RESISTANCE OF THIN PLATES AND WIRES IN A STRONG MAGNETIC FIELD

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A method is presented for obtaining an exact solution of the problem of the distribution of current and field in a wire of arbitrary shape in a strong magnetic field for any arrangement and shape of the contacts. The mean free path of the conduction electrons is assumed to be infinite. For a plane parallel plate an exact solution of this same problem is given without making any assumptions about the magnitude of the magnetic field. It is shown that in general the resistance is very sensitive to the nature of the contacts. It becomes infinite in the case of point contacts; at the same time, the ratio of the potential differences near the contacts (infinitely close to them in the limit) to the current strength is stable with respect to the type of contact. As the shape and size of the contacts are varied, the dependence of the resistance on the strong magnetic field changes from saturation to a quadratic growth. A static skin effect (damping of the current in the bulk of the sample) occurs in the strong magnetic field. The whole current is localized in a layer whose thickness if of the order of the Larmor radius. For a symmetric arrangement of contacts, the static skin effect appears only in conductors with equal numbers of holes and electrons, and does not affect the dependence of the resistance on the magnetic field.

1. INTRODUCTION

THE present paper (for the first time in the literature) presents a method for exact solution of the problem of the resistance of a thin wire in the case of infinite mean free path l of the conduction electrons [1)] for arbitrary cross section, arbitrary (generally, asymmetric) orientation of the magnetic field and arrangement of contacts with appropriate current leads (the finite ratio of wire diameter to mean free path can be taken into account without difficulty). It turns out that even the character of the dependence of the resistance on the magnetic field and its order of magnitude change when one changes the relative values of the linear dimensions of the contacts, the Larmor radius r, the size of the sample, and the distance between contacts. In particular, if the dimensions of the contacts are small compared to r, the formulas for the resistance are "unstable," the resistance is determined mainly by the area of the contacts and, if they are point contacts, the resistance is infinite (cf. the end of this section).

An extremely important point in getting the solution was to write correctly the complete system of equations (which are essentially different in the microscopic and phenomenological theories) and the boundary conditions for the distribution function of the conduction electrons. En passant it was necessary to clarify the degree of accuracy at which it makes sense to consider the resistance as the quantity which permits one uniquely (for a given conductor) to determine, for example, the Joule heat from the difference in potential across the contacts.

[1)]The proposed method enables one to obtain the solution also for finite mean free paths l (cf. Sec. 4). This will be done in a report dealing with bulk samples, where $l \neq \infty$ must be taken into account. For this same reason the problem of a strong magnetic field parallel to the surface of the plate will be treated there.

2. FORMULATION OF THE PROBLEM

We shall be interested in the resistance of a bounded metal sample, i.e., the relation between the current strength J flowing through the sample and the difference of potential of the points of cur-

rent feed.[2)]

This problem splits into two parts. First it is necessary to find the connection between the potential φ and the current density $\mathbf{j}$ and the density of uncompensated charge ρ', which is expressed in well known fashion in terms of the electron distribution function $n(\mathbf{r}, \mathbf{p})$. For the determination of the distribution function we have the kinetic equation, which in the absence of collisions is exact.

The boundary condition for the kinetic equation is determined by the character of the reflection of the electrons from the metal surface. In good metals (with a number of free electrons on the order of one per atom) the reflection is diffuse, (i.e., the distribution function for the electrons reflected at the wire surface) is independent of the angle of emergence of the electrons. Since in good metals the only important electrons are those with energy ϵ close to the limiting Fermi energy ϵ_0, the distribution function for the reflected electrons can depend only on the coordinates of points $\mathbf{r}_S$ on the surface of the metal.

It is always assumed in the literature that the distribution function for the electrons reflected at the surface is simply the equilibrium Fermi function $n_0(\epsilon)$. (In particular, the solution obtained by Chambers[1] corresponds to this condition.) This is however incorrect in the general case, since it does not take into account the difference in the numbers of electrons moving along and opposite to the electric field, so that the number of electrons impinging on the surface (which, because of conservation of charge, is also the number of electrons reflected from the surface) can in external fields differ from its equilibrium value. Thus at the surface of a metal the distribution function for the reflected electrons should be given in terms of the distribution function for the electrons striking the surface by using the law of conservation of charge. The law of conservation of charge obviously states that the current density normal to the surface, j_n^S, is equal to the current density $i(\mathbf{r}_S)$ entering at the point through the contact:

$$j_n^S = i(\mathbf{r}_S), \tag{1}$$

where $\mathbf{n}$ is the internal normal to the surface, and the symbol S will be used to indicate that the corresponding quantity is taken on the surface of the conductor. Away from currents with input current leads, $j_n^S = 0$. (Equation (1) is the familiar continuity condition at the surface for the normal component of the current density.)

Thus the kinetic equation automatically guarantees conservation of charge in the interior of the metal while the boundary condition guarantees its conservation at the surface.

The second part of the problem is to determine the distribution of the potential φ in the conductor. For this purpose we have the equation

$$\operatorname{div} \mathbf{D} = 4\pi\rho' \tag{2}$$

with a boundary condition for the electric induction $\mathbf{D}$. For conductors, however, this equation simplifies. From the definition of ρ' it is clear that $\rho' = e^2\nu\hat{L}\varphi$, where $\nu \sim n/\epsilon_1$ is a characteristic density of states (n is the charge density and ϵ_1 is a characteristic energy), while $\hat{L}$ is a linear operator, $\hat{L}\varphi \sim \varphi$. This estimate can be obtained, for example, by comparing the definitions of ρ' and $\mathbf{j}$, (cf. also formulas (5) and (12) of this paper). Since $\operatorname{div} \mathbf{D} \sim \Delta\varphi$, if φ changes over distances of order λ, it follows from (12) that $\hat{L}\varphi \sim (\lambda_0/\lambda)^2\varphi$, where $\lambda_0 = (e\sqrt{4\pi\nu})^{-1}$. In good metals with a number of electrons of order one per atom, λ_0 is of the order of the interatomic spacing; in semimetals like bismuth $\lambda_0 \sim 10^{-7}$—10^{-6} cm, in semiconductors, even at such low temperatures that degeneracy becomes important, for $n \sim 10^{12}$ cm^{-3} and an effective mass $m \sim 10^{-29}$ g, $\lambda_0 \sim 10^{-6}$ cm (since $\lambda_0 \sim m^{-1/2}n^{-1/6}$, its dependence on n is very weak). This means that, in using the kinetic equation, keeping the right hand side is an unjustified increase in accuracy (even in the region where one "formulates" a boundary condition for the distribution function), so Eq. (2) reduces to $\hat{L}\varphi = 0$. The latter equation is formally equivalent to the equation

$$\rho' = 0, \tag{3}$$

which will serve for the determination of the potential φ. (In variable fields the replacement of (2) by (3) is equivalent to neglecting the displacement current.)

