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SCATTERING OF pμ ATOMS BY PROTONS

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Further experimental studies are presented on pμ-atom scattering by protons. These experiments have been carried out to determine the spin state of the pμ atom prior to muon decay or muon capture by the proton. The range distributions of the pμ atoms are obtained with the help of a diffusion chamber at various hydrogen densities and impurity concentrations (carbon, oxygen). An analysis of the distributions and the data on the pμ-atom lifetimes shows that they can be satisfactorily described by choosing for the "effective" cross section the value $\sigma_{p\mu+p} = (1.73 \pm 0.19) \times 10^{-19}\ \mathrm{cm}^2$. The best agreement is attained if it is assumed that the pμ atom is produced with an energy of several eV. An analysis of the data based on familiar theoretical expressions for the scattering cross sections in various spin states of the pμ atom leads to the following conclusions. 1) The cross section for elastic scattering of pμ atoms by protons is $(1.67 \pm 0.30) \times 10^{-19}\ \mathrm{cm}^2$ for the singlet spin state of the pμ atom ($F = 0$). This value is much greater than that predicted by the theory. Such intensive resonance scattering may be due to the existence of a low-energy virtual level in the pμp system. 2) The most probable transition rate from the triplet state of the pμ atom to the singlet state is $\sim 10^{10}\ \mathrm{sec}^{-1}$ (for liquid-hydrogen density) and as a result muon capture by protons in liquid as well as in gaseous hydrogen should proceed in a pμ atomic state with $F = 0$. Only at gas pressures of the order of an atmosphere may the muon depolarization prior to capture be incomplete, owing to inverse transitions which are possible under these conditions. 3) The rate of formation of ppμ mesic molecules in the para state is negligibly small compared to their rate of formation in the ortho state, although it is higher by an order of magnitude than the value calculated by Cohen et al.[4] It is also shown that transition of the muon from the proton to carbon and oxygen nuclei occurs predominantly on the higher Cμ and Oμ atomic orbits and the probability for direct transition to the 1s level is smaller than 3%.

1. INTRODUCTION

In our previous studies of negative muons stopping in a hydrogen diffusion chamber, we observed events with displacements of the start of the decay electron track with respect to the point of stopping of the muon by 0.5—3 mm, and we showed that these displacements are due to the ranges of the neutral pμ atom. On the basis of the measured values of the mean square range in the diffusion approximation we have found the cross section for scattering of pμ atoms by protons

$$p\mu + p \to p\mu + p, \qquad (1)$$

which turned out to be $\sim 10^{-19}\ \mathrm{cm}^2$.[1] According to the theory, the cross section for process (1) should depend strongly on the spin state of the pμ atom (in the ground state the pμ atom can be either in a triplet state with the combined spin of the proton and μ meson, $F = 1$, or in a singlet state with $F = 0$). For a statistical mixture of these states the scattering cross section is $\sigma_{1,0} \sim 10^{-18}\ \mathrm{cm}^2$,[2] while for the singlet state $\sigma_0 \lesssim 10^{-20}\ \mathrm{cm}^2$.[2-4] Since the experimental value of $\sigma_{p\mu+p}$ obtained by us[1] is intermediate between $\sigma_{1,0}$ and σ_0, it was impossible to draw definite conclusions as to the spin state of the pμ atom before the decay of the muon. This fact is particularly important in connection with the problem of studying muon capture by protons in gaseous hydrogen, since the capture probability depends considerably on the spin orientation of the proton and muon.[5]

In the present work we have studied process (1) in more detail with an order of magnitude better statistics. The cross sections were determined from analysis of the distributions of the number of events as a function of pμ-atom range under different operating conditions.

2. EXPERIMENTAL SETUP AND RESULTS

A diffusion chamber in a magnetic field of 7000 G was exposed in a beam of negative mesons with an initial momentum of 260 MeV/c which had been slowed down and stopped in the gas of the chamber. The contamination of π-meson stoppings was 4.5%. Detailed descriptions of the experimental arrangement and of the experimental conditions in the meson beam from the JINR synchrocyclotron have been given previously.[1,6,7]

In order to discover how the observed effects depend on the density of the hydrogen and on impurities with other Z (atoms of carbon and oxygen are always present in the working volume of the chamber in the form of alcohol vapor), we made four experiments. The experimental conditions and the results of identification of the events are listed in Table I. In experiments 1 and 2, methyl alcohol was used as the working liquid. In experiment 3 the density of C and O atoms was reduced by use of normal propyl alcohol.[7] The total density of C and O atoms given in Table I for experiments 1—3 was evaluated at the mean effective height of the sensitive layer. For the conditions of experiment 4 the impurity concentration was sharply increased by addition of carbon dioxide and ethane in a ratio corresponding to the same number of atoms of carbon and oxygen.

