REGGE POLES IN A POTENTIAL OF THE COULOMB WELL TYPE

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The energy dependence of the motion of the Regge poles is investigated on the basis of an exact solution of the Schrödinger equation for a potential of the Coulomb-well type, which is identical with a Coulomb potential at short distances and which cuts off like a rectangular-well potential at large distances. It is shown that the trajectories of the poles in a Coulomb-well potential have a Regge character at small and medium energies. The asymptotic pole trajectories at high energies coincide with those for the Coulomb case.

INTRODUCTION

Much attention has been paid recently to the analytic properties of the scattering amplitude in the complex angular-momentum plane. Regge[1] has shown that in nonrelativistic quantum mechanics, the only singularities of the amplitude for potentials that are superpositions of Yukawa potentials, are simple moving poles (Regge poles). Regge's hypothesis that the singularities of the scattering amplitude in the complex orbital angular-momentum plane are simple has been extended further to relativistic quantum field theory. This has stimulated interest in the behavior of the Regge-pole trajectories for other types of potentials. This question is considered in a number of papers on the basis of the first or second perturbation-theory approximation, while in other papers it is considered by investigating the potentials for which the Schrödinger equation has an analytic solution. The number of such potentials is small. Thus, Singh[2] investigated the motion of the poles in a Coulomb potential, Bollini et al.[3,4] in a square-well potential, and Arutyunyan et al.[5] in a δ-function potential.

We note that the dependence of the aforementioned potentials on the distance differs greatly from that of the Yukawa potential, which is the most enticing from the physical point of view. In this paper we investigate the motion of the poles in a potential of the type of a Coulomb well, which is similar in many respects to the Yukawa potential. In the first section we derive the equation and discuss the general structure of the Regge-pole trajectory; in the second we obtain the locations of the Regge poles at high energies; in the third we investigate the relative pole motion; in the fourth we study the motion of the poles at medium and low energies. In the last section we discuss the results.

1. DERIVATION OF THE REGGE-POLE EQUATION. GENERAL STRUCTURE OF THE TRAJECTORIES

We define the Coulomb-well potential in the following manner:

$$V=\begin{cases} g/2mr, & \mu r<1 \\ 0, & \mu r>1 \end{cases}, \tag{1.1}$$

where g is the coupling constant and m the particle mass. The solution of the equation in the potential (1.1) is of the form

$$\psi=\frac{\sqrt{\pi}\,e^{-ikr}}{\Gamma(1+\nu)}\left(\frac{kr}{2}\right)^{\nu+1/2}F(\lambda,2\nu+1,2ikr),\quad \mu r<1, \tag{1.2}$$

$$\psi_s\sim(\pi kr/2)^{1/2}H_\nu^{(1)}(kr),\quad \mu r>1, \tag{1.3}$$

where $\nu = l + 1/2$, $\lambda = \nu + 1/2 + g/2ik$, k is the particle momentum, $F(\lambda, 2\nu + 1, 2ikr)$ the confluent hypergeometric function, $H_\nu^{(1)}(kr)$ the Hankel function of the first kind, and $\Gamma(1 + \nu)$ the Euler gamma-function.

The equation for the Regge poles can be obtained in this case by joining together the logarithmic derivatives of the wave functions at the point $\mu r = 1$:

$$(2kr)^{-1}+\frac{H_\nu^{(1)\prime}(kr)}{H_\nu^{(1)}(kr)}=-i+\frac{\nu+1/2}{kr}+2i\frac{F'(\lambda,2\nu+1,z)}{F(\lambda,2\nu+1,z)}, \tag{1.4}$$

where $z = 2ikr$. An equation equivalent to (1.4) can also be obtained on the basis of the S-matrix in the Lipman-Schwinger form [6]. In such an approach we can expand an integral rather than the special functions, which is a more direct operation.

Let us show how to gain an understanding of the over-all structure and behavior of the Regge trajectories from simple physical considerations. After the substitution ($r = e^x$, $\psi \sim \chi\sqrt{r}$), the Schrödinger equation for the potential (1.1) takes the form

$$\chi'' - \nu^2\chi = [gr - (kr)^2]\chi, \qquad \mu r < 1,$$
$$\chi'' - \nu^2\chi = -(kr)^2\chi, \qquad \mu r > 1. \qquad (1.5)$$

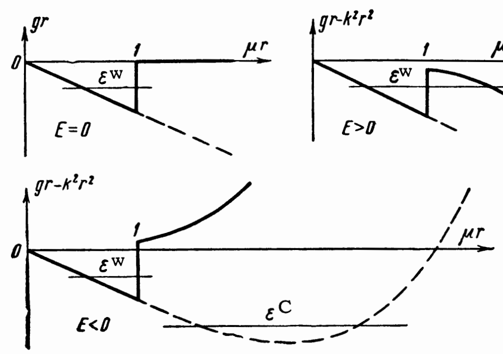

FIG. 1

Figure 1 shows plots of the effective potential "energy" for three values of $E = k^2/2m$ and for a coupling constant $g < 0$. The plot of the effective potential "energy" for a Coulomb-well potential (solid line) differs from the corresponding plot for a Coulomb potential (dotted line) only at large distances $\mu r > 1$. The plot of the effective potential "energy" in a Coulomb potential with $E = 0$ is a straight line. It is known that there are no bound states in such potentials. Therefore all the Coulomb trajectories go to infinity as $E \to 0$. On the other hand, the curve corresponding to the Coulomb well has a trough. Therefore, if the trough is sufficiently deep, bound-state levels with energy $\mathscr{E}_n^W$ are produced, meaning that Regge poles corresponding to these bound states appear on the real positive axis of the ν plane. It is obvious that the number of Regge poles coincides with the number of the levels. Let us trace the motion of the Regge poles as the energy E is varied.

If $E < 0$ and $k^2 \ll g\mu$ (this means that the intercept of the Coulomb potential with the abscissa axis r_0 is far from the edge of the well μ^{-1}), then the difference between the Coulomb potential and the Coulomb-well potential type is large. In this case the Coulomb energy levels $\mathscr{E}_n^C$ begin much lower than the bottom of the well, and consequently also much lower than the well levels $\mathscr{E}_n^W$. With increasing binding energy E, the point r_0 approaches μ^{-1}, and the minimum of the Coulomb potential rises. When $\mu r_0 \sim 1$ the two potentials become approximately equal, and therefore their energy levels should coincide approximately ($\mathscr{E}_n^W \sim \mathscr{E}_n^C$). Thus, at large E, the Coulomb trajectories are the asymptotes of the bound-state trajectories for the Coulomb well.

When $E > 0$ a particle situated at the bound level $\mathscr{E}_n^W$ can penetrate into the region $\mu r > 1$, and therefore the Regge poles shift in this case from the real axis to the upper half plane of ν.

2. LOCATIONS OF THE REGGE POLES AT HIGH ENERGIES

Let us investigate on the basis of Eq. (1.4) the locations of the trajectories of the bound states as $E \to \infty$. To this end we use the asymptotic values of the Hankel function and of the confluent hypergeometric function as $z \to \infty$. As shown in the preliminary analysis, the solution should tend to the Coulomb solution as $E \to \infty$. Therefore it is convenient to separate directly the pure Coulomb term in (1.4):

$$\Gamma\left(\nu + \frac{1}{2} - \frac{g}{2ik}\right)\Big/\Gamma(\lambda) = \Big\{2U_2(\lambda, 2\nu+1, z) - \left[1 + \frac{H_{\nu+1}^{(1)}(kr)}{iH_\nu^{(1)}(kr)}\right] U_2(\lambda, 2\nu, z)\Big\}$$
$$\times \Big\{\left[1 + \frac{H_{\nu+1}^{(1)}(kr)}{iH_\nu^{(1)}(kr)}\right] U_1(\lambda, 2\nu+1, z) - 2\left(\nu + \frac{1}{2} - \frac{g}{2ik}\right) U_1(\lambda, 2\nu+2, z)\Big\}^{-1}, \qquad (2.1)$$

where U_1 and U_2 are confluent hypergeometric functions of the third kind.

The poles $\Gamma(\lambda)$ determine the positions of the bound levels in a Coulomb potential. We therefore seek a solution of (2.1) in the form $\lambda = -N + \delta$, where N is an integer and $|\delta| \ll 1$. Iterating with respect to δ, we get

$$\nu = -\frac{1}{2} - N - \frac{gr}{4\varkappa} + \frac{gr\, e^{-4\varkappa}(4\varkappa)^{-2-gr/2\varkappa}}{\Gamma(1+N)\Gamma(-N - gr/2\varkappa)}. \qquad (2.2)$$

It follows from the solution (2.2) that at high energies ($\kappa \to \infty$) the Regge poles tend to half-integer negative points. Figure 2 shows a plot of the motion of the Regge poles at negative energies. The ordinates represent the reciprocal energy E^{-1} in arbitrary units, and the abscissas Re ν. The straight lines drawn from the half-integer points are the Coulomb solutions, which are the asymptotes when E is large. The points where the curves cross the lines Re ν = 1/2, 3/2, etc. determine the binding energies of the levels.

It follows form (2.2) that when $g/ik = N_0$, where N_0 is some integer, all the curves with $N \geq N_0$ have a deviation $\delta = 0$. This means that all curves with $N \geq N_0$ pass simultaneously through integer or half-integer negative values of ν, and coincide exactly at that instant with the corresponding points that lie on the Coulomb lines. These cases are marked in Fig. 2 by full points on the curves.

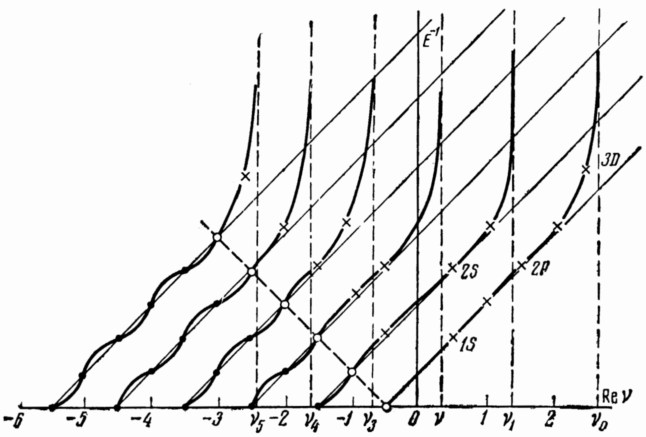

FIG. 2

On the basis of (2.1) we can show that poles possess this property in the exact solution. In fact, the points

$$\begin{aligned} g/ik &= 0, \quad & \nu &= -1/2,\ -3/2,\ -5/2,\ \ldots, \\ g/ik &= 1, \quad & \nu &= -1,\ -2,\ -3,\ \ldots, \\ g/ik &= 2, \quad & \nu &= -3/2,\ -5/2,\ -7/2,\ \ldots \end{aligned} \tag{2.3}$$

etc. are essentially singularities of the second kind for the left side of the exact equation (2.1), i.e., the left side of (2.1) can assume an arbitrary value, depending on the manner in which it tends to these points. On the other hand, the right side of (2.1) has no singularities at these points, i.e., it is continuous in the vicinity of the points (2.3). The points (2.3) will therefore be roots of the exact equation (2.1).

3. RELATIVE MOTION OF THE REGGE POLES

Let us discuss another important property of the pole motion—the reciprocity property. We shall prove that if a Regge pole passes through a point $\nu = N/2$ (N is an integer), then a second point passes simultaneously through the point $\nu = -N/2$. We note that the converse is not true.

To prove the reciprocity property, we first transform Eq. (1.4) with the aid of the recurrence relations into

$$+\frac{H_{\nu+1}^{(1)}(kr)}{iH_\nu^{(1)}(kr)} = \frac{\nu + 1/2 - g/2ik}{\nu + 1/2}\,\frac{F(\lambda, 2\nu+2, z)}{F(\lambda, 2\nu+1, z)}. \tag{3.1}$$

We then rewrite the left side of (3.1), taken at the point $\nu = -N/2$, for the positive values $\nu = N/2 \equiv \tilde{\nu}$:

$$1 + \frac{H_{\nu+1}^{(1)}(kr)}{iH_\nu^{(1)}(kr)} = 1 + \frac{H_{1-\tilde{\nu}}^{(1)}(kr)}{iH_{\tilde{\nu}}^{(1)}(kr)} = 1 + \frac{2i\tilde{\nu}}{kr} + \frac{H_{1+\tilde{\nu}}^{(1)}(kr)}{iH_{\tilde{\nu}}^{(1)}(kr)}. \tag{3.2}$$

The transformation of the right side of the equation is made complicated by the fact that as $2\nu \to -N$ the confluent hypergeometric functions have singularities. It can be shown, however, that the hypergeometric function $F(\lambda, c, z)$ has the following property:

$$\lim_{(c+N)\to 0} (c+N)F(\lambda, c, z) = -\frac{(-z)^{N+1}\Gamma(1+\lambda+N)}{\Gamma(\lambda)\Gamma(1+N)\Gamma(2+N)} \times F(1+\lambda+N, 2+N, z). \tag{3.3}$$

Therefore the ratio of the hypergeometric functions contained in the right side of (3.1) is transformed to

$$\lim_{2\nu\to -N}\frac{F(\lambda, 2\nu+2, z)}{F(\lambda, 2\nu+1, z)} = -\frac{2\tilde{\nu}(2\tilde{\nu}-1)F(\tilde{\lambda}-1, 2\tilde{\nu}, z)}{(\tilde{\lambda}-1)zF(\tilde{\lambda}, 2\tilde{\nu}+1, z)},$$

$$\tilde{\lambda} = \tilde{\nu} + \frac{1}{2} + \frac{g}{2ik}. \tag{3.4}$$

Substituting (3.2) and (3.4) in (3.1) and applying to the hypergeometric function $F(\tilde{\lambda} - 1, 2\tilde{\nu}, z)$ the shift theorem with respect to the second index, we obtain an equation for the Regge poles at the point $\nu = \tilde{\nu}$. By definition, this equation is assumed to be valid at the point $\tilde{\nu}$, and must therefore be valid also at the point $-\tilde{\nu}$. This proves the reciprocity property. The conjugate points $\tilde{\nu}$ and $-\tilde{\nu}$ are marked in Fig. 2 by crosses.

Let us determine the positions of the bound levels in S-states, i.e., in those cases when the Regge-pole trajectories cross the vertical line $\mathrm{Re}\,\nu = 1/2$. In this case the general equation (3.1) takes the form

$$F(g/2ik, 1, 2ikr) = 0. \tag{3.5}$$

It is clear that when the energy increases the solution of (3.5) will tend to the Coulomb solution $gr = nz$. The thin straight lines in Fig. 3 are the Coulomb solutions corresponding to different values of the principal quantum number n. The point of intersection of the line gr = const with the n-th curve on Fig. 3 gives the position of the nS-level.

Let us separate the roots of (3.5) for large energies. Using the asymptotic value of the confluent hypergeometric point, we obtain the following formula for the solution of (3.5):

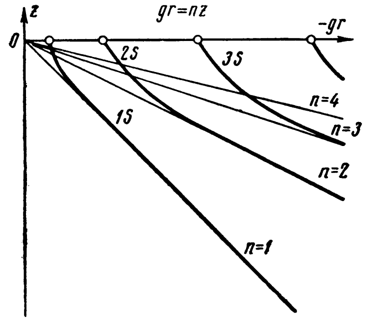

FIG. 3

$$gr = nz - z^{2n}e^{z} / \Gamma^2(n), \qquad l = 0, \tag{3.6}$$

where $z = -4\kappa$. Expression (3.6) coincides with (2.2) if we put in the latter $\nu = 1/2$. From the curves of Fig. 3, and coincidentally also from formula (3.6), it follows that only Coulomb poles can pass through the point $\nu = 1/2$ from the right half-plane of ν to the left one. The reason is that all the curves have only pure Coulomb asymptotes.

4. MOTION OF REGGE POLES AT MEDIUM AND LOW ENERGIES

Let us find the positions of the Regge poles at low energies. It follows from (3.1) that when E = 0 their position is determined by the equations

$$J_{2\nu+1}(2\sqrt{-gr}) = 0, \qquad \operatorname{Re}\nu < 0, \tag{4.1}$$

$$J_{2\nu-1}(2\sqrt{-gr}) = 0, \qquad \operatorname{Re}\nu > 0. \tag{4.2}$$

The roots of (4.1) and (4.2) are designated in Fig. 2 by $\nu_N(0)$. The vertical lines through $\nu = \nu_N(0)$ are asymptotes for the curves $\nu_N(\kappa)$.

It is difficult to obtain from (3.1) the expansion of the pole trajectories in the vicinity of the point E = 0, since the expansion of the function F, whose index contains the momentum p in the denominator, is quite cumbersome. We shall therefore study the pole trajectories on the basis of the S-matrix representation in the Lipman-Schwinger form. If we apply to it the integral Mellin transformation, then the equation for the Regge poles will take the form

$$1 = \int_{\Omega} \frac{d\sigma}{2\pi i} \varkappa^{2\sigma} \Gamma(\nu - \sigma) \Gamma(-\sigma) R(\sigma, \nu), \tag{4.3}$$

where $\kappa = k/2i\mu$ and the contour Ω separates the poles of the interaction function $R(\sigma, \nu)$ from the poles of the Γ-functions.

Omitting the cumbersome calculations, we present the final form of the equation for the Regge poles, for a Coulomb-well potential, in first order in κ^2:

$$\varkappa^{-2\nu} = \frac{\Gamma(-\nu)}{\Gamma(\nu)} \frac{(1+A)J_{2\nu+1} + BJ_{2\nu+2}}{(1+C)J_{2\nu-1} + DJ_{2\nu-2}}, \tag{4.4}$$

where

$$A = -\frac{14\nu + 11}{3(\nu+1)}\varkappa^2 - \frac{2\varkappa^2}{gr}\left(2\nu^2 + 5\nu + 5 - \frac{1}{\nu+1}\right),$$

$$B = 2\varkappa^2(-gr)^{-1/2}\left(\frac{3\nu+2}{\nu+1} + \frac{14\nu+9}{3gr}\right),$$

$$C = -\frac{\varkappa^2}{\nu-1} + \frac{2\varkappa^2}{3gr(\nu-1)}(10\nu^2 - 9\nu - 4)$$

$$+ \frac{2\varkappa^2}{3(gr)^2}(8\nu^3 + 20\nu^2 + 22\nu - 17),$$

$$D = (-gr)^{1/2}\frac{2\varkappa^2}{3gr}\left[\frac{7-4\nu}{\nu-1} - (gr)^{-1}(4\nu^2 + 12\nu + 17)\right].$$

Solving (4.4) with respect to ν, we obtain two types of pole trajectories. We consider first the bound-state poles. We assume that when $\kappa = 0$ the poles are located at the points $\nu_N(0)$, where N is the number of the root of the Bessel function (4.1) or (4.2). To obtain the behavior of the poles near the value $\kappa = 0$, it is sufficient to expand in (4.4) in terms of the index of the Bessel functions

$$J_{2\nu\pm1} \approx (\nu - \nu_N(0))\,\partial J_{2\nu\pm1}/\partial\nu.$$

We then obtain

$$\nu - \nu_N(0) \approx -\left[CJ_{2\nu-1} + DJ_{2\nu-2} + \varkappa^{2\nu}\frac{\Gamma(1-\nu)}{\Gamma(1+\nu)}J_{2\nu+1}\right] \times \left(\frac{\partial J_{2\nu-1}}{\partial\nu}\right)^{-1}, \qquad \operatorname{Re}\nu_N(0) > 1; \tag{4.5}$$

$$\nu - \nu_N(0) \approx -\left[AJ_{2\nu+1} + BJ_{2\nu+2} + \varkappa^{-2\nu}\frac{\Gamma(1+\nu)}{\Gamma(1-\nu)}J_{2\nu-1}\right] \times \left(\frac{\partial J_{2\nu+1}}{\partial\nu}\right)^{-1}, \qquad \operatorname{Re}\nu_N(0) > -1. \tag{4.6}$$

We note that the terms proportional to $\kappa^{\pm 2\nu}$ in (4.5) and (4.6) are directly connected with the scattering phase shift and determine the deviation of the trajectory from the real axis for real E > 0. We can show that

$$\operatorname{Im}(\nu - \nu_N(0)) = -\frac{2\nu_N\varkappa^{2\nu_N}}{z}\frac{\Gamma(1-\nu_N)}{\Gamma(1+\nu_N)}\left(\frac{d\nu_N(0)}{dz}\right) > 0, \qquad \operatorname{Re}\nu_N(0) > 1, \tag{4.7}$$

$$\operatorname{Im}(\nu - \nu_N(0)) = \frac{2\nu_N\varkappa^{-2\nu_N}}{z}\frac{\Gamma(1+\nu_N)}{\Gamma(1-\nu_N)}\left(\frac{d\nu_N(0)}{dz}\right) > 0, \qquad \operatorname{Re}\nu_N(0) > -1. \tag{4.8}$$

The coefficients of κ^2 in (4.5) and (4.6), as follows from physical considerations, are always real and negative. Therefore the trajectories can

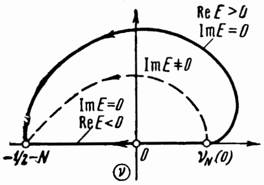

FIG. 4

go out of the points $\nu_N(0)$ only to the upper half-plane of ν (Fig. 4). It follows from (2.2) that at higher energies the trajectories are likewise in the upper half-plane. At medium energies these two asymptotes join together. The over-all picture of the bound-state trajectories in the complex ν plane is shown in Fig. 4. When $E < 0$ the Regge pole shifts from the point $\nu = \nu_N(0)$ to the point $\nu = -1/2 - N$. When E is complex, the trajectories bend out in the upper half plane of ν. Finally, for real $E > 0$, the poles describes arcs of maximum radius.

In a potential of the Coulomb-well type, there are in addition to the bound-state trajectories also threshold-condensation trajectories or Gribov-Pomeranchuk trajectories.[7] If we put $|\nu| \ll 1$, then the terms proportional to κ^2 in the right side of (4.4) should be discarded compared with the term $\kappa^{-2\nu}$. Expanding the remaining terms in powers of ν, we obtain

$$\varkappa^{-2\nu} = \exp\left[2\nu(a_1 + \nu a_2)\right] \approx 1 + 2\nu[a_1 + (a_1^2 + a_2)\nu], \tag{4.9}$$

where a_1 and a_2 are some coefficients that depend on the coupling constant g.

Equation (4.9) defines an entire bundle of trajectories that emerge from the origin:

$$\nu \approx -i\pi p / \ln \varkappa + \dots \qquad (p = 0, \pm 1, \dots). \tag{4.10}$$

The asymptotic behavior of these trajectories at high energies can be easily obtained from first-order perturbation theory. After suitable manipulations we obtain

$$\varkappa^{-2\nu} \approx -\frac{g\Gamma(-\nu)}{4\mu\Gamma(\nu)(\nu + {}^1/_2)(\nu + g/2\mu)}. \tag{4.11}$$

When ν is small, as can be readily seen, Eq. (4.11) has the same form as (4.9), but with different coefficients a_1 and a_2. Thus, Eqs. (4.9) and (4.11) ensure a continuous joining of the trajectories on going from low to high energies. It can be shown that the corresponding solution of (4.11) goes off to infinity parallel to the direction defined by the angle

$$\varphi = \pm\pi/2 + \alpha, \tag{4.12}$$

where α is the phase of the momentum κ, the plus pertains to the upper half-plane, and the minus to the lower. Figure 5 shows the trajectories of the

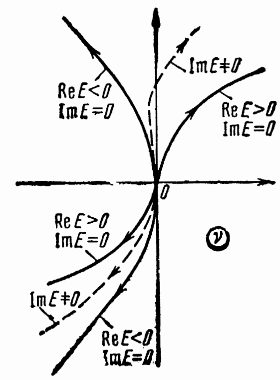

FIG. 5

poles for different complex values of the energy. The trajectories corresponding to negative energies ($\alpha = 0$) are complex conjugates. With increasing phase α, the trajectories of the upper half plane go over to the first quadrant, and the trajectories of the lower half-plane always remain in the third quadrant. When the energy becomes positive ($\alpha = \pi/2$), the trajectories go off to infinity parallel to the real axis.

We conclude the study of the Regge-pole trajectories by constructing an over-all picture of the motion of all the poles, both as a function of the coupling constant g and as a function of the energy. We take an infinite number of sheets of the ν plane and renumber their indices, in steps of one, from $-\infty$ to $+\infty$ (Fig. 6). On all sheets with $p \geq 1$ we place only the threshold-condensation trajectories, and on the main sheet with $p = 0$ we place only the trajectory of the first bound state, frequently called the leading trajectory. On the sheets with $p \leq -1$ we place trajectories of both types. The bound-state trajectory on the sheet with $p = -N$ begins at some point $\nu = \nu_N(0)$ and ends at the point $\nu = -1/2 - N$. Although the surface constructed in this manner is not a Riemann surface of the function $\kappa(\nu, g)$, which is the inverse of $\nu = \nu(\kappa, g)$, it nevertheless presents a clear picture of the character of the motion of all the poles.

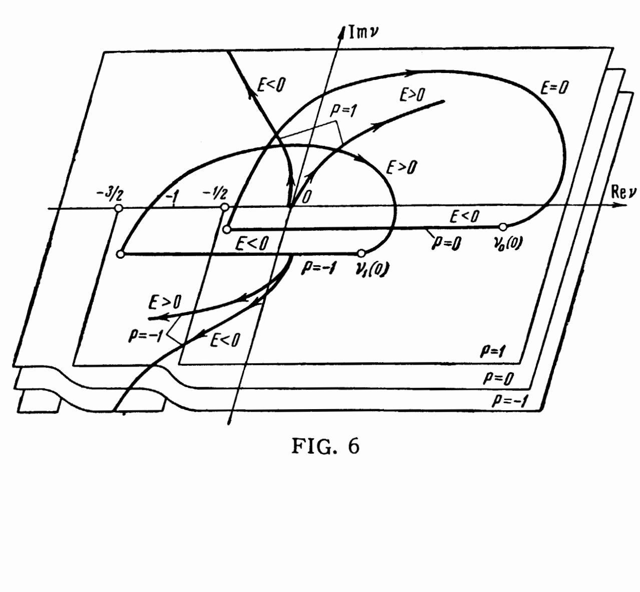

FIG. 6

5. DISCUSSION OF RESULTS

We have investigated the behavior of the Regge poles as a function of the energy, for a potential of the Coulomb-well type. The Coulomb-well potential coincides with the Coulomb potential at small distances and cuts off like a square-well potential at large distances. We note that the Yukawa potential, which is of greatest interest from the physical point of view, has in principle the same dependence on r as the Coulomb-well potential, but of course there is the difference that at large distances the Yukawa potential falls off more slowly, exponentially. Therefore, for example, the positions of the first levels and some other physical quantities are nearly equal in both potentials. Mathematically, however, the Coulomb-well and the Yukawa potentials are far from equivalent. Whereas the Coulomb-well potential leads to a solution of the Schrödinger equation in analytic form, the solution for the Yukawa potential cannot be expressed in terms of any known special functions. In addition, both potentials (and incidentally any other potential) can claim only an approximate physical description of the interaction. Therefore it is convenient to use the Coulomb-well potential in lieu of the Yukawa potential when describing the interaction in various physical problems. The difference in the physical results obtained on the basis of the two potentials is slight. For example, the conditions for the appearance of the 1S level in these potentials are respectively $gr = -1.65$[8] and $gr = -1.45$. We note that, as follows from the foregoing analysis, the non-analyticity of the Coulomb-well potential introduces no mathematical complications.

We note that the Coulomb-well potential is more diverse in its analytic properties than the square-well potential. For example, at high energies a Coulomb-well potential has Regge poles at the half-integer negative points $\nu = l + 1/2$, unlike the square-wave potential. This is connected with different behavior of the potentials at short distances. At low energies $k \rightarrow 0$ there exist bound states in a Coulomb-well potential, while in a Coulomb potential these bound states go to infinity, owing to the slow decrease of the Coulomb potential as $r \rightarrow \infty$.

We see therefore that consideration of a Coulomb-well potential serves as a good illustration of the general properties of Regge-pole trajectories.

[1] T. Regge, Nuovo Cimento **14**, 951 (1959) and **18**, 947 (1960).

[2] V. Singh, Phys. Rev. **127**, 632 (1962).

[3] C. G. Bollini and J. J. Giambiagi, Nuovo Cimento **26**, 619 (1962) and **28**, 341 (1961).

[4] Patashinskiĭ, Pokrovskiĭ, and Khalatnikov, JETP **43**, 117 (1962) and **44**, 2062 (1963), Soviet Phys. JETP **16**, 84 (1963) and **17**, 1387 (1963).

[5] Arutyunyan, Gol'dman, and Nagorskiĭ, JETP **45**, 246 (1963), Soviet Phys. JETP **18**, 172 (1964).

[6] V. A. Kolkunov, JETP **45**, 1123 (1963), Soviet Phys. JETP **18**, 775 (1964).

[7] V. N. Gribov and I. Ya. Pomeranchuk, JETP **43**, 308 (1962), Soviet Phys. JETP **16**, 220 (1963).

[8] Ahmadzadeh, Burke, and Tate, Preprint UCRL-10216, 1962.

Translated by J. G. Adashko
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