It should be emphasized that ρ' differs from the really very small quantity $\hat{L}\varphi$ by a large factor, so to define it one must, after obtaining the field strength from (3), make use of Eq. (2). (This corresponds to successive approximations in the solution of (2).)

Replacement of (2) by (3) also has a fundamental significance. The point is that λ_0 obviously determines the depth at which external fields are damped

[2)] We recall that in constant fields, curl $\mathbf{E} = 0$ and the electric field strength is $\mathbf{E} = -\nabla\varphi$. The equation curl $\mathbf{H}_1 = 4\pi\mathbf{j}/c$ serves to determine the weak magnetic field $\mathbf{H}_1$ produced by the current. The continuity equation (which should be satisfied automatically in a microscopic theory) can be regarded as the condition for the existence of a solution of the inhomogeneous equation when the corresponding homogeneous equation curl $\mathbf{H}_1 = 0$ has a nontrivial solution.

out. This means that it is only to an accuracy determined by the ratio λ_0/λ that we can speak of the resistance as an internal characteristic, independent of the external fields. (For example, by placing the conductor between the plates of a capacitor and varying the applied potential, one can to this accuracy change the distribution of Joule heat in the conductor for a given current.) Mathematically, having determined ρ' inside the sample as a function of φ and the external currents at the surface, we come from (2) to the Poisson equation, for whose unique solution we have to assign the potential on the surface. Thus a complete solution requires either simultaneous assignment on the surface of the conductor of the "outside" current density and potential, or the "output" outside the conductor and the treatment of the internal problem.[3)]

We note that there is a difference in principle between the equations of the phenomenological and microscopic theories. In the first case the relation between $\mathbf{j}$ and $\nabla\varphi$ is given; the basic equation is div $\mathbf{j} = 0$ with the boundary condition $j_n^S = i(\mathbf{r}_S)$ and ρ' determined by the formula $\rho' = (4\pi)^{-1}$ div $\mathbf{D}$. In the second case the relations div $\mathbf{j} = 0$ and $\mathbf{j}^S = i(\mathbf{r}_S)$ are satisfied automatically, the fundamental equation for determining φ is $\rho'\{\varphi\} = 0$, while ρ' is again defined in terms of φ by formula (2).

We now proceed to the solution of the problem. We write the distribution function for the conduction electrons in the form

$$n(\mathbf{r}, \mathbf{p}) = n_0(\varepsilon) - \frac{\partial n_0}{\partial \varepsilon} e\psi(\mathbf{r}, \mathbf{p}) \approx n_0(\varepsilon) + \delta(\varepsilon - \varepsilon_0) e\psi(\mathbf{r}, \mathbf{p}), \tag{4}$$

where $\delta(\epsilon - \epsilon_0)$ is a delta function, and e is the charge of the electron. (All the further arguments can be taken over for semiconductors; the only change is that one does not replace $\partial n_0/\partial\epsilon$ by the δ function.) Then in terms of the variables ϵ, p_z (z is the direction of the constant magnetic field H, while $\mathbf{p}$ is the quasimomentum of the conduction electron) and t (the period of the electron in its orbit) the current density and charge have the form

$$\mathbf{j} = \frac{2e^3H}{ch^3}\iint dp_z\, dt\, \mathbf{v}\psi \equiv \langle \mathbf{v}\psi\rangle = [\langle\langle \mathbf{v}\psi\rangle\rangle],$$

$$\rho' = \frac{2e^3H}{ch^3}\int\int \psi\, dp_z dt \equiv \langle\psi\rangle \tag{5}$$

(the symbol $\langle\langle \ldots \rangle\rangle$ denotes a time average), while the kinetic equation, in the approximation linear in the electric field, is written as

$$\frac{\partial\psi}{\partial\mathbf{r}}\mathbf{v} + \frac{\partial\psi}{\partial t} = \mathbf{vE} = -\mathbf{v}\frac{\partial\varphi}{\partial\mathbf{r}}, \tag{6}$$

where $\mathbf{v} = \partial\epsilon/\partial\mathbf{p}$ is the electron velocity. (Equation (6), because of (5), automatically guarantees div $\mathbf{j} = 0$.)

We write the boundary condition for Eq. (6). According to (1) and (5),

$$j_n^S = \langle v_n\psi^S\rangle \equiv \langle v_n\psi^S\rangle_+ + \langle v_n\psi^S\rangle_- = i(\mathbf{r}_S),$$

where the plus and minus subscripts denote integration over electron states with $v_n > 0$ and $v_n < 0$, respectively. According to the condition for diffuseness of the reflection (cf. the beginning of this section) ψ^S for $v_n > 0$ depends only on $\mathbf{r}_S$ and can be removed from the brackets; as a result we get the boundary condition (which automatically guarantees diffuse reflection):

$$\psi^S|_{v_n>0} = \frac{\langle v_n\psi^S\rangle_-}{\langle v_n\rangle_-} + \frac{i(\mathbf{r}_S)}{\langle v_n\rangle_+}. \tag{7}$$

(Here we have used the fact that $\langle v_n\rangle = 0$ because of the central symmetry of the dispersion law: $\epsilon(-\mathbf{p}) = \epsilon(\mathbf{p})$.)

The general solution of (6) has the form

$$\psi(\mathbf{r}, \mathbf{p}) = f(\mathbf{r} - \mathbf{r}(t);\ p_z) - \varphi(\mathbf{r}), \quad \mathbf{r}(t) = \int^t \mathbf{v}(t')\,dt', \tag{8}$$

where f is an arbitrary function of its arguments. According to (8) and (7), $f(\mathbf{r}_S - \mathbf{r}(t))|_{v_n>0}$ depends only on $\mathbf{r}_S$ (i.e., is constant along the curve $\boldsymbol{\rho} = \mathbf{r}_S - \mathbf{r}(t)$).

We use the notation

$$f(\mathbf{r}_S - \mathbf{r}(t))_{v_n(t)>0} = h(\mathbf{r}_S). \tag{9}$$

Relation (9), which is a consequence only of the isotropic reflection from the surface, enables one to express $f(\mathbf{r} - \mathbf{r}(t))$ at any point inside the metal in terms of the function h:

$$f(\mathbf{r} - \mathbf{r}(t)) \equiv f(\boldsymbol{\rho}_S - \mathbf{r}(\lambda))|_{v_n(\lambda)>0} = h(\boldsymbol{\rho}_S), \quad \boldsymbol{\rho}_S = \boldsymbol{\rho}_S(\mathbf{r}, \mathbf{p}), \tag{10}$$

where λ is the instant of reflection of the electron by the surface, given by the equations

$$\mathbf{r} - \mathbf{r}(t) = \boldsymbol{\rho}_S - \mathbf{r}(\lambda),$$

$$\mathcal{G}(\boldsymbol{\rho}_S) = 0, \quad \mathbf{v}(\lambda)\mathbf{n}(\boldsymbol{\rho}_S) > 0, \quad \lambda < t; \tag{11}$$

$\mathcal{G}(\boldsymbol{\rho}_S)$ is the equation of the surface of the conductor.

In order to stay at all times in the interior of the conductor (only there is the problem defined),

[3)] In particular, in the absence of external fields (aside from an external emf) for $\mathbf{D} = \mathbf{E}$, because of the continuity of φ and $\partial\varphi/\partial n$ which follows from the Maxwell equations, to define φ it is sufficient to require the vanishing of φ (satisfying the equation $\Delta\varphi = 4\pi\rho'$) at infinity. At the same time $\lim_{\lambda_0\to 0} \lambda_0\partial\varphi/\partial n$ is finite.

we should choose for λ the largest root of (11). The solution of (11) exists and is unique under these conditions; actually it corresponds to the first intersection of the surface of the conductor (at point ρ_S at instant λ) by the trajectory of the electron going from the surface at point $\mathbf{r}$ at instant t.

Thus we find from (8) and (10):

$$\psi(\mathbf{r}, \mathbf{p}) = h(\rho_S(\mathbf{r}, \mathbf{p})) - \varphi(\mathbf{r}). \tag{12}$$

Substituting (12) in (7) and using the definition of ρ_S, we get an equation for determining the function $h(\mathbf{r}_S)$:

$$h(\mathbf{r}_S) - \frac{\langle \mathbf{v}(\mathbf{p})\mathbf{n}(\mathbf{r}_S) h(\mathbf{r}_S + \mathbf{r}(\lambda_S) - \mathbf{r}(t))\rangle_-}{\langle \mathbf{v}(\mathbf{p})\mathbf{n}(\mathbf{r}_S)\rangle_-} = \frac{i(\mathbf{r}_S)}{\langle \mathbf{v}(\mathbf{p})\mathbf{n}(\mathbf{r}_S)\rangle_+}; \tag{13}$$

$$\mathcal{G}(\mathbf{r}_S + \mathbf{r}(\lambda_S) - \mathbf{r}(t)) = 0, \qquad \lambda_S = \lambda_S^{max}(\mathbf{r}_S, \mathbf{p}) \leqslant t,$$
$$\mathbf{v}(\lambda_S)\mathbf{n}(\mathbf{r}_S(\lambda_S) - \mathbf{r}(t)) > 0. \tag{14}$$

Finally, Eqs. (3) and (5), allow us, after substituting the function (13) in them, to relate the potential and current density at any point $\mathbf{r}$ to the function $h(\mathbf{r}_S)$:

$$\varphi(\mathbf{r}) = \langle h(\mathbf{r} + \mathbf{r}(\lambda) - \mathbf{r}(t))\rangle / \langle 1\rangle, \tag{15}$$

$$\mathbf{j}(\mathbf{r}) = \langle \mathbf{v} h(\mathbf{r} + \mathbf{r}(\lambda) - \mathbf{r}(t))\rangle, \tag{16}$$

where

$$\lambda = \lambda^{max}(\mathbf{r}, \mathbf{p}) \leqslant t, \qquad \mathcal{G}(\mathbf{r} + \mathbf{r}(\lambda) - \mathbf{r}(t)) = 0,$$
$$\mathbf{v}(\lambda)\mathbf{n}(\mathbf{r} + \mathbf{r}(\lambda) - \mathbf{r}(t)) > 0. \tag{17}$$

The homogeneous equation corresponding to (13) has the nontrivial solution $h(\mathbf{r}_S) = \text{const}$. But this solution is orthogonal to the right hand side (because the total current entering the conductor is equal to the total outgoing current), so the solution of the inhomogeneous equation (13) is determined to within an additive constant—a solution of the homogeneous equation, which is natural, since only the difference of potentials has a physical significance.

We shall now prove (and make use of in our calculation) the identical nature of the singularities of the functions $\varphi(\mathbf{r}_S)$ and $i(\mathbf{r}_S)$. Let us set

$$h(\mathbf{r}_S) = i(\mathbf{r}_S) + g(\mathbf{r}_S) \tag{18}$$

in Eq. (13). We find

$$g(\mathbf{r}_S) - \frac{\langle \mathbf{v}(\mathbf{p})\mathbf{n}(\mathbf{r}_S) i(\mathbf{r}_S + \mathbf{r}(\lambda_S) - \mathbf{r}(t))\rangle_-}{\langle \mathbf{v}(\mathbf{p})\mathbf{n}(\mathbf{r}_S)\rangle_-} = \frac{\langle \mathbf{v}(\mathbf{p})\mathbf{n}(\mathbf{r}_S) i(\mathbf{r}_S + \mathbf{r}(\lambda_S) - \mathbf{r}(t))\rangle_-}{\langle \mathbf{v}(\mathbf{p})\mathbf{n}(\mathbf{r}_S)\rangle_-} \tag{19}$$

and $h(\mathbf{r}_S)$, according to (18), has the same singularities as $i(\mathbf{r}_S)$, whereas the right side of (19), because of the double integration, has a much weaker singularity than $i(\mathbf{r}_S)$. Repeating a substitution analogous to (18), i.e., setting the unknown function equal to the right hand side plus an unknown correction, one can weaken the singularity to an arbitrary extent by such corrections.

According to (15) the potential on the surface is

$$\varphi(\mathbf{r}_S) = {}^1/_2\{h(\mathbf{r}_S) + \langle h(\mathbf{r}_S + \mathbf{r}(\lambda_S) - \mathbf{r}(t))\rangle_- / \langle 1\rangle_-\} \tag{20}$$

and also has the same singularity as $i(\mathbf{r}_S)$ [cf. (18)]. This means that if the contacts are pointlike and the current $i \to \pm\infty$ at the feed points, a finite current strength corresponds to an infinite potential difference, i.e., to an infinite resistance.

3. TRANSFORMATION OF THE FUNDAMENTAL EQUATION

Thus, according to Sec. 2 the problem reduces to solving Eqs. (13) and (14) for the function $h(\mathbf{r}_S)$, given on the surface of the conductor, and permitting one, by using (15)—(17) to compute the current density and potential. If λ_S and $\mathbf{n}$ did not depend on $\mathbf{r}_S$, the integral equation (13) could be solved by Fourier series. This is precisely the case for the infinite plate. Thus the problem of the infinite plate allows an explicit expression for the exact solution for any arrangements of the contacts. In particular the relation between the Larmor radius r and the plate thickness d can be arbitrary: when $d > r$ Sondheimer[2,3] oscillations of resistance with magnetic field appear (and can be calculated exactly). In the general case of a wire, the solution can be effectively constructed when the dimensions of the conductor are large compared to the Larmor radius.

Simply to make the presentation easy we shall consider a conductor having the shape of a right cylinder of arbitrary cross section (Fig. 1). We choose the coordinate axes as follows: axis ζ along the generator of the cylinder, axis η is the projection of the z axis (which is along H) on the plane of the transverse cross section perpendicular to η (obviously then the x axis is perpendicular to ζ and z), the y axis is perpendicular to x and z. The origin is chosen at the point on the cylinder surface with minimum projection η. The transverse cross section is shown separately in Fig. 2. Here

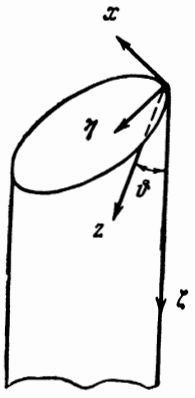

FIG. 1.

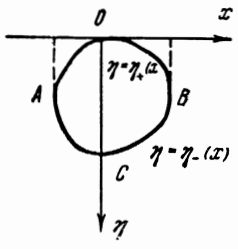

FIG. 2.

$\eta = \eta_+(x)$, $h = h_+(\zeta, x)$ for the segment AOB and $\eta = \eta_-(x)$, $h = h_-(\zeta, x)$ on the segment ACB. The position of the contacts with their input leads will be indicated in each individual case.

Since the wire is cylindrical, λ_S and $\mathbf{n}$ do not depend on ζ, and consequently for any dependence of i on ζ equation (13) can be written for the Fourier components of the function h in the variable ζ.

In order to make all the computations much less complicated, we make still another simplifying assumption which does not affect the basic results: we shall assume that the Fermi surface is closed and that $v_\eta \neq 0$ for any p_z and t. It is understood that this is possible only in exceptional cases, for example for plates with a magnetic field normal to the surface and with a quadratic dispersion law for the conduction electrons. But the inclusion of sections in which $v_\eta = 0$ only makes it necessary to take account of the existence of electrons which, after emerging from the surface, collide with it on the same revolution—cf. Fig. 3.

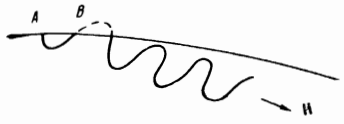

FIG. 3.

From this figure it is already clear how one should take account of such electrons (cf. also[4-8]). The whole calculation is then repeated step by step, but the number of terms in the formula doubles each time (since one must describe separately electrons like those on the segment AB (Fig. 3), and the rest.)

In the absence of orbits with $v_\eta = 0$, Eq. (14) for the determination of λ_S is easily seen to take the form (everywhere except, possibly, in a neighborhood of size r around the points A and B of Fig. 2)

$$\bar{v}_z > 0: \ \lambda_S = \lambda_S^+(x), \quad \eta_-(x) - \eta_+(x + x(\lambda_S^+) - x(t)) = \eta(t) - \eta(\lambda_S^+); \tag{21}$$

$$\bar{v}_z < 0: \ \lambda_S = \lambda_S^-(x), \quad \eta_+(x) - \eta_-(x + x(\lambda_S^-) - x(t)) = \eta(t) - \eta(\lambda_S^-). \tag{22}$$

Equation (13) consequently is written for the function $h_+(x)$ on the segment AOB and $h_-(x)$ on the segment ACB (Fig. 2) in the form

$$h_\mp(\xi; x) - \frac{\langle \mathbf{v}(t)\, \mathfrak{1}_\pm(x)\, h_\mp(\zeta + \zeta(\lambda_S^\mp) - \zeta(t);\ x + x(\lambda_S^\mp) - x(t))\rangle_\mp}{\langle \mathbf{v}(t)\, \mathbf{n}_\pm(x)\rangle_-} = -\frac{i_\pm(\zeta; x)}{\langle \mathbf{v}(t)\, \mathbf{n}_\pm(x)\rangle_\mp} \tag{23}$$

(where each time one takes either the upper or the lower sign).

For the case where the contacts for the current leads are on the opposite infinitely distant end faces of the wire at $\zeta = \pm\infty$, i = 0 in Eq. (23) and E_ζ = const. Then because of (15) we can set

$$h_\pm(\zeta; x) = -E_\zeta \zeta + E_\zeta \eta_\pm(x) \operatorname{ctg}\vartheta + G_\pm(x) E_\zeta, \tag{24}*$$

and the right side of the equation for $G_\pm$ is of the order of the Larmor radius r (for the notation for the angles, cf. Fig. 1).

We point out that (24) is the solution of the homogeneous equation corresponding to Eq. (23) (since i = 0). It is important to note that the solution (24) increases (in absolute value) to infinity with ζ and therefore for the existence of a solution of the inhomogeneous equation (23) it is not necessary that its right hand side be orthogonal to the solutions of the homogeneous equation.

Now let us consider the general case. If the wire is convex its radius of curvature is of the order of its linear dimensions, i.e., by assumption it is much greater than the Larmor radius r. Thus everywhere except in the immediate neighborhood of the points A and B (with dimensions of order r), the functions η_+ and η_- in formulas (21) and (22) can be expanded in series and, in the approximation of interest to us, we can stop at the linear terms. In other words, over intervals of x large

*ctg ≡ cot.

compared to r, but small compared to the characteristic diameter d, one can replace curved segments by straight lines (whose slopes vary slowly with x).

Thus to solve the problem it is sufficient to assume that $\eta_+(x)$ and $\eta_-(x)$ are linear functions. If the corresponding straight lines are not parallel, by introducing a local origin at the point of intersection of these lines (it is clear that Eqs. (21)—(22) do not depend on the choice of the reference origin) and writing their equations

$$\eta_\pm(x) = a_\pm x = x \operatorname{tg} \Phi_\pm, \qquad (25^*)$$

we get:

$$(a_- - a_+)x = \eta(t) - \eta(\lambda_S^+) + a_+(x(\lambda_S^+) - x(t)), \quad (21a)$$

$$(a_+ - a_-)x = \eta(t) - \eta(\lambda_S^-) + a_-(x(\lambda_S^-) - x(t)). \quad (22a)$$

From this it is clear that $x(\lambda_S^\pm)$ is a periodic function of x with period

$$\Delta x = \frac{|\langle\langle v_\eta\rangle\rangle T|}{a_+ - a_-} = \frac{|\langle\langle v_z\rangle\rangle| T \sin\vartheta}{a_+ - a_-}. \qquad (26)$$

Here $T = 2\pi m^*c/eH$ is the period of revolution of the electron in the magnetic field (m^* is the effective mass), $\langle\langle\ \rangle\rangle$ denotes a time average. We remind the reader that the basic equation for determining the time dependence has the form $\mathbf{p} = ec^{-1}\,\mathbf{v} \times \mathbf{H}$, and for closed Fermi surfaces $\mathbf{r}(t + T) = \mathbf{r}(t) + \langle\langle\mathbf{v}\rangle\rangle T$ and $x(t) \sim y(t) \sim r$, since $\langle\langle v_x\rangle\rangle = \langle\langle v_y\rangle\rangle = 0$.

We emphasize that for the case where the cross section of the wire is a polygon, Eqs. (21a)—(22a) are exact.

If the lines $\eta = \eta_\pm(x)$ are parallel, their equations can be written in the form

$$\eta_\pm = ax \mp b/2 = x \operatorname{tg} \Phi \mp b/2. \qquad (27)$$

It is convenient to introduce new axes: the ξ axis, directed along the normal to the line (27) perpendicular to the ζ axis, and the μ axis, parallel to the line (27). (For the choice of axes, cf. Fig. 4.)

Then Eqs. (21a)—(22a) take the form

$$\xi(\lambda_S^\pm) - \xi(t) = \mp d, \qquad d = b\cos\Phi, \qquad (28)$$

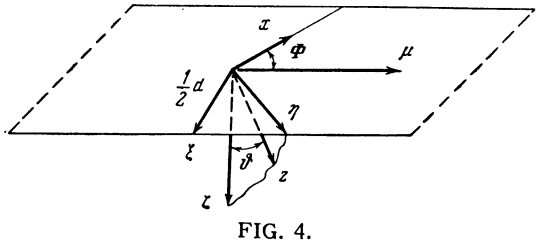

FIG. 4.

*tg = tan.

and $\lambda_S^\pm$ depends only on t and p_z and is independent of the coordinates. (This condition remains when we include orbits with $v_\eta = 0$.) In the approximation we are considering the solution of the homogeneous equation (13) exists for i = 0:

$$h_\pm(x) = -E_\xi\zeta \mp \tfrac{1}{2}bE_\xi \operatorname{ctg}\vartheta + A_\pm - Bx/\cos\vartheta \qquad (29)$$

($A_\pm$ and B are constants; the cases of $\Phi = \pi/2$ and $\vartheta = 0$, i.e., magnetic field parallel to the surface, are singular).

Equations (23), after substitution of i = 0, coincide, and one can express $A_+ - A_-$, which is needed for determining the potential difference, in terms of E_ζ and E_μ. Considering that

$$\zeta = \xi \operatorname{ctg}\vartheta/\cos\Phi - y/\sin\vartheta + x \operatorname{ctg}\vartheta \operatorname{tg}\Phi,$$

using Eq. (28) and the inequality

$$v_\xi(t)\,dt = v_\xi(\lambda_S^\pm)\,d\lambda_S^\pm,$$

which is a consequence of it, taking account of the fact that x(t) and y(t) are periodic, and also the presence of a time average in the triangular brackets, we get (i = 0!)

$$A_+ = A_- = A. \qquad (30)$$

We now proceed to treat various special cases.

4. RESISTANCE OF PLATES. CONTACTS ON THE END FACES AT INFINITY

As already appeared in the preceding Section, for plates in the general case one can get an exact solution.[4)] If contacts are placed on the side surfaces of the plate at $\zeta = \pm\infty$, then i = 0 and the solution has the form (29). Since curl $\mathbf{E} = 0$, $E_\mu = \text{const}$, and according to (29) and (15), $B = E_\mu$. (From this it is clear that the solution (29) actually is, for plates, the general solution of the homogeneous equation, since it contains just the two constants needed for physical questions: E_μ and E_ζ.) Consequently, according to (29) and (30),

$$g_\pm = A - E_\zeta\xi\frac{\operatorname{ctg}\vartheta}{\cos\Phi} \mp \frac{b}{2}E_\zeta \operatorname{ctg}\vartheta - K(t); \qquad (31)$$

$$K(t) = -E_\zeta\frac{y(t)}{\sin\vartheta} + \frac{x(t)}{\cos\Phi}(E_\mu - E_\zeta \operatorname{ctg}\vartheta\sin\Phi). \qquad (32)$$

[4)]It can be obtained also for finite $l \ll d$, when there is no correlation between the plate surfaces and it is sufficient to consider a half-space. Inasmuch as $\langle\psi\rangle = 0$, only a term ψ/t_0 is added to the kinematic equation (t_0 is the mean-free-path time; we confine ourselves to this approximation for simplicity). Then Eq. (3) is of the Wiener-Hopf type.

Substituting (31) in formula (16) for the current density, we get

$$\mathbf{j} = \langle \mathbf{v}(t)\,(K(t) - K(\lambda))\rangle. \qquad (33)$$

Here we have used the fact that $\langle \mathbf{v} \rangle = 0$ and that Eqs. (17) in the present case take the form

$$\xi(\lambda^{\pm}) - \xi(t) = \mp d/2 - \xi. \qquad (34)$$

From (33) and the equality $v_\xi(t)dt = v_\xi(\lambda^{\pm})d\lambda^{\pm}$, for fixed ξ it follows from (34) that, as it should, $j_\xi(\xi) = 0$. From (33) we also find that

$$j_\alpha(\xi) = \sigma_{\alpha\beta}(\xi)E_\beta, \qquad (35)$$

where α and β take on the values μ, ζ, and that $\sigma_{\alpha\beta}(\xi) \sim r$.

To calculate the current density according to (33), it is convenient to use the fact that $K(\lambda)$, according to (32), is periodic in λ with period T, while λ, according to (34), changes by an amount T when ξ changes by $\langle\!\langle v_\xi \rangle\!\rangle T$. This means that $K(\lambda)$ is periodic in ξ with period $\overline{v}_\xi T$. We expand $K(\lambda^{\pm})$ in Fourier series in ξ:

$$K(\lambda^{\pm}) = \sum_{n=-\infty}^{+\infty} \varkappa_n^{\pm} \exp(2\pi i n\xi/\langle\!\langle v_\xi \rangle\!\rangle T). \qquad (36)$$

We get [for the notation cf. (5) and (33)]:

$$j_\alpha = \sum_{m=-\infty}^{\infty} j_m^{\alpha}, \quad j_0^{\alpha} = [\langle\!\langle v_\alpha K\rangle\!\rangle - \langle\!\langle v_\alpha\rangle\!\rangle\langle\!\langle v_\xi K\rangle\!\rangle/\langle\!\langle v_\xi\rangle\!\rangle]; \qquad (37)$$

$$j_m^{\alpha} = \frac{1}{2\pi i m}\left[T\exp\left(\frac{\pi i n d}{|\langle\!\langle v_\xi\rangle\!\rangle T|}\right)\left\langle\!\!\left\langle v_\alpha(t)\exp\left(-\frac{2\pi i m\xi(t)}{\langle\!\langle v_\xi\rangle\!\rangle T}\right)\right\rangle\!\!\right\rangle \times \left\langle\!\!\left\langle \dot{K}(t')\exp\left(\frac{2\pi i m\xi(t')}{\langle\!\langle v_\xi\rangle\!\rangle T}\right)\right\rangle\!\!\right\rangle\right]. \qquad (37a)$$

Noting that

$$\langle\!\langle v_x x\rangle\!\rangle = \langle\!\langle v_y y\rangle\!\rangle = 0, \qquad \langle\!\langle v_x y\rangle\!\rangle = -\langle\!\langle v_y x\rangle\!\rangle \sim n_1 - n_2,$$

where n_1 and n_2 are respectively the numbers of electrons and holes, we find for $d \gg r$:

$$j_0^{x,\eta} = \mp\frac{(n_1 - n_2)ec}{H\sin\vartheta}E_{\eta,x}. \qquad (38)$$

Let us make clear the physical meaning of this result in the case of a strong magnetic field $d \gg r$. If in the metal the numbers of holes and electrons are the same, then $j_0^{\alpha} = 0$, and the current at any depth is related to terms like (37a). But these terms, because of the integration over p_z, oscillate and fall off rapidly with distance from the surface $\xi = \pm d/2$ of the plate (because $d \gg r$). Furthermore, the main contribution to the total current also comes just from a presurface layer with thickness of order r in the vicinity of the surface (to see this it is sufficient to calculate the total current in a region far

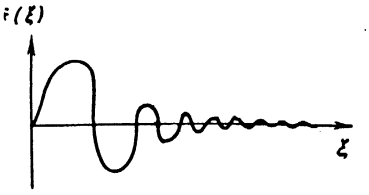

FIG. 5.

from the surface of the plate). Then in the basic approximation in terms of r/d (i.e., for $d \to \infty$), the current density at the surface is zero (this is most clearly seen from formula (33)), at a depth of order r it increases to a value of order $neceE_\xi/H$, and at large depths $\xi \gg r$ it damps out, while oscillating, as $(r_0/\xi)\cos(\xi/r_0)$, where $r_0 = (\langle\!\langle v_\xi\rangle\!\rangle T)_{max}$ (cf. Fig. 5). For just this reason the total current when $n_1 = n_2$ is independent of the plate thickness and is proportional to r^2. The current in the "deep" layers of the metal gives only small Sondheimer oscillations, which have been treated in detail in[3] and for which one can give an exact formula. When $d \gg r$ calculating them presents no difficulties: the main contribution in (37a) comes from the region near the ends of the interval of integration, i.e., near the turning points (points A and B in Fig. 6), where $v_\xi(p_z) \approx v'_\xi(p_0)(p_z - p_0)$, $v_z(p_0) = v(p_0) = v_0$. This is the static skin effect pointed out earlier.[4,5]

It is important to note that, first, the static skin effect occurs only for $n_1 = n_2$, second, with the contacts placed on the side faces the dependence of the resistance on magnetic field is not changed (the reason being the uniformity of the field in the sample in the leading approximation; the role of the field uniformity was pointed out in[4]). In fact, let us determine the resistance of the plate. To do this we must consider that although the plate is large in the μ direction (even compared to $d \gg r$), it is still bounded, so that the total current can flow only in the ζ direction, while $J_\mu = 0$ (this follows immediately from the continuity equation in integral form, taking account of the boundaries of the plate in the $\pm\mu$ directions). Since

$$J_{x,\eta} = \mp\frac{(n_1 - n_2)ec}{H\sin\vartheta}dE_{\eta,x} + r^2 v_0 a_\beta^{x,\eta}E_\beta, \qquad (39)$$

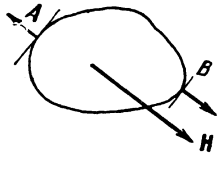

FIG. 6.

$\rho = E_\zeta d/J_\zeta$ has the form

$$\rho(H,d) \sim \begin{cases} \rho(0,\ d) \sim d^{-1}, & n_1 \neq n_2 \\ \rho(0,\ d)\,(d/r)^2 \sim r^{-2}d, & n_1 = n_2 \end{cases}. \quad (40)$$

For a symmetric arrangement of the contacts, we must correct the graphs in[4,5] in accordance with these formulas.

We emphasize that we have restricted our treatment to a strong magnetic field only to be specific; formulas (31a)—(37a) are valid for any ratio d/r (but, naturally, with $l \gg d, r$).

5. RESISTANCE OF A PLATE. ASYMMETRIC ARRANGEMENT OF CONTACTS.

The main idea for obtaining the solution for the case of an arbitrary arrangement of contacts is the same as in the preceding section. The basic equations have already been written: these are Eqs. (16), (20), (23), (28) and (34). The homogeneous solution (29) (where E_ζ, naturally, is simply an arbitrary constant) of Eq. (13) should be dropped, since for contacts placed at finite distance from one another, the potential cannot increase without limit at infinity (since the potential difference between infinitely distant points of the plate cannot increase without limit). Since all the functions in (23) damp out at infinity, while λ_S and $\mathbf{n}_\pm$ do not depend on the points on the planes, Eq. (23) can be solved by expanding all the functions in Fourier integrals.

Instead of using the coordinates x, ζ, it is more convenient to describe points on the surface of the plate by the coordinates x, y (cf. Fig. 4), which is possible in all cases except the case we have not considered, where the magnetic field is parallel to the surface of the plate. The connection of ζ with x and y is given by the formula

$$\zeta = \xi \operatorname{ctg}\vartheta / \cos\Phi - y/\sin\vartheta + x \operatorname{ctg}\vartheta \operatorname{tg}\Phi.$$

On the surfaces $\xi = \pm 1/2b \cos\Phi$, and

$$y = -\zeta \sin\vartheta - x\cos\vartheta \operatorname{tg}\Phi \mp {}^1\!/_2 b \cos\vartheta. \quad (41a)^*$$

Introducing the vectors $\mathbf{k} = (k_x, k_y)$, $\mathbf{R} = (x, y)$ and denoting the Fourier components of a function by capital letters, we find from (23):

$$H_\pm(\mathbf{k}) - a_\pm(\mathbf{k}) H_\pm(\mathbf{k}) = i_\pm(\mathbf{k}) / \langle v_\xi \rangle^\mp \equiv K_\pm(\mathbf{k}); \quad (41)$$

$$a_+(\mathbf{k}) = a_-^*(\mathbf{k}) = \langle v_\xi(t) \exp\{i\mathbf{k}(\mathbf{R}(\lambda_S^\mp) - \mathbf{R}(t))\}\rangle_- / \langle v_\xi \rangle_- = a(\mathbf{k}). \quad (42)$$

*tg ≡ tan.

Just as in the preceding section, by taking account of the periodicity of x(t) and y(t), we can expand the function in triangular brackets in Fourier series in d [treating d as a parameter; cf. (28)] and obtain:

$$a_+(\mathbf{k}) = \sum_{n=-\infty} a_n(\mathbf{k}), \quad (43)$$

$$a_n(\mathbf{k}) = \left\langle \frac{1}{\langle\!\langle v_\xi \rangle\!\rangle} \exp\left(\frac{2\pi i n d}{\langle\!\langle v_\xi T \rangle\!\rangle}\right) \times \left| \left\langle\!\left\langle v_\xi(t) \exp\left\{-i\mathbf{k}\mathbf{R}(t) + \frac{2\pi i n \zeta(t)}{\langle\!\langle v_\xi \rangle\!\rangle T}\right\}\right\rangle\!\right\rangle \right|^2 \right\rangle_-. \quad (44)$$

The formulas given are valid for any ratio of d and r, and in particular allow a limiting transition to the case where there is no magnetic field.[5] We shall, however, restrict ourselves to the more interesting case of $d \gg r$. Then it is sufficient to keep only the leading term with n = 0 in (43), so that in the leading approximation in r/d,

$$a_+(\mathbf{k}) \approx \left[\frac{1}{\langle\!\langle v_\xi \rangle\!\rangle} \left| \langle\!\langle v_\xi(t) e^{-i\mathbf{k}\mathbf{R}(t)} \rangle\!\rangle \right|^2\right] = a_-(\mathbf{k}) \approx a(\mathbf{k}). \quad (45)$$

Obviously $a_+(\mathbf{k})$ is an even function of $\mathbf{k}$. (For small kr_0 the quantity $a_+ \approx 1$, and it is important to keep the next approximation in kr_0, since for $a_+ = 1$ Eqs. (41) have no solution. This could mean that one needed to keep terms with $n \neq 0$. But as one sees from (44), for small kr_0 they are also proportional to k^2 and are small compared to the correction to the value 1 for a_0).

According to (41), (45), and (20),

$$\varphi_\pm(\mathbf{r}_S) = \frac{1}{2}\int e^{i\mathbf{k}\mathbf{R}}\, d\mathbf{k} \left\{ \frac{K_+(\mathbf{k}) + K_-(\mathbf{k})}{1 - a(\mathbf{k})} - (1 - b_\mp(\mathbf{k})) \frac{K_\mp(\mathbf{k}) + a(\mathbf{k}) K_\pm(\mathbf{k})}{1 - a^2(\mathbf{k})} \right\}, \quad (46)$$

where

$$b_\mp(k) = \langle \exp\{ik(R(\lambda_S^\pm) - R(t))\}\rangle_\mp / \langle 1 \rangle_\mp = \sum_{n=-\infty}^{\infty} \frac{1}{\langle 1 \rangle_\mp} \left[\frac{1}{\langle\!\langle v_\xi \rangle\!\rangle} \exp\left(\frac{2\pi i n d}{\langle\!\langle v_\xi \rangle\!\rangle T}\right) \left\langle\!\left\langle \exp\left\{-ikR(t) + \frac{2\pi i n \xi(t)}{\langle\!\langle v_\xi \rangle\!\rangle T}\right\}\right\rangle\!\right\rangle \left\langle\!\left\langle v_\xi(t') \exp\left\{ikR(t') - \frac{2\pi i n \xi(t')}{\langle\!\langle v_\xi \rangle\!\rangle T}\right\}\right\rangle\!\right\rangle \right]_\mp, \quad (47)$$

and by the same argument as for $a_\pm(\mathbf{k})$,

$$b_\mp(\mathbf{k}) = \frac{1}{[1]_\mp} \left[\frac{1}{\langle\!\langle v_\xi \rangle\!\rangle} \langle\!\langle e^{-i\mathbf{k}\mathbf{R}(t)} \rangle\!\rangle \langle\!\langle v_\xi(t') e^{i\mathbf{k}\mathbf{R}(t')} \rangle\!\rangle \right]_\mp. \quad (48)$$

[5] In particular, for example, if H = 0,

$$f = h\left(x - \frac{v_x}{v_z} z - \frac{v_x}{2|v_z|} d,\ y - \frac{v_y}{v_z} z - \frac{v_y}{2|v_z|} d\right).$$

We note that although $(1 - a(\mathbf{k})) \sim k^2$ for small k, there is no divergence of (46), since $(1 - b \mp (\mathbf{k})) \sim k$, $d\mathbf{k} \sim k\, dk$, while $K_+(\mathbf{k}) + K_-(\mathbf{k}) \sim k$. The last relation is connected with the conservation of charge:

$$K_+(0) + K_-(0) \sim \int d\mathbf{r}_S \{i_+(\mathbf{r}_S) + i_-(\mathbf{r}_S)\} = 0$$

(the total current entering the plate is equal to the total current emerging). Expressing [by formula (41a)] y on the surface of the plate in terms of ζ, we get the required potential in the region of the contact, which permits us to find the resistance, and the Joule heat according to the formula

$$Q = -\int \mathbf{j}\nabla\varphi dv = -\int \operatorname{div}(\mathbf{j}\varphi)\, dv = \int \mathbf{j}\varphi d\mathbf{S}$$

etc. (There is, of course, no difficulty in determining, from the formulas given earlier, the potential at any point, the current distribution over the sample, as well as the quantity introduced earlier—the ratio of the potential difference infinitely close to the contacts to the current strength.)

Since the problem contains seven quantities with dimensions of length (the two linear dimensions a_1, a_2 for each of the contacts, the separation of the contacts D, the plate thickness d and the Larmor radius r), an extremely large number of different limiting cases are possible, each of which can be analyzed. But the basic purpose of the present paper is to demonstrate the technique and write the general formulas in closed form. We shall therefore limit ourselves to a few remarks, which follow at once from our formulas.

1. For point or line contacts the resistance is infinite, while the ratio of the difference of potential infinitely close to the contacts to the current strength increases quadratically with the magnetic field in the first case and linearly in the second case, for arbitrary ratio of the number of electrons n_1 and holes n_2.

2. For dimensions of the contacts small compared to r, the resistance is inversely proportional to the area of the contacts, proportional to H^2 for any n_1 and n_2, and is independent of d and D; if the dimensions in one direction are small compared to r and large in the other, then $\rho \sim H$.

3. The dependence of ρ on H varies from saturation to a quadratic growth.

6. RESISTANCE OF WIRES. CONTACTS AT INFINITY ON THE END FACES

We shall show how the method presented above is generalized by using the example of a wire with contacts on the end faces. According to Sec. 3, the problem reduces to solving Eq. (23) for i = 0, (24), (21a), (22a), (25). We set $G_\pm$ in (24) equal to

$$G_\pm = A_\pm - Bx + w_\pm(x). \tag{49}$$

Using the fact that

$$\zeta = \eta \operatorname{ctg}\vartheta - y/\sin\vartheta,$$

$$(\eta(\tau) - a_\mp x(\tau))\Big|_{\lambda_S^\pm}^{t} = \pm(a_+ - a_-)x,$$

we get

$$w_\pm(x) - \frac{\langle v_{n\pm}(t)\, w_\mp(x + x(\lambda_S^\mp) - x(t))\rangle_\mp}{\langle v_{n\pm}\rangle_\mp} = \mp(A_+ - A_-)$$

$$+ \frac{\langle v_{n\pm}(t)\,(K_1(t) - K_1(\lambda_S^\mp))\rangle_\mp}{\langle v_{n\pm}\rangle_\mp} = P_\mp, \tag{50}$$

where each time we take either the upper or the lower sign; the signs in the angular bracket refer to $\bar{v}_z$, while $\lambda_S^\pm$ is given by (21a), (22a), and (25). The function $K_1(t)$ has the form

$$K_1(t) = Bx(t) - y(t)/\sin\vartheta, \tag{51}$$

so that the right sides in (50) are periodic in x with period (26), as is $x(\lambda_S^\pm)$. Accordingly we can expand $K_1(\lambda_S^\pm)$ in a Fourier series in x in exactly similar fashion to (36)—(37) (using formulas (21a)—(22a) to get $dx/d\lambda$). We then find for $P_\mp$

$$P_\mp = \mp(A_+ - A_-) + \frac{\sin(\Phi_- - \Phi_+)}{\sin^2\vartheta\cos\Phi_+\cos\Phi_-}\frac{\langle yv_x\rangle_+}{\langle v_z\rangle_+}$$

$$+ \sum_{n=-\infty}^{+\infty}{}' P_n^\mp \tag{52}$$

(the prime on the sum indicating that the term with n = 0 is omitted), where, because of

$$v_{n\pm} = (v_\eta - a_\mp v_x)\cos\Phi_\pm \mp (a_+ - a_-)v_x\cos\Phi_\pm)$$

the quantity $P_n^\mp$ is equal to

$$P_n^\mp = \frac{a_- - a_+}{2\pi i n\langle v_\eta\rangle_\mp}\left[T\exp\left\{\frac{2\pi i n(a_- - a_+)x}{\langle\langle v_\eta\rangle\rangle T}\right\}\right.$$

$$\left.\times\langle\langle v_x\alpha_\pm\rangle\rangle\langle\langle \dot{K}_1\alpha_\mp\rangle\rangle\right]_\mp,$$

$$\alpha_\pm(t) = \exp\left\{\frac{\mp 2\pi i n}{\langle\langle v_\eta\rangle\rangle T}(\eta(t) - a_\mp x(t))\right\}. \tag{53}$$

We set

$$A_+ - A_- = \frac{\sin(\Phi_- - \Phi_+)}{\sin^2\vartheta\cos\Phi_+\cos\Phi_-}\frac{\langle yv_x\rangle_+}{\langle\langle v_z\rangle\rangle_+} \sim \frac{n_1 - n_2}{H}, \tag{54}$$

so that only oscillating terms are left on the right in (50). (The quantity B is still undetermined.) For $x \gg r$, i.e., far from the point of intersection of the lines (in general this means far from the points A and B of Fig. 2), it is easy to calculate the asymptotic form of the oscillating right hand side of (50).

In the lowest approximation in $r/|a_- - a_+|x$, the right side, using (54), has the form

$$P_\mp \approx \sum_{n=1}^{\infty} \Lambda_n^\mp \frac{q^2}{nx} \sin\frac{2\pi nx}{q}, \quad \Lambda_n^\mp \sim 1; \tag{55}$$

$$q = \left| \frac{\langle\langle v_z \rangle\rangle T \sin\vartheta}{a_- - a_+} \right|_{p_z = p_0}. \tag{56}$$

Here $p_z = p_0$ corresponds to the turning points (A or B in Fig. 6).

In this same approximation we can set

$$w_\pm(x) = \sum_{n=-\infty}^{+\infty}{}' w_n^\pm \frac{q^2}{nx} \sin\frac{2\pi nx}{q}. \tag{57}$$

Substituting (57) in (50) and again, of course, keeping only the lowest approximation, we see that $w_n^\pm \sim 1$ (when $n \gg 1$ we have $w_n^\pm \approx \Lambda_n^\pm$).

The accuracy thus obtained for $h_\pm(x)$ is sufficient for calculating both the monotonic part of the current density and the total current and the Sondheimer oscillations. There is also no difficulty in finding the higher approximations for $h_\pm(x)$.

The solution given is still determined only to within the constant B. Its physical meaning will become clear if we turn to the case of a plane parallel plate which was treated in Sec. 4. In an infinite plate the current can flow in two directions and, accordingly, one can assign in addition to E_ζ a second independent constant. The solution found above is a "local" one, valid in a region large compared to r but small compared to d (cf. Sec. 3); we must also take account of the dependence of $\Phi_\pm$ on x. Thus this solution "doesn't know" whether the curve $\eta = \eta(x)$ is closed or goes off to infinity (as, for example, $\eta_+(x) = \cos x$, $\eta_- = 2 + \sin x$) and is "prepared" to supply the additional constant (corresponding to E_μ in the plane parallel case).

If the curve $\eta = \eta(x)$ is closed, the constant B is determined from the condition for matching of the solutions $h_\pm(x)$ in the region $x \sim r$, i.e., in the case of two lines, in the region near their intersection, where we cannot use the asymptotic form (53), while in the general case it is determined from conditions near the points A and B of Fig. 2, where $\eta_\pm(x)$ cannot be replaced by a pair of straight lines, since $\eta_\pm(x) \sim |x - x_{A,B}|^{1/2}$.

For actually finding B it is, however, more convenient, instead of matching the solutions, to use the continuity equation in integral form (since charge conservation near the points A and B was not guaranteed, while the integral form is more convenient for this case): the total current J_x along the x axis is zero ($J_x = 0$).

All the further considerations, which are completely analogous to those of Sec. 4, enable one to get a closed form for the resistance for any geometry of the wire surface, and lead qualitatively to the same results for the monotonic part of the current. For $n_1 \neq n_2$ the current is distributed uniformly over the wire, the average specific resistance tends toward a saturation of the same order as in the absence of the field, and is thus inversely proportional to the characteristic wire diameter d.

For $n_1 = n_2$ the current damps rapidly into the wire according to the formula $(r_0/\xi)\cos(\xi/r_0)$, as in the case of plates: the total current is concentrated within a layer of thickness of order r near the surface, internal or external ("static skin effect"), and is proportional to the perimeter of the cross section and the square of the Larmor radius; as compared to the resistance in the absence of the field, the specific resistance now increases like $(d/r)^2$, i.e., proportional to H^2. Thus for any n_1 and n_2, the specific resistance is the same in order of magnitude as in an infinite sample with free path d.

The Sondheimer oscillations occur with a period equal to unity in the parameter $d_0/\langle v_z T\rangle_{p_z = p_0} \sin\vartheta \sim H$, where d_0 is the extremal diameter of the wire in the η direction.

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