A. Lifetime of the pμ Atom

The lifetime τ of the pμ atom depends substantially on the amount of impurity. If we neglect the small contribution of muon capture by deuterons[1,8] (which occurs under the conditions of these experiments), the lifetime τ and its reciprocal λ (the total rate of inelastic processes) will be determined by the expression

$$1/\tau = \lambda = \lambda_0 + \lambda_Z' C_Z + \lambda_{pp\mu}', \qquad (2)$$

where λ_0 is the decay rate of a free muon, $\lambda_Z' C_Z$ is the rate of transfer of the muon to complex nuclei whose concentration is C_Z, and $\lambda_{pp\mu}'$ is the rate of formation of the mesic molecule ppμ in gaseous hydrogen.

In this work we have used a method of determining λ based on comparison of the yield of stars with visible prongs, due to muon capture by carbon and oxygen nuclei, for different total concentrations of these nuclei. Let Y_1 and Y_2 be the yields of stars per stopping muon in experiments with impurity concentrations of C_1 and C_2, respectively. Then, if $C_1 < C_2 < 1$ so that we can neglect the direct transfer of muons to Cμ and Oμ orbits (without transition from a pμ atom), λ will be given (for a concentration C_1) by the expression

$$\lambda = \frac{(\lambda_0 + \lambda_{pp\mu}') Y_2 (1 - C_1/C_2)}{Y_2 - Y_1}. \qquad (3)$$

It should be noted that for the case $C_1 \ll C_2$ which exists in our experimental conditions, if we compare experiments 1—3 with experiment 4, the value of λ is only slightly sensitive to the uncertainty in the impurity concentration.

In sampling the stars we used the following selection criteria: 1) The length of each prong of the star must be greater than 0.4 mg/cm^2 of hydrogen; 2) for single-prong stars with a prong length less than 2 mg/cm^2, the emission angle must be greater than 20°; 3) the track length of the stopping muon must be greater than 2 cm.

Separation of muon stars from pion stars, which were present to the greatest extent in experiment 4, was made on the basis of a relative measurement of the masses of the stopping mesons. The measurements were made in a reprojector by means of templates of variable curvature, prepared according to the parametric equation of the curve for heavy particles stopping in matter in the presence of a magnetic field. For a given density of hydrogen and a constant magnetic field, the equation involves only a parameter α connected with the mass of the particle by the relation $\alpha \sim m^{-0.75}$. Measurements were made only for that part of the events in which the primary track of the stopping particle was greater than 40 mm. In addition, only one- and two-prong stars were measured. This resulted in rejection of about 50% of the pion stars and only 8% of the muon stars.

Figure 1 shows a distribution of the primary

Table I

Experiment number	Hydrogen pressure, atm	Working liquid	Total number of C and O atoms per cm^3 (10^{19})	Number of photographs	Number of muon stoppings	Number of muon stars	Number of $\mu \to e$ decays	Number of events with visible pμ-atom ranges	Final number of events
1	23.2	CH_3OH	0,15±0.05	18 780	2540	28±5	1860	85	997
2	4.8	CH_3OH	0.15±0.05	32 910	2100	31±6	810	226	475
3	4.6	C_3H_7OH	0,03±0,01	31 840	3300	17±4	682	118	182
4	23.0 *	CH_3OH	2,4±0.3	37 800	7050	135±14	4750	—	—

*0.7% C_2H_6 + 1.3% CO_2 were added.

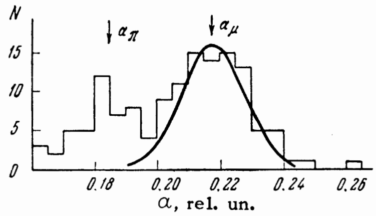

FIG. 1. Distribution of primary tracks in pion and muon stars as a function of the parameter α, for experiment 4. The smooth curve is the resolution function for muon stars.

tracks in stars as a function of the parameter α for experiment 4. Also shown is the resolution function for muon stars, found by measuring muon tracks in $\mu \rightarrow e$ decays. Identification of the stars in experiment 4 showed that of the total number of 211 stars due to particles stopping in the chamber gas, 76 ± 11 are attributed to pion stars.[1)] For experiments 1 and 2 the total contribution of pion and muon stars from direct transfer was 10%, while for experiment 3 it did not exceed 4%. Table I lists the final numbers of muon stars for each experiment.

Using the yield of stars in experiment 4 and the combined yield of stars in experiments 1 and 2, we have found from expression (2) that the average rate of "inelastic" processes in the experiments with methyl alcohol is

$$\lambda = (1.4 \pm 0.3) \cdot 10^6 \text{ sec}^{-1}.$$

The small contribution to λ from formation of the mesic molecule $pp\mu$ was determined on the basis of the data for liquid hydrogen[8,9] by scaling the density.

It follows from the values of λ found in the experiments with methyl alcohol that the rate of transfer of a muon from a proton to C and O nuclei, converted to the density of liquid hydrogen, is

$$\lambda_{\text{C, O}}{}^{p} = (2.2 \pm 0.9) \cdot 10^{10} \text{ sec}^{-1}.$$

This value of $\lambda^{p}_{\text{C,O}}$ is in good agreement with the value found in our earlier experiments.[1,8] For experiment 3 the total rate of inelastic processes is close to the decay rate of a free muon and amounts to

$$\lambda = (0.67 \pm 0.07) \cdot 10^6 \text{ sec}^{-1}.$$

The latter value takes into account the different capture probability of muons by carbon and oxygen nuclei.

The yield of events in which the beginning of the electron track is accompanied by a visible "blob" (Auger electron track)[7] for experiment 3 was roughly two times smaller than for experiments 1 and 2, which we should expect from the values of λ for these experiments.

The results of our experiments also provide a quantitative evaluation of the probabilities of transfer of a muon from a $p\mu$ atom directly to the ground state of a $Z\mu$ atom with release of the proton binding energy, i.e., the probability of the reactions

$$p\mu + \text{C} \rightarrow \text{C}\mu + p + 97 \text{ keV} \tag{4}$$

$$p\mu + \text{O} \rightarrow \text{O}\mu + p + 175 \text{keV}. \tag{5}$$

In Fig. 2 we have shown for the experiments at low hydrogen pressure (experiments 2 and 3) the distribution of the number of events as a function of the size of the "blob," measured in the direction of its greatest extent. The arrows in this figure indicate the expected ranges of protons in reactions (4) and (5). If we consider the efficiency for detecting the protons, we find that the contribution of reactions (4) and (5) to the process of transfer of a muon to a $Z\mu$ atom does not exceed 3%.

B. Distribution in the Projected Ranges of the $p\mu$ Atoms

In the photographs taken during experiments 1—3 we observed events due to the formation and resulting range of a $p\mu$ atom, i.e., events in which the beginning of the decay-electron track was displaced with respect to the point of stopping of the muon. For the subsequent measurements we selected only those events in which the beginning of the decay-electron track was accompanied by a

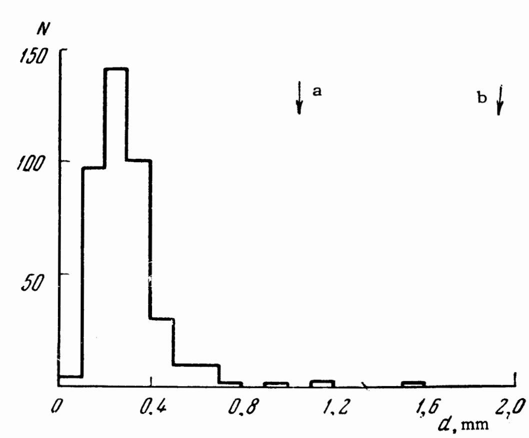

FIG. 2. Distribution of number of events as a function of the size of the "blob" d, in events with visible $p\mu$-atom range. Arrows a and b indicate the expected values of proton ranges in reactions (4) and (5), respectively.

[1)]The yield of pion stars is in good agreement with that expected from the Z law for the probability of direct transfer of a π meson to a shell of a πZ atom.

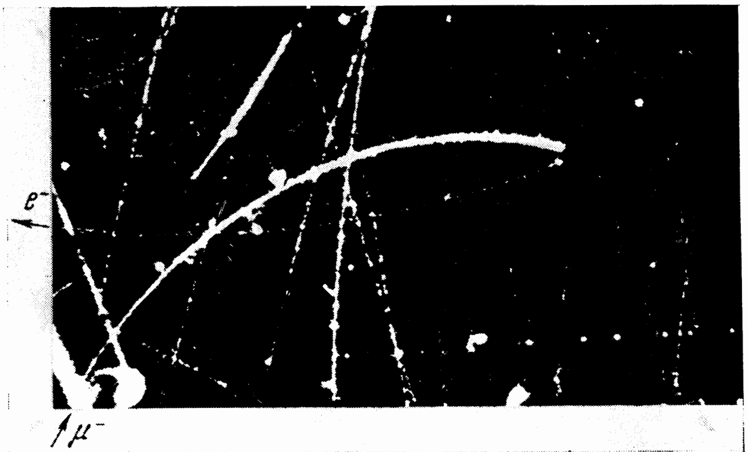

FIG. 3. Photograph of an event with a visible $p\mu$-atom range.

visible "blob" (Auger electron track). A photograph of such an event is shown in Fig. 3. With this selection we rejected roughly 50% of the events with visible ranges (displacements) of $p\mu$ atoms for experiments 1 and 2 and about 70% for experiment 3, but in the latter case the range could be more clearly identified and its length accurately measured.

As in our previous work[1,7] we used a UIM-22 microscope to select the events and measure the projected length l on a horizontal plane of the ranges of the $p\mu$ atoms. A distribution of the events in l was plotted in 0.5 mm intervals. Figure 4 shows the distribution in l for experiment 2, from which we can see that the background of spurious events in the interval 0.5—3 mm is negligibly small. In the distribution in l for experiment 1, in which the deuterium concentration was known and amounted to 0.007%, we took into account the background of events due to formation and range of the $d\mu$ atom. This background amounted to 5% in the interval Δl = 0.5—2 mm.

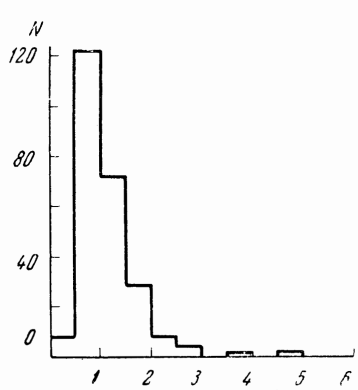

FIG. 4. Distribution of events as a function of $p\mu$-atom range for experiment 2 (without corrections for detection efficiency).

In plotting the distributions we introduced for each interval Δl corrections taking into account the different efficiency for detection of events with different values of l, resulting from the finite width of the muon track. As in our previous work,[7] this was done by measuring the angular characteristics of the event. The corrections amounted on the average to 15% of the total number of detected events. In the interval Δl = 0—0.5 mm we have included also events without visible displacements. The number of such events was estimated on the basis of the total number of $\mu \to e$ decays and the probability of transfer to impurities.

The distributions in l obtained for experiments 1—3 are shown in Fig. 5. The errors for each point include, in addition to the statistical errors, the uncertainties in the corrections. The last column of Table I lists the total number of events for each experiment after introduction of all corrections.

3. ANALYSIS OF THE EXPERIMENTAL DATA

In the analysis we used the results of theoretical calculations of the cross section for process (1).[2-4,10] The cross section for scattering of a $p\mu$ atom by a proton, for a $p\mu$-atom energy considerably greater than the energy of the hyperfine splitting, is given by the expression[2-4]

$$\sigma_{1,0} = 4\pi[{}^3/_4 a_u{}^2 + {}^1/_4 a_g{}^2 / (1 + k^2 a_g{}^2)]. \qquad (6)$$

This cross section is the sum of the cross sections for coherent scattering (without change of spin) and incoherent scattering (with change of spin). The incoherent-scattering cross section, i.e., the cross section for the transition F = 1 → F = 0, has the form[10]

$$\sigma_{1\to 0} = 4\pi \frac{3(a_u - a_g)^2}{16 + k_0{}^2(3a_u + a_g)^2}. \qquad (7)$$

For $p\mu$ atoms in the state F = 0 for mesic atom

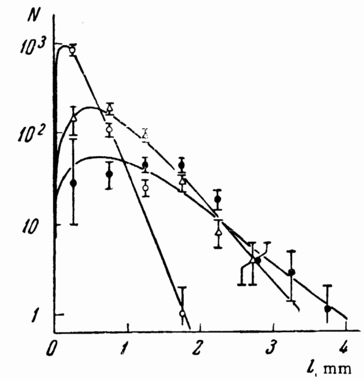

FIG. 5. Experimental distributions of events in range for $p\mu$ atoms: o – in experiment 1, Δ – in experiment 2, • – in experiment 3. The smooth curves are calculated by the Monte-Carlo method with the parameters listed in Table III.

energies $E < \Delta E$ ($\Delta E \approx 0.2$ eV is the energy of the hyperfine splitting), the elastic scattering cross section is

$$\sigma_0 = 4\pi[(3a_u + a_g)/4]^2. \tag{8}$$

In expressions (6)—(8), $k^2 = 2ME/\hbar^2$, $k_0^2 = 2M\Delta E/\hbar^2$, a_g and a_u are the scattering lengths for mesic-atom potentials V_g and V_u, corresponding to symmetric and antisymmetric states of the $p\mu p$ system with respect to exchange of the spatial coordinates of the two protons. For the quantity a_u different authors[3,4] have obtained nearly identical values of ≈ 5 (in mesic-atom units, $\hbar^2/m_\mu e^2 = 2.56 \times 10^{-11}$ cm), while for a_g the computed values vary considerably: $a_g = -17$ [3] and $a_g = -11$.[4]

It is obvious that if the time of transfer from the state $F = 1$ to state $F = 0$ is appreciably less than the lifetime of the $p\mu$ atom, the experimental distributions in $p\mu$-atom ranges will be described only by the one cross-section value for the state with $F = 0$. In the opposite case it is necessary to take into account the contribution of all three scattering processes.

In addition, the analysis must take into account the possibility of the existence in the $p\mu$ atom of an initial energy greater than thermal energy. According to Leon and Bethe[11] and Wightman,[11] after capture of the muon into a high orbital of the $p\mu$ atom (with a principal quantum number $n \geq 14$) part of the excitation energy can go into chemical dissociation of the H_2 molecule, so that in the transition from the initial state to the ground state (after a time of $\sim 10^{-11}$ sec) the $p\mu$ atom acquires a kinetic energy of the order of 1 eV.

It is necessary also to take into account the contribution of the cross section for the process

$$p\mu + Z \rightarrow p\mu + Z. \tag{9}$$

In analysis of the $N(l)$ distributions it was assumed that the cross section for process (9) was equal to the cross section for elastic scattering of $d\mu$ atoms by complex nuclei determined by us earlier[7] and amounting to $(1.2 \pm 0.3) \times 10^{-18}$ cm^2.

On undergoing collisions with protons, the $p\mu$ atom is slowed down to thermal energies. In the low-energy region in which the $p\mu$ atom moves ($\lesssim 1$ eV), the scattering occurs not on free protons but on H_2 molecules. The cross section for the process $p\mu + H_2$, reduced to a per-proton basis, should differ from the cross section for $p\mu + p$ and should depend strongly on energy.

In accordance with what we have said above, the analysis of the experimental distributions was carried out sequentially in several steps:

1) It was assumed that for description of all of the experimental distributions $N(l)$ we can use a single "effective" cross section for a constant velocity of the $p\mu$ atom (the diffusion approximation).

2) The same as in 1) but taking into account the initial velocity of the $p\mu$ atom (the Monte-Carlo method).

3) The analysis of the distributions took into account expressions (6)—(8) in order to determine the contribution of the cross sections $\sigma_{1,0}$ and σ_0.

In all of the steps the calculation was carried out by the method of least squares with an electronic computer, and the distributions were compared for experiments 1, 2, and 3, with appropriate values for the density of protons n_p, the density of complex nuclei n_Z, and the lifetime τ of the $p\mu$ atom.

A. The Diffusion Approximation

In this case it can be shown that the distribution in range r for $p\mu$ atoms has the form

$$N(r) \sim re^{-r/L}, \tag{10}$$

and the distribution in projected range is

$$N(l) \sim lK_0(l/L) \tag{11}$$

($K_0(l/L)$ is a Bessel function). Here L is the diffusion length, defined as

$$L = \left(\frac{1}{3n_p\Sigma}\bar{v}_{p\mu}\tau\right)^{1/2}, \tag{12}$$

where Σ is the "transport cross section":

$$\Sigma = A\sigma_{p\mu+p}(1 - \overline{\cos\vartheta}), \tag{13}$$

$A = \sigma^{\text{bind}}_{p\mu+p}$ is a coefficient taking into account the difference in the cross sections for the processes $p\mu + p$ and $p\mu + H_2$; $\overline{\cos\vartheta}$ is the average cosine of the scattering angle; τ is the lifetime of the $p\mu$ atom; $\bar{v}_{p\mu}$ is the average relative velocity of the $p\mu$ atom and the H_2 molecule, which under our conditions ($T = 240°K$) is 2.7×10^5 cm/sec.

In subsequent calculations we made use of a certain analogy in the behavior of $p\mu$ atoms and neutrons, and transfer from the real $p\mu + H_2$ scattering process to the process $p\mu + p$ was made by the method of Krieger and Nelkin,[12,13] which is one of the approximate methods of calculating the scattering of slow neutrons by molecules. These methods were based on the use of the Fermi pseudopotential. The conditions for applicability of these calculations are fulfilled in our case, since the scattering length of the $p\mu$ atom ($\sim 10^{-10}$ cm) is much less than the wavelength of the $p\mu$ atom and the internuclear distance in the H_2 molecule ($\sim 10^{-8}$ cm). In the method of Krieger and Nelkin the exact quantum-mechanical treatment of all possible transitions between rotational and vibrational levels of the molecule in the scattering process is replaced by a procedure of averaging on the basis of the mass-tensor approximation and the approximation of short scattering times. Such calculations are legitimate in the region where the neutron energies are large in comparison with the distance between the rotational levels of the molecule (0.01 eV) but small in comparison with the excitation energy of the vibrational levels (~ 1 eV); in this energy region the results of such calculations agree very well with experiment.

The specified condition for the energies is fulfilled in the present experiments, since the thermal motion energy of a $p\mu$ atom, averaged over a Maxwellian spectrum, amounts to 0.03 eV for $T = 240°K$. In view of the fact that at these temperatures some rotational transitions take part in the scattering, in the calculations we have neglected correlations (due to the Pauli principle) between the rotational state of the molecule and its spin value (we do not distinguish between scattering by ortho and para hydrogen). Following the work of Krieger and Nelkin,[12,13] we can obtain the expression for the differential scattering cross section of $p\mu$ atoms by bound protons for the singlet state of the $p\mu$ atom in the form

$$\left[\frac{d\sigma(\vartheta)}{d\Omega}\right]^{\text{bind}}_{p\mu+p} = \sigma_{p\mu+p}\frac{4.25}{\pi E_0^{1/2}}\int\limits_{-E_0}^{\infty} d\varepsilon\left\{\left[\frac{\varepsilon+E_0}{q}\right]^{1/2} e^{-0.62q^2} \times \exp\left[-\frac{14.4}{q^2}(\varepsilon+0.84q^2)^2\right]\right\}, \tag{14}$$

where $\epsilon = E - E_0$ and $q = |\mathbf{k} - \mathbf{k}_0|$ is the change of energy and momentum of the $p\mu$ atom in the collision ($E_0 = 0.03$ eV, $k = \sqrt{2ME}/\hbar$); the numerical coefficients given in (14) are calculated for the present experimental conditions. Equation (14) was obtained by assuming a Maxwellian motion of the H_2 molecules and is expressed in the laboratory system of coordinates. Numerical integration of (14) over angle gives $\sigma^{\text{bind}}_{p\mu+p} = 1.6\ \sigma_{p\mu+p}$, i.e., $A = 1.6$. The expression for the differential cross section for scattering in the triplet spin state is somewhat more complicated; however, estimates show that the value of A changes by not more than 7%. Using (14), we can easily calculate the average cosine of the scattering angle

$$\overline{\cos\vartheta} = \int\limits_0 \cos\vartheta\frac{d\sigma(\vartheta)}{d\Omega}d\vartheta \bigg/ \int\limits_0 \frac{d\sigma(\vartheta)}{d\Omega}d\vartheta. \tag{15}$$

It turns out to be 0.40.

In approximating the experimental distribution in l by function (11) by the method of least squares, it was found that $\Sigma = (1.21 \pm 0.17) \times 10^{-19}$ cm^2. The value of $\sigma_{p\mu+p}$ determined from relation (13) is listed in the first line of Table II. Here is also listed the value of χ^2, from which we can see that it is considerably larger than the expected value of χ^2, which is 17 (18 experimental points, one parameter being varied). Thus, the simple diffusion approximation agrees poorly with the experimental data.

B. The Monte-Carlo Method

In this step of the analysis it was assumed that the $p\mu$ atom has an initial energy $E_{\text{init}} = 1$ eV. In this case the distribution functions of projected ranges of the mesic atoms were calculated by the Monte-Carlo method, as we had done previously[7] for calculating the ranges of $d\mu$ atoms. In the mesic-atom energy region $E_{\text{therm}} < E_{p\mu} \le E_{\text{init}}$ it was assumed that the $p\mu$ atom is scattered by free protons. In this case the angular distribution was taken as isotropic in the center-of-mass system. For thermal energies of the mesic atom it was assumed that the cross section increases by a factor of 1.6, and the angular distribution in the laboratory system was taken in the form of (14), i.e., the results of the Krieger-Nelkin calculation were used. Comparison of the theoretical distributions with the experimental data was done by a χ^2 analysis similar to that used by us earlier.[7] Table II lists the value of $\sigma_{p\mu+p}$ obtained and the value of χ^2_{min}, which in this case is close to the expected value. Thus, we can conclude that the experimental data are well described in the approximation of a single cross section. We wish to emphasize that this occurs only if we take into account the existence in the $p\mu$ atom of an initial

Table II

Variant	Results	χ^2_{min} ($\overline{\chi^2} = 17$)
Diffusion approximation	$\sigma_{p\mu+p} = (1.26 \pm 0.18) \cdot 10^{-19}$ cm^2	38
Monte-Carlo method. Approximation of one cross section	$\sigma_{p\mu+p} = (1.73 \pm 0.19) \cdot 10^{-19}$ cm^2	24
Monte-Carlo method. Inclusion of pμ-atom spin states	$a_g = +(3 \pm 2)$ (mesic-atom units)	25
	$a_g = -(33 \pm 2)$ (mesic-atom units)	22

energy considerably larger than the thermal energy.

C. Determination of the Scattering Length a_g

The further analysis consisted of determining the contribution of the scattering cross sections $\sigma_{1,0}$ and σ_0 to the measured cross section $\sigma_{p\mu+p}$. For this purpose we used the dependence of the cross sections $\sigma_{1,0}$, $\sigma_{1\to 0}$, and σ_0 on the scattering lengths, determined by expressions (6)—(8). If we consider the value of the scattering length a_u as known from theory and given by $a_u = 5$,[3,4] then all three cross sections will be functions of the single quantity a_g.

Figure 6 shows the dependence of the quantity a_g on the cross sections $\sigma_{1,0}$ and σ_0, and also on the function $\gamma = \sigma_{1\to 0}/\sigma_{1,0}$, which represents the probability of the transition of a pμ atom from the triplet state to the singlet state in one collision. All the cross sections are given for an energy of 1 eV. In this figure the straight line parallel to the abscissa represents the experimental cross section $\sigma_{p\mu+p}$ obtained from analysis of the distributions by the Monte-Carlo method. It is evident that only two values of the scattering length a_g correspond to the experimental value of $\sigma_{p\mu+p}$: one near $a_g \approx +5$, the other near $a_g \approx -35$.

A more accurate determination of the possible values of a_g was carried out by the Monte-Carlo method with the program described earlier for a determination of $\sigma_{p\mu+p}$, but including simulation of the transition between the spin states of the pμ atom. Analysis of the experimental distributions by the method of least squares gives in this case two values: $a_g = +(3 \pm 1)$ and $a_g = -(33 \pm 1)$. Figure 7 shows the behavior of χ^2 as a function of a_g, which illustrates the existence of the two solutions.

We carried out also an additional analysis of the values found for a_g, consisting of variation of the initial energy specified in the Monte-Carlo method in calculation of the range distributions. The initial energy was varied between 0.03 eV and 5 eV. The final values of a_g taking into account the uncertainty due to inexact knowledge of the initial energy are listed in Table II. The corresponding values of χ^2 are also given.

4. RESULTS OF ANALYSIS AND DISCUSSION

Table II summarizes the results of the analysis of experimental distributions on ranges of pμ atoms. As we have already noted, the distributions measured for different hydrogen densities can be described, in a simultaneous analysis of all the data, with use of only one "effective" scattering cross section. Considerably better agreement with experiment is achieved here if we assume that the pμ atom acquires in its formation a kinetic energy

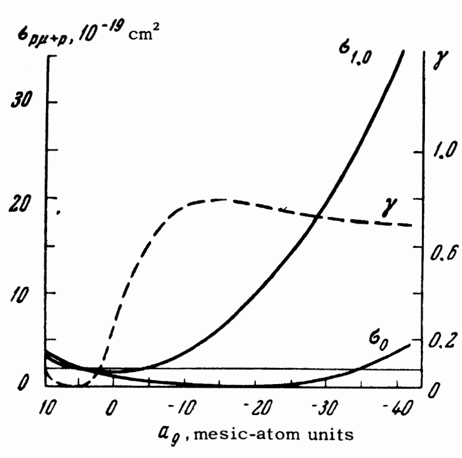

FIG. 6. The cross sections $\sigma_{1,0}$ and σ_0 and the parameter $\gamma = \sigma_{1\to 0}/\sigma_{1,0}$ as a function of the scattering length a_g.

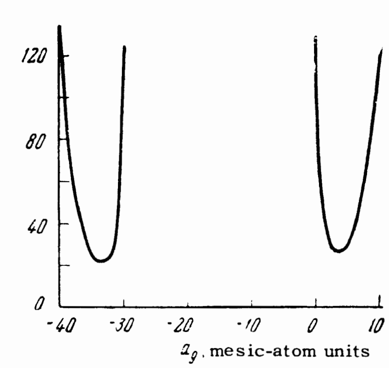

FIG. 7. Plot of χ^2 for the l distributions of experiments 1–3, as a function of the scattering length a_g, computed by the Monte-Carlo method.

Table III

Number of experiment	Parameters corresponding to minimum χ^2: $n_p\sigma_{p\mu+p}$, cm^{-1}*	$n_Z\sigma_{p\mu+Z}$, cm^{-1}	λ, 10^6 sec^{-1}	$\bar{v}_{p\mu}$, 10^5 cm/sec	Average number of collisions	Number of trials	χ^2_{min}
1	360	1.80	1.40	2.88	84.5	3000	5.3
2	75	1.80	1.40	3.36	16,8	1500	3.5
3	72	0.36	0,67	2,94	30,4	1000	13.4

*Here the cross section $\sigma_{p\mu+p}$ is the scattering cross section in the singlet state of the pμ atom at thermal energy, i.e., $\sigma_{p\mu+p} = 1.6\,\sigma_0$.

of several electron volts. It must be noted that the latter fact agrees with the theoretical discussion of Wightman and of Bethe and Leon[11] of the initial stage of formation of mesic hydrogen atoms, in which the consideration of the initial velocity of the order of 10^6 cm/sec turns out to be important for interpretation of the experimental data on capture of π^- and K^- mesons by protons or deuterons. The existence of an initial velocity of the pμ atom different from the velocity of the thermal motion has the result that at low hydrogen density (5 atm) the pμ atom does not succeed in being thermalized during its lifetime. This explains the fact that the values of the cross section $\sigma_{p\mu+p}$ found previously by us[1] with the diffusion-approximation method were a function of the hydrogen density.

In Table III, along with the parameters corresponding to the minimum χ^2, are listed also the average velocity and average number of collisions of the pμ atom with H_2 molecules, obtained by the Monte-Carlo method.

The fact that the distributions in l are described by a single value of the scattering cross section means either that the rate of transition from the triplet state to the singlet state is very great and the measured cross section is the scattering cross section in the state F = 0, or that these transitions are almost completely absent. The same fact shows up in that, for a fixed scattering length $a_u = +5$, two values of the scattering length a_g equally well fit the experimental data (Table II). Corresponding to this we obtained two sets of cross sections determined from the two different scattering lengths a_g and the single fixed scattering length a_u. These two sets are listed in Table IV, which also lists the theoretical results of Gershteĭn and Zel'dovich[2,3] and Cohen et al.[4]. (The scattering length a_g and the corresponding cross sections for the work of Cohen et al.[4] were determined by us from the scattering phase shifts listed in their papers, for a pμ-atom energy less than the energy of the hyperfine splitting.) The last line of Table IV lists the transition rate of the pμ atom from the state F = 1 to the state F = 0 for liquid-hydrogen density, calculated from the relation $W_{1\to 0} = \sigma_{1\to 0} n_p \bar{v}_{p\mu}$ for the corresponding values of the scattering lengths.

The problem arises as to which of the sets has the greater basis for existence. On the basis of the discussion given below, we have been able to decide this question in favor of the first set.

As we have indicated previously,[1] the discrepancy in the theoretical values of the scattering length a_g can be caused by the existence in the pμp system for a potential V_g of a virtual level with an energy close to zero. Under these conditions the scattering has a resonance nature and the value of the scattering length a_g depends strongly on the variation of the parameters of the potential V_g and consequently on the accuracy and rigor of the theoretical calculations of this potential, which have been carried out with a number of approximations.[3,4] It is sufficient to state that in the Morse function approximating the potential V_g

$$U = U_0\left(e^{-2b(R-R_0)} - 2e^{-b(R-R_0)}\right)$$

it is necessary to increase by only 5% the depth of the potential well U_0 or to change the parameter b = 0.67 to b = 0.65 in order to change the value of a_g from −17 to −33. From this point of view it is quite possible that the value $a_g = -33$ found on

Table IV

Quantity	Experiment: Solution for $a_g = -(33\pm2)$	Experiment: Solution for $a_g = +(3\pm2)$	Theory: Refs. 2 and 3, $a_g = -17$	Theory: Ref. 4, $a_g = -11$
$\sigma_{1,0}$, cm^2	$(24\pm5)\cdot10^{-19}$	$(1.74\pm0.30)\cdot10^{-19}$	$6\cdot10^{-19}$	$3,5\cdot10^{-19}$
σ_0, cm^2	$(1,67\pm0.30)\cdot10^{-19}$	$(1,61\pm0.30)\cdot10^{-19}$	$2\cdot10^{-21}$	$8\cdot10^{-21}$
$\sigma_{1\to0}$, cm^2	$(16,8\pm3.0)\cdot10^{-19}$	$(4.9\pm1.3)\cdot10^{-21}$	—	—
$W_{1\to0}$, sec^{-1}	$(1,6\pm0.3)\cdot10^{10}$	$(2.0\pm0.5)\cdot10^{7}$	$5\cdot10^{9}$	$2\cdot10^{9}$

the basis of the experimental data and both theoretical values lie within the limits determined by the accuracy of the theoretical calculations. It should be noted that these calculations have usually been carried out in first-order perturbation theory in the parameter m_μ/m_p. Inclusion of higher orders should lead to reduction of the principal term and consequently to improved agreement with experiment. We note also that the more exact calculations recently completed by Wessel and Phillipson[14] by the variation method of the levels in the $p\mu p$ system lead to increase of the well depth U_0 in comparison with preceding calculations.

On the other hand, the value $a_g = +3$ requires not only an extremely substantial change of the potential V_g (such that the parameter b changes from 0.67 to 0.40) but also the assumption of the existence in the $p\mu p$ system of a bound level (with an energy of about 90 eV), which theoretically is extremely improbable.

Thus, the scattering length value $a_g = -33$ does not contradict theory but is a direct reflection of the fact that the scattering in the $p\mu p$ system has a strongly resonant nature.

From this arises the following conclusion of our work, which is important for understanding of the mechanism of muon capture by protons: Since for the value $a_g = -33$ the transition rate of the $p\mu$ atom from the state with the total spin $F = 1$ to the state with $F = 0$ turns out to be large, $W_{1\to 0} = (1.6 \pm 0.3) \times 10^{10}\ \text{sec}^{-1}$ (see Table IV), complete depolarization of muons should be observed not only in liquid hydrogen but also in gaseous hydrogen at pressures of the order of 20 atm, and therefore muon capture by protons should occur practically always from the state with $F = 0$. Only at pressures of the order of an atmosphere and below can the depolarization be incomplete, since the inverse transitions will be possible in that case as the result of the presence in the $p\mu$ atom of an initial velocity and as the result of the small number of collisions.

It should be emphasized that this conclusion does not contradict the well known experimental data on measurement of muon depolarization in liquid hydrogen[15] and on determination of the probability of muon capture from the $pp\mu$-molecule state.[16]

The existence in the $p\mu p$ system close to zero of a virtual level and an intense resonance scattering could lead to a substantial increase in the rate of formation of mesic $pp\mu$ molecules in the para state.[3,17] Cohen et al.[4] give an estimate of the value of this rate for liquid-hydrogen density and show that its order of magnitude is $10^2\ \text{sec}^{-1}$. The estimate was based on a scattering length $a_g = -11$. It can also be shown that the rate of formation of $pp\mu$ molecules by E0 transition (electric monopole) to a state with zero momentum will be proportional to a_g^2 (for $a_g \gg 1$). Under these conditions replacement of the theoretical value $a_g = -11$ by the experimental value $a_g = -33$ involves an increase in the rate of formation of mesic molecules in the para state altogether by about an order of magnitude, i.e., up to $10^3\ \text{sec}^{-1}$, which nevertheless is negligibly small in comparison with the rate of formation of molecules in the ortho state. The latter, according to theory [3-4] and experiment,[8-9] amounts to about $2 \times 10^6\ \text{sec}^{-1}$.

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