STRUCTURE OF DOMAIN BOUNDARY IN A FERROMAGNETIC OF FINITE THICKNESS

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Submitted to JETP editor February 20, 1965

J. Exptl. Theoret. Phys. (U.S.S.R.) **49**, 599-608 (August, 1965)

The structure of the domain boundary of a ferromagnetic crystal of finite thickness 2d is determined by perturbation theory and by taking into account the surface anisotropy β'. A uniform boundary and a boundary which is longitudinally periodic are considered. When $\beta' \neq 0$ there exists a range of thicknesses $d'_c < d < d_c$ in which a Neel boundary can exist; when $\beta' > \beta_c$ the Bloch boundary is energetically more favorable at any thickness. General equations are derived for the period and shape of the periodic boundary and are investigated in two limiting cases. Depending on the magnitude of β', the Bloch boundary may become narrower or broaden toward the crystal surface.

INTRODUCTION

THE structure and the energy of the domain boundary in an infinite uniaxial ferromagnetic crystal were first analyzed rigorously by Landau and Lifshitz [1] and then by Shirobokov [2]. Similar problems for crystals with another type of magnetic anisotropy were solved by many authors [3-5]. The structure of the domain boundary near the Curie temperature was investigated by Bulaevskiĭ and Ginzburg [6]. In the places where the boundary emerges to the crystal surface, magnetic poles are formed. The influence of the demagnetizing fields produced by these poles on the structure of the domain boundary in a bulk crystal is small, and distorts the latter only near the crystal surface; for thin films, however, it can change the structure of the boundary radically. Neel [7] was the first to show that, starting with some critical values of the film thickness, a boundary in which the vector of the magnetic moment turns without going outside the plane of the crystal is energetically more favored; such boundaries have been named Neel boundaries.

The energies of the Bloch and Neel boundaries in thin films (and the critical thickness of the film) were later determined more accurately by many workers [8-12]. The influence of the demagnetizing fields on the structure of the Bloch boundary in a thin film was investigated by Hoffman [12] and by Muller and Dawson [13]. Shtrikman and Treves [14] and Bhide and Shenoy [15] investigated whether a longitudinally-periodic boundary (i.e., broken up into sections of opposite magnetization) can be produced under the influence of the demagnetizing fields.

None of these investigations, however, took into account the general boundary conditions for the magnetization vector **M** on the surface of the crystal, i.e., the effect of surface anisotropy. The surface anisotropy, as shown by experiments on spin-wave resonance, can be large; its nature and magnitude were discussed by many authors [16-18]. The purpose of the present paper is to investigate the structure of the domain boundary in a uniaxial ferromagnetic crystal. The solution of such a problem is at present of special interest in view of the possibilities that are opening up for experimentally investigating the structure of the boundary on the surface of the crystal [19,20].

1. GENERAL EQUATIONS AND BOUNDARY CONDITIONS

For temperatures far from the Curie temperature, the volume density of the Hamiltonian for a uniaxial ferromagnetic crystal inside the crystal can be written in the form

$$\mathscr{H}_V = \frac{1}{2}\alpha\left[\left(\frac{\partial \mathbf{M}}{\partial x}\right)^2 + \left(\frac{\partial \mathbf{M}}{\partial y}\right)^2 + \left(\frac{\partial \mathbf{M}}{\partial z}\right)^2\right] + \frac{1}{2}\beta[\mathbf{M}^2 - (\mathbf{M}\mathbf{l})^2] + \frac{\mathbf{H}^2}{8\pi}, \tag{1.1}$$

where **l** is a unit vector in the direction of the easy magnetization axis, α the exchange constant (we neglect the difference between the constants α_1 and α_2 in the expression for the exchange energy), and β the anisotropy constant. The Landau-Lifshitz equation in the static case takes the form

$$[\mathbf{MH}^e] = 0, \qquad (1.2)^*$$

where

$$\mathbf{H}^e = -\frac{\delta\mathcal{H}}{\delta\mathbf{M}} = -\frac{\partial\mathcal{H}}{\partial\mathbf{M}} + \frac{\partial}{\partial x_k}\frac{\partial\mathcal{H}}{\partial(\partial\mathbf{M}/\partial x_k)}.$$

In our case

$$\mathbf{H}_V{}^e = \alpha\nabla^2\mathbf{M} + \beta\mathbf{l}(\mathbf{Ml}) + \mathbf{H}, \qquad (1.3)$$

where $\mathbf{H}$ is the demagnetizing field, which should be determined from the solution of the boundary value problem of magnetostatics.

The volume density of the Hamiltonian near the surface is of the form

$$\mathcal{H}_S = \frac{1}{2}\beta[\mathbf{M}^2-(\mathbf{Ml})^2] - \frac{1}{2}\beta'(\mathbf{Mn})^2 + \frac{\alpha'}{a}\mathbf{M}\frac{\partial\mathbf{M}}{\partial n} + \frac{1}{2}\alpha\left[\left(\frac{\partial\mathbf{M}}{\partial x}\right)^2 + \left(\frac{\partial\mathbf{M}}{\partial y}\right)^2 + \left(\frac{\partial\mathbf{M}}{\partial z}\right)^2\right] + \frac{\mathbf{H}^2}{8\pi}, \qquad (1.4)$$

where $\mathbf{n}$ is the outward normal to the surface, α' and β' are the exchange and anisotropy surface constants (in accordance with experimental data on spin-wave resonance, we shall assume α' to be a positive quantity), and a is the lattice constant. The effective field near the surface is

$$\mathbf{H}_S{}^e = \beta\mathbf{l}(\mathbf{Ml}) + \beta'\mathbf{n}(\mathbf{Mn}) - \frac{\alpha'}{a}\frac{\partial\mathbf{M}}{\partial n} + \alpha\nabla_n{}^2\mathbf{M} + \mathbf{H}, \qquad (1.5)$$

where the index n denotes the absence of differentiation with respect to the coordinate that is normal to the surface.

Equation (1.2) with a field in the form (1.5) is the boundary condition of the problem. Both the equations and the boundary conditions are simultaneously satisfied on the surface, so that we can write the boundary conditions in the form [21]

$$[\mathbf{M}(\mathbf{H}_S{}^e - \mathbf{H}_V{}^e)] = 0,$$

or in our case

$$\left[\mathbf{M}, \frac{\alpha'}{a}\frac{\partial\mathbf{M}}{\partial n} - \beta'\mathbf{n}(\mathbf{Mn})\right] = 0. \qquad (1.6)$$

We shall henceforth consider a crystal in the form of a layer of thickness 2d, with an easy axis parallel to the surface of the layer and coinciding with the y axis (Fig. 1). In terms of projections along the coordinate axes, noting that only two out of the three equations in (1.2) are independent, we obtain the system

$$m_x\nabla^2 m_y - m_y\nabla^2 m_x + \sigma^2 m_x m_y = \alpha^{-1}(m_y h_x - m_x h_y),$$
$$m_z\nabla^2 m_y - m_y\nabla^2 m_z + \sigma^2 m_z m_y = \alpha^{-1}(m_y h_z - m_z h_y),$$
$$m_x{}^2 + m_y{}^2 + m_z{}^2 = 1 \qquad (1.7)$$

$^*[\mathbf{MH}^e] \equiv \mathbf{M}\times\mathbf{H}^e$.

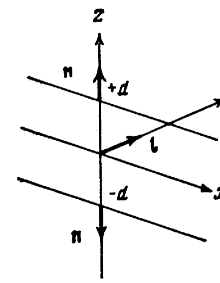

FIG. 1

with boundary conditions

$$m_y\frac{\partial m_x}{\partial n} - m_x\frac{\partial m_y}{\partial n} = 0,$$
$$m_y\frac{\partial m_z}{\partial n} - m_z\frac{\partial m_y}{\partial n} - \frac{\beta' a m_y m_z}{\alpha'} = 0 \qquad (1.8a)$$

when $z = \pm d$ and

$$m_x, m_z \to 0, \quad m_y \to \pm 1 \qquad (1.8b)$$

when $x \to \pm\infty$, where

$$\mathbf{m} = \mathbf{M}/M, \quad \mathbf{h} = \mathbf{H}/M, \quad \sigma^2 = \beta/\alpha.$$

The h_i in the right side of (1.7) must be determined from the solution of the boundary problem of magnetostatics ($\mathbf{h} = -\operatorname{grad}\psi^i$):

$$\nabla^2\psi^e = 0 - \text{ outside the layer},$$
$$\nabla^2\psi^i = -4\pi\operatorname{div}\mathbf{m} - \text{ inside the layer} \qquad (1.9)$$

with conditions on the boundary of the layer:

$$\partial\psi^i/\partial x - \partial\psi^e/\partial x = 0, \quad \partial\psi^i/\partial z - \partial\psi^e/\partial z = 4\pi m_z.$$

In the general case, Eqs. (1.7) and (1.9) constitute the complete system of equations of the problem. In view of its exceeding complexity, all the attempts at its solution were based heretofore on some method of approximately decoupling the systems (1.7) and (1.9).

2. HOMOGENEOUS BOUNDARY

It is simplest to separate the systems in the case when $d \to \infty$. Then $h_y = h_z = 0$ and $h_x = -4\pi m_x$, the functions m do not depend on y or z, and the system (1.7) with boundary conditions (1.8b) has two solutions:

a) Bloch boundary:

$$m_x = 0, \quad m_y = \operatorname{th}\sigma x, \; m_z = \operatorname{sech}\sigma x, \; \sigma^2 = \beta/\alpha; \quad (2.1a)^*$$

b) Neel boundary:

$$m_x = \operatorname{sech}\sigma_1 x, \quad m_y = \operatorname{th}\sigma_1 x,$$
$$m_z = 0, \quad \sigma_1{}^2 = (\beta + 4\pi)/\alpha. \qquad (2.1b)$$

Substituting these solutions in (1.1) and calcu-

*th ≡ tanh.

lating the surface density of the boundary energy $\gamma = \int \mathcal{H} dx$, we obtain for the Bloch and Neel boundaries, respectively,

$$\gamma_B = 2M^2(\alpha\beta)^{1/2}, \quad \gamma_N = 2M^2[\alpha(\beta + 4\pi)]^{1/2};$$

we see that the Bloch boundary is energetically more favorable.

In the limiting case $d \to 0$, the systems also separate; then (when $\beta' = 0$)

$$\gamma_B = 2M^2[\alpha(\beta + 4\pi)]^{1/2}, \quad \gamma_N = 2M^2(\alpha\beta)^{1/2},$$

i.e., $\gamma_N < \gamma_B$.

For a finite value of d, the problem can be simplified by assuming that the boundary is longitudinally homogeneous. Then in the system (1.7) the derivatives with respect to y and the fields h_y vanish. However, the system (1.7)–(1.9) does not separate. Making the change of variables $\eta = \sigma x$ and $\xi = \sigma z$, we rewrite the system (1.7) in the form

$$m_x \frac{\partial^2 m_y}{\partial \eta^2} - m_y \frac{\partial^2 m_x}{\partial \eta^2} + m_x m_y = \frac{1}{\beta} m_y h_x + m_y \frac{\partial^2 m_x}{\partial \xi^2} - m_x \frac{\partial^2 m_y}{\partial \xi^2},$$

$$m_z \frac{\partial^2 m_y}{\partial \eta^2} - m_y \frac{\partial^2 m_z}{\partial \eta^2} + m_z m_y = \frac{1}{\beta} m_y h_z + m_y \frac{\partial^2 m_z}{\partial \xi^2} - m_z \frac{\partial^2 m_y}{\partial \xi^2} \quad m_x^2 + m_y^2 + m_z^2 = 1. \tag{2.2}$$

We seek its solution by a perturbation method:

$$m_i = m_i^0(\eta) + \Delta m_i(\eta, \xi).$$

It is easy to see that in this case the small parameter will be $4\pi/\beta$. The zeroth-approximation solution, satisfying the system (2.2) without the right side, is of the form

$$m_x^0 = c \operatorname{sech} \sigma x, \quad m_y^0 = \operatorname{th} \sigma x, \quad m_z^0 = s \operatorname{sech} \sigma x, \tag{2.3}$$

where c and s are constants, and $c^2 + s^2 = 1$. It satisfies also the boundary conditions if $\beta' \lesssim (4\pi\alpha/a^2)^{1/2}$.

If we put $c = \cos\varphi$ and $s = \sin\varphi$, then $\pi/2 -$ is the angle between the plane of the boundary and the plane in which M rotates inside the boundary ($\varphi = 0$ corresponds to the Neel boundary, $\varphi = \pi/2$ to the Bloch boundary). For any value of φ, the energy of the boundary in the zeroth approximation is $\gamma_0 = 2M^2(\alpha\beta)^{1/2}$, i.e., the zeroth-approximation solution is degenerate in φ.

The system of first-approximation equations is

$$L\mu^+ = \beta^{-1}(h_x c + h_z s), \quad L\mu^- = \beta^{-1}(h_x s - h_z c), \tag{2.4}$$

where

$$L \equiv \partial^2/\partial\eta^2 + \partial^2/\partial\xi^2 + (2\operatorname{sech}^2\eta - 1),$$

and the first-approximation corrections are expressed in terms of the functions μ^+ and μ^- by

$$\Delta m_x = -s\mu^- - c\mu^+ \operatorname{th}\eta, \quad \Delta m_y = \mu^+ \operatorname{sech}\eta, \quad \Delta m_z = c\mu^- - s\mu^+ \operatorname{th}\eta. \tag{2.5}$$

The boundary conditions for $\xi = \pm\sigma d$ are of the form

$$\frac{\partial\mu^+}{d\xi} = \mp s^2 \frac{\beta' a}{\alpha'\sigma} \operatorname{th}\eta \operatorname{sech}\eta, \quad \frac{\partial\mu^-}{d\xi} = \pm cs \frac{\beta' a}{\alpha'\sigma} \operatorname{sech}\eta. \tag{2.6}$$

The magnetic fields h_x and h_z in (2.4) were obtained from (1.9), in which the zeroth-approximation solution (2.3) was substituted. With the aid of the Fourier transformation with respect to η we obtain

$$h_x = -4\pi c \int_0^\infty \operatorname{sech}\frac{l\pi}{2}[1 - e^{-l\delta}\operatorname{ch} l\xi]\cos l\eta\, dl + 4\pi s \int_0^\infty \operatorname{sech}\frac{l\pi}{2} e^{-l\delta}\operatorname{sh} l\xi \sin l\eta\, dl,$$

$$h_z = 4\pi c \int_0^\infty \operatorname{sech}\frac{l\pi}{2} e^{-l\delta}\operatorname{sh} l\xi \sin l\eta\, dl - 4\pi s \int_0^\infty \operatorname{sech}\frac{l\pi}{2} e^{-l\delta}\operatorname{ch} l\xi \cos l\eta\, dl, \tag{2.7*}$$

where $\delta = \sigma d$.

The limiting energy with account of the first approximation is of the form

$$\gamma = \gamma_0 + \Delta\gamma_V + \Delta\gamma_S,$$

$$\Delta\gamma_V = \frac{1}{2\sigma\delta}\int_{-\delta}^{+\delta} d\xi \int_{-\infty}^{+\infty} d\eta \mathcal{H}_V',$$

$$\Delta\gamma_S = \frac{a}{2\delta}\int_{-\infty}^{+\infty} d\eta \mathcal{H}_S' = \frac{aM^2}{4\delta}\int_{-\infty}^{+\infty} d\eta \left\{\left[-\beta' m_z^2 - \frac{\alpha'}{a}\mathbf{m}\frac{\partial\mathbf{m}}{\partial n}\right]_{\xi=+\delta} + \left[-\beta' m_z^2 - \frac{\alpha'}{a}\mathbf{m}\frac{\partial\mathbf{m}}{\partial n}\right]_{\xi=-\delta}\right\},$$

where $\mathcal{H}_V'$ and $\mathcal{H}_S'$ are the first-approximation corrections to the Hamiltonians. It is easy to show that $\Delta\gamma_V$ and $\Delta\gamma_S$ do not depend on μ^+ or μ^-, and that

$$\Delta\gamma_V = -\frac{M^2}{2\sigma}\int_{-\infty}^{+\infty} \operatorname{sech}\eta(\tilde{h}_x c + \tilde{h}_z s)\, d\eta, \quad \Delta\gamma_S = -s^2\beta' a M^2/\delta, \tag{2.8}$$

where $\tilde{h}_i$ are the values of the fields (2.7) averaged over ξ.

From the condition $\partial(\Delta\gamma_V + \Delta\gamma_S)/\partial\varphi = 0$ it follows that φ can assume only two values, zero and $\pi/2$, i.e., the degeneracy in φ is lifted when the first approximation is taken into account; the

*sh = sinh.

boundary can be either of the Bloch or of the Neel type. For a Bloch boundary ($c = 0$, $s = 1$) we have

$$\Delta\gamma_B = -\frac{M^2}{2\sigma}\int_{-\infty}^{+\infty} \tilde{h}_z \operatorname{sech}\eta\, d\eta - \beta' M^2 a/\delta \equiv \frac{4\pi M^2}{\sigma}\left[B(\delta) - \frac{a}{d}\frac{\beta'}{4\pi}\right], \tag{2.9}$$

for a Neel boundary ($c = 1$, $s = 0$)

$$\Delta\gamma_N = -\frac{M^2}{2\sigma}\int_{-\infty}^{+\infty} \tilde{h}_x \operatorname{sech}\eta\, d\eta \equiv \frac{4\pi M^2}{\sigma}[1 - B(\delta)] \tag{2.10}$$

Using the Parseval inequality, the double integral $B(\delta)$ in these expressions was transformed into

$$B(\delta) = \frac{\pi}{2}\int_0^\infty \operatorname{sech}^2\frac{l\pi}{2}\, e^{-l\delta}\,\frac{\operatorname{sh} l\delta}{l\delta}\, dl$$

and calculated with an electronic computer. The results for $\Delta\gamma_N$ and $\Delta\gamma_B$ yielded the plots of Fig. 2.

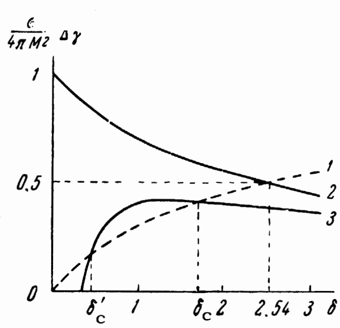

FIG. 2. The corrections $\Delta\gamma_B$ and $\Delta\gamma_N$ to the boundary energy as functions of the thickness of the layer: curve 1 – $\Delta\gamma_N$, curves 2 and 3 – $\Delta\gamma_B$ for $\beta' = 0$ and $\beta' \approx 0.3\ (4\pi/a\sigma)$ respectively.

The critical value of the film thickness, below which the Neel boundary becomes more favorable, corresponds at $\beta' = 0$ to $\delta_c = 2.54$. However, when $\beta' \neq 0$, as can be seen from Fig. 2, $\Delta\gamma_B$ can cross $\Delta\gamma_N$ at two points, $\delta = \delta_c$ and $\delta = \delta'_c$. The Neel boundary can exist only in the thickness interval $\delta'_c < \delta < \delta_c$; as β' increases, this interval decreases. This can be seen especially clearly on Fig. 3, obtained from the equality $\Delta\gamma_B = \Delta\gamma_N$. When $\beta' > \beta'_c \approx 0.4(4\pi/a\sigma)$ the Bloch boundary is energetically more favored at any crystal thickness.

We note that the fields $\tilde{h}_x$ and $\tilde{h}_z$ can be obtained approximately with the aid of the demagnetizing factors, by approximating the boundary by means of an elliptic cylinder; it is not clear, however, what should be assumed for the effective width of a magnetic pole in the form $\operatorname{sech}\sigma x$. However, if $\Delta\gamma_B$ and $\Delta\gamma_N$, expressed in terms of the

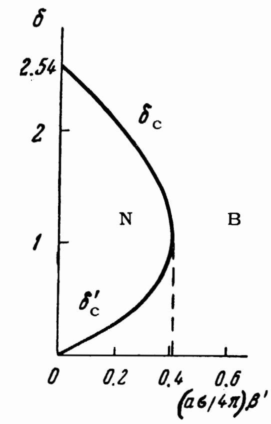

FIG. 3. N and B – regions of the existence of Neel and Bloch boundaries, respectively.

demagnetizing factors, are compared with the corresponding computer expressions (Fig. 2), it is easy to verify that this width should be taken equal to $5/\sigma$. Thus, (2.9) and (2.10) take the approximate form

$$\Delta\gamma_B \approx \frac{4\pi M^2}{\sigma}\left[\frac{1}{1+0.4\delta} - \frac{a}{d}\frac{\beta'}{4\pi}\right], \qquad \Delta\gamma_N \approx \frac{4\pi M^2}{\sigma}\frac{0.4\delta}{1+0.4\delta}. \tag{2.11}$$

The change in the form of the Bloch boundary is determined in the first approximation by the system (2.4) with boundary conditions (2.6) and with $c = 0$ and $s = 1$. Such a problem, with $\beta' = 0$, was solved by Muller and Dawson[13]; the solution was obtained in the form of multiple series. The general character of the variation of the effective width of the Bloch boundary, as a function of z for $\beta' = 0$, is shown in Fig. 4a. In addition to a decrease in the effective width of the boundary at the surface, a component $m_x(x) = -m_x(-x)$ appears.

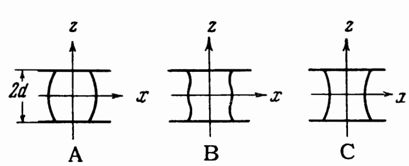

FIG. 4

Our approximate analysis for $\beta' \neq 0$ has shown that if β' is sufficiently large there can occur the cases shown in Figs. 4b and 4c, i.e., the effective width of the boundary can increase at the surface of the ferromagnet.

The effective width b of the boundary, averaged over the thickness of the film, can be obtained either from the first-approximation equations for a Bloch boundary by averaging them over ξ, or from expression (2.11) for $\Delta\gamma_B$:

$$b \approx \frac{5}{\sigma}\left(1+\frac{4\pi}{\beta}\frac{1}{1+0.4\delta}-\frac{a}{d}\frac{\beta'}{\beta}\right)^{-1/2}. \qquad (2.12)$$

We see therefore that, depending on the relation between the corrections due to the demagnetizing fields and the surface anisotropy, the average effective width of the Bloch boundary in a thin film can be either larger or smaller than the width of the boundary $\sim 5/\sigma$ in the bulk material.

The change in the form of the Neel boundary is determined in first approximation by the system (2.4) with boundary conditions (2.6) and with c = 1, s = 0.

3. PERIODIC BOUNDARY

There is no justification for assuming that the boundary is longitudinally uniform along the y axis in a crystal of finite thickness; this assumption was employed in most papers and in the preceding section only for the sake of simplicity. It is there that a boundary with variable polarity along its length may be energetically more favored, since this leads to a decrease in the energy of the demagnetizing fields (but simultaneously to an increase in the exchange energy). For sufficiently large d, we can regard the boundary as a thin film broken up into domains, and apply to it the results of the earlier papers [14,22].

For a complete solution of the problem of the boundary in a layer, it is necessary to find a solution periodic in y for the system (1.7) with boundary conditions (1.8). The expected form of the solution is shown in Fig. 5. Such a behavior of the solution is well approximated by the Jacobi elliptic functions cn(2K/T)y and sn(2K/T)y, which were used by Shirobokov [2] to investigate a periodic domain structure. The form and the period of the solution can vary over a wide range, depending on the value of T and on the modulus k of the complete elliptic integrals **K** and **E**.

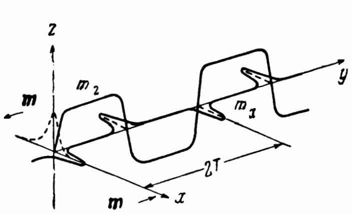

FIG. 5

If we write the solution in the form

$$m_x = \operatorname{cn} u \operatorname{sech}\eta + \Delta m_x(\eta, u, \xi),$$

$$m_y = \operatorname{th}\eta + \Delta m_y(\eta, u, \xi),$$

$$m_z = \operatorname{sn} u \operatorname{sech}\eta + \Delta m_z(\eta, u, \xi), \qquad (3.1)$$

where u = (2K/T)y, then the problem can be solved by perturbation theory (with small parameter $4\pi/\beta$), subject to the condition

$$\rho^2 \equiv (2\mathbf{K}/\sigma T)^2 \lesssim 4\pi/\beta.$$

In this case the zeroth approximation is the system (2.2) without the right side; thus, the energy in the zeroth approximation is $\gamma_0 = 2M^2(\alpha\beta)^{1/2}$, i.e., this solution is degenerate in the values of T and k.

The first-approximation system is

$$L_\mu^+ = \beta^{-1}[(h_x c + h_z s)\operatorname{th}\eta - h_y \operatorname{sech}\eta] - \rho^2(1-k^2s^2)\operatorname{th}\eta\operatorname{sech}\eta,$$

$$L_\mu^- = \beta^{-1}(h_x s - h_z c) + \rho^2 k^2 sc\operatorname{sech}\eta, \qquad (3.2)$$

and the first-approximation corrections Δm_i are expressed by relations (2.5), but now c ≡ cn u and s ≡ sn u. The boundary conditions are of the form (2.6). The fields h_i in (3.2) were obtained from (1.9), in which the zeroth approximation solution was substituted. Using Fourier transformations in η and in u, we obtain

$$h_i = \frac{1}{2\pi}\sum_{p=-\infty}^{+\infty}\exp\{i(2p-1)\pi u/2\mathbf{K}\}\int_{-\infty}^{+\infty} h_i^* e^{il\eta}\,dl,$$

$$h_x^* = \frac{2\pi^3}{k\mathbf{K}}\operatorname{sech}\frac{l\pi}{2}\left[(e^{-\lambda\delta}\operatorname{ch}\lambda\xi - 1)\frac{l^2}{\lambda^2}\operatorname{sech}\theta - \frac{l}{\lambda}e^{-\lambda\delta}\operatorname{sh}\lambda\xi\operatorname{cosech}\theta\right],$$

$$h_y^* = \frac{2\pi^3}{k\mathbf{K}}\frac{(2p-1)\pi}{\sigma T\lambda}\operatorname{sech}\frac{l\pi}{2}\left[(e^{-\lambda\delta}\operatorname{ch}\lambda\xi - 1)\frac{l}{\lambda}\operatorname{sech}\theta - e^{-\lambda\delta}\operatorname{sh}\lambda\xi\operatorname{cosech}\theta\right],$$

$$h_z^* = -i\frac{2\pi^3}{k\mathbf{K}}\operatorname{sech}\frac{l\pi}{2}e^{-\lambda\delta}\left[\frac{l}{\lambda}\operatorname{sh}\lambda\xi\operatorname{sech}\theta - \operatorname{ch}\lambda\xi\operatorname{cosech}\theta\right],$$

$$\theta = (2p-1)\pi\mathbf{K}'/2\mathbf{K}, \quad \lambda^2 = l^2 + [(2p-1)\pi/\sigma T]^2. \qquad (3.3)$$

The boundary energy, taking the first approximation into account, is

$$\gamma = \gamma_0 + \Delta\gamma_V + \Delta\gamma_S,$$

where

$$\Delta\gamma_V = \frac{1}{8\delta\sigma\mathbf{K}}\int_{-\infty}^{+\infty}d\eta\int_{-\delta}^{+\delta}d\xi\int_{-2\mathbf{K}}^{+2\mathbf{K}}du\,\mathcal{H}_V',$$

$$\Delta\gamma_S = \frac{a}{8\delta\mathbf{K}}\int_{-\infty}^{+\infty}d\eta\int_{-2\mathbf{K}}^{+2\mathbf{K}}du\,\mathcal{H}_S'. \qquad (3.4)$$

Just as in Sec. 2, we can readily show that $\Delta\gamma_V$ and $\Delta\gamma_S$ do not depend on μ^+ and μ^-, and that

$$\Delta\gamma_V = \frac{4\alpha M^2}{\sigma T^2}\mathbf{KE} - \frac{M^2}{8\sigma\mathbf{K}}\int_{-\infty}^{+\infty} d\eta \int_{-2\mathbf{K}}^{+2\mathbf{K}} du\,(\tilde{h}_x \mathbf{c} + \tilde{h}_z \mathbf{s})\operatorname{sech}\eta,$$

$$\Delta\gamma_S = -\frac{aM^2\beta'}{\sigma d}\frac{\mathbf{K}-\mathbf{E}}{k^2\mathbf{K}}, \qquad (3.5)$$

where $\tilde{h}_x$ and $\tilde{h}_z$ are the fields (3.3) averaged over ξ.

From the condition for the energy minimum we obtain the system of equations

$$\partial(\Delta\gamma_V + \Delta\gamma_S)\,/\,\partial T = 0, \qquad \partial(\Delta\gamma_V + \Delta\gamma_S)\,/\,\partial k = 0, \qquad (3.6)$$

from which we can obtain the period T and the modulus k as a function of the layer thickness 2d and the constants of the material; thus, the degeneracy with respect to T and k is lifted in the first approximation of perturbation theory.

The system (3.6) can be solved by computer methods. To obtain an idea of the character of the functions T(d) and k(d), the system was investigated in two limiting cases: $k \approx 1$ and $k \approx 0$. When $k \approx 1$ (corresponding to the values $\delta \gg \delta_c$, i.e., larger than the thickness of the crystal), we obtain

$$T \approx 0.16\delta\alpha^{1/2}\left(\frac{4\pi}{1+(4\pi/\beta)^{1/2}} + \frac{a}{d}\beta'\right)^{1/2},$$

$$k \approx 1 - 8\exp\left\{-0.16\delta\left(\frac{4\pi}{1+(4\pi/\beta)^{1/2}} + \frac{a}{d}\beta'\right)\right\}. \qquad (3.7)$$

With decreasing crystal thickness, the length of the Bloch sections of the boundary decreases, and that of the Neel sections increases very slowly, so that the period decreases. An increase in β' leads to an increase in the period, owing to the change in the relation between the energies of the Bloch and Neel sections (in analogy with the results of Sec. 2). When $\beta' = 0$, the value $k = 0$ (i.e., identical length of Bloch and Neel sections of the boundary) corresponds to $\delta = \delta_c$; the period then reaches a minