$$

and $\Omega_B = (4\pi e^2 n_B)^{1/2} m^{-1/2}$ is the plasma frequency for the electron flux (we take account of the fact that $\omega^2\epsilon/c^2k^2 \ll 1$). According to (8), the damping decrements decrease with increasing $\mathbf{k}\cdot\mathbf{u}$, vanishing when $\mathbf{k}\cdot\mathbf{u} = (\mathbf{k}\cdot\mathbf{u})_{1,2}$. With further increase in $\mathbf{k}\cdot\mathbf{u}$, the spin waves cease to be attenuated and start to grow.

We now determine the correlator of the magnetic induction in an antiferromagnet with directional electron motion. Using (5) and (6), we obtain

$$\langle B_i B_j^*\rangle_{\mathbf{k}\omega} = 2(2\pi)^2 T_e(\Omega_2^2 - \omega^2) \times \left\{\frac{n_{ij}\delta(\omega^2 - \omega_1^2)}{(\mathbf{ku})_1 - \mathbf{ku}\,\mathrm{sign}\,\omega} + \frac{\delta_{i2}\delta_{2j}\delta(\omega^2 - \omega_2^2)}{(\mathbf{ku})_2 - \mathbf{ku}\,\mathrm{sign}\,\omega}\right\},$$

$$n_{ij} = \delta_{ij} - k^{-2}k_ik_j - \delta_{i2}\delta_{2j}. \quad (10)$$

Knowing the correlator of the magnetic induction, we can determine with the aid of Maxwell's equations the correlators of the other electromagnetic quantities. In particular, for the correlator of the magnetic moment $\mathbf{M}$, taking (7) and (8) into account, we get

$$\langle M_i M_j^*\rangle_{\mathbf{k}\omega} = \frac{1}{2}T_e(\Omega_2^2 - \Omega_1^2) \times \left\{\frac{\delta_{i1}\delta_{1j}\delta(\omega^2 - \omega_1^2)}{(\mathbf{ku})_1 - \mathbf{ku}\,\mathrm{sign}\,\omega} + \frac{\delta_{i2}\delta_{2j}\delta(\omega^2 - \omega_2^2)}{(\mathbf{ku})_2 - \mathbf{ku}\,\mathrm{sign}\,\omega}\right\}. \quad (11)$$

Noting that in an equilibrium antiferromagnet

$$\langle |M_{1,2}|^2\rangle_{\mathbf{k}\omega} \sim \max\{\hbar|\omega|;\,T\}\,\Omega_1^{-1}(\Omega_2^2 - \Omega_1^2)\,\delta(\omega^2 - \omega^2_{1,2})$$

(T is the temperature of the sample), we see that in the presence of directional electron motion the square of the amplitude of the random spin waves can exceed the equilibrium level by a factor A, where

$$A \sim \frac{T_e}{\hbar\omega}(e^{\hbar\omega/T} - 1)\frac{\gamma^0}{\gamma}. \quad (12)$$

We emphasize that relations (10)—(12) [and (17) and (18) as well] have been obtained within the framework of the linear theory and are valid in the stability region $[|\mathbf{k}\cdot\mathbf{u}| < (\mathbf{k}\cdot\mathbf{u})_{1,2}]$. It follows from these relations that in an antiferromagnet with directional motion of the electrons the correlation functions are proportional to $[1 - |\mathbf{k}\cdot\mathbf{u}| \times (\mathbf{k}\cdot\mathbf{u})^{-1}_{1,2}]^{-1}$, and increase strongly on approaching the limit of the instability region, determined from the condition $\gamma(\mathbf{k}) \to 0$. According to these relations, the amplitude of the random spin waves becomes infinite on the border of the stability region; the limitations on the growth of the fluctuation amplitude as the limit of stability is approached are imposed only by the nonlinear effects.

4. We proceed to consider the interaction between a current of charged particles and spin waves in a ferromagnet. In this case the nonvanishing components of the magnetic permeability tensor are $\mu_{11} = \mu_{22} \equiv \mu_1$ as well as μ_{12}, μ_{21} and μ_{33}; in this case (see [6])

$$\mu_1 = [\Omega(\Omega + 4\pi g M_0) - \omega^2](\Omega^2 - \omega^2)^{-1}, \quad \mu_{33} = 1,$$
$$-{}^1\!/_2 i(\mu_{12} + \mu_{21}^*) \equiv \mu_2 = 4\pi g M_0\omega(\Omega^2 - \omega^2)^{-1};$$
$$\mathrm{Re}\,\Omega = gM_0(\alpha k^2 + \beta + H_0/M_0), \quad (13)$$

where α and β are constants characterizing the exchange interaction and the magnetic anisotropy [the external magnetic field H_0 is applied along the easiest magnetization axis (the 3-axis)].

In order not to complicate the formulas that follow, we shall assume that $kr_L \gg 1$, where $r_L^2 = T_e mc^2(eB_0)^{-2}$. Recognizing that in this case according to (2),

$$4\pi\,\mathrm{Im}\,\varkappa_{ij}^e = (\delta_{ij} - k^{-2}k_ik_j)4\pi\,\mathrm{Im}\,\varkappa_t^e = (\delta_{ij} - k^{-2}k_ik_j)\Omega_B^2(\omega k)^{-1}(\pi m/2T_e)^{1/2},$$

we can rewrite (3) in the form

$$B_1 = 4\pi(ck)^{-1}\cos\theta\,D^{-1}(Y_2 - i\mu_2 Y_1/\mu_1);\; B_3 = -B_1\,\mathrm{tg}\,\theta;$$
$$B_2 = -i\mu_2 B_1/\mu_1 + 4\pi(ck\sin\theta)^{-1}\mu_1(1 - \mu_2^2/\mu_1^2)Y_3;$$
$$D = \sin^2\theta + \mu_1^{-1}\cos^2\theta - (\omega/ck)^2 \times[\varepsilon + (1 - \mathbf{ku}/\omega)4\pi\varkappa_t^e] \times[1 + \cos^2\theta + \mu_1^{-1}(\mu_1^2 - \mu_2^2)\sin^2\theta] \quad (14)$$

(we assume that $T_e \gg mu^2$ and $u \gtrsim \omega/k$).

Solving the equation $D = 0$, which determines the frequency and the damping decrement of the natural oscillations of the ferromagnet with directional electron motion, we obtain

$$\omega_s(\mathbf{k}) = \Omega^{1/2}(\Omega + 4\pi g M_0\sin^2\theta)^{1/2},$$
$$\gamma_s(\mathbf{k}) = \gamma_s^0(\mathbf{k})\{(\mathbf{ku})_s - \mathbf{ku}\}\{(\mathbf{ku})_s - \omega_s\}^{-1}, \quad (15)$$

where γ_S^0 is the spin-wave damping decrement in the free ferromagnet, and

$$(\mathbf{ku})_s = \omega_s\{1 + \gamma_s^0 k(2T_e/\pi m)^{1/2}(ck/\Omega_B)^2\Omega(2\pi g M_0)^{-1}(\omega_s^2 + \Omega^2\cos^2\theta)^{-1}\} \quad (16)$$

[in the derivation of (14)—(16) we took into account that $\omega^2\epsilon/c^2k^2 \ll 1$]. According to (15), the damping decrement decreases with increasing $\mathbf{k}\cdot\mathbf{u}$, vanishing when $\mathbf{k}\cdot\mathbf{u} = (\mathbf{k}\cdot\mathbf{u})_S$. With further increase of $\mathbf{k}\cdot\mathbf{u}$, the spin waves cease to attenuate and start to grow.

We now determine the correlator of the magnetic induction in a ferromagnet with directional motion of the electrons. Using (5) and (14) we obtain

$$b_{ij} = \delta_{ij} - k_i k_j / k^2 \quad (i, j = 1, 3); \quad b_{12} = -b_{21} = i\omega / \Omega;$$

$$b_{23} = -b_{32} = i\omega \operatorname{tg} \theta / \Omega, \quad b_{22} = \omega^2 / \Omega^2 \cos^2 \theta. \quad (17)$$

For the magnetic-moment correlator, taking (13) and (15) into account, we find

$$\langle M_i M_j^* \rangle_{\mathbf{k}\omega} = 2\pi T_e r_{ij} g M_0 \Omega [(\mathbf{ku})_s - \mathbf{ku} \operatorname{sign} \omega]^{-1} \delta(\omega^2 - \omega_s^2),$$

$$r_{11} = 1; \quad r_{12} = -r_{21} = i\omega / \Omega; \quad r_{22} = \omega^2 / \Omega^2;$$

$$r_{3i} = r_{i3} = 0 \quad (i = 1, 2, 3). \quad (18)$$

Noting that in an equilibrium ferromagnet

$$\langle |M_{1,2}|^2 \rangle_{\mathbf{k}\omega} \sim \max \{\hbar|\omega|;\ T\} g M_0 \delta(\omega^2 - \omega_s^2),$$

we see that in the presence of directional motion of the electrons the square of the amplitude of the random spin waves can exceed the equilibrium level by a factor A, where A is determined by formula (12).

We note in conclusion of this section that all the relations obtained are valid for not too large k, $\hbar k^2/m \ll \omega(\mathbf{k})$, when quantum effects can be disregarded in the determination of the electric susceptibility of the electrons.

5. Anomalous fluctuations in ferro- and antiferromagnets can be observed by the same methods as equilibrium fluctuations, for example, with the aid of slow-neutron scattering. The differential cross section of this process, $d\sigma$, is proportional, as is well known, (see, for example, [7]) to the correlator of the magnetic moment and differs from zero if the condition $|\mathbf{k} \cdot \mathbf{v}| = \omega(\mathbf{k})$ is satisfied, a condition equivalent to the conservation of the energy and momentum in the scattering (we assume for concreteness that $\omega \gg \hbar k^2/m_n$, where m_n is the neutron mass and $\mathbf{v}$ its initial velocity). The neutron scattering cross section increases sharply when the change in the neutron momentum upon scattering, $\hbar\mathbf{k}$, approaches the critical value satisfying relations (9) or (16). Then $d\sigma$ exceeds in order of magnitude the scattering cross section in an equilibrium ferro- or antiferromagnet by a factor of A where A is given by (12).

In conclusion I thank A. I. Akhiezer and V. G. Bar'yakhtar for valuable advice and a discussion.

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Translated by J. G. Adashko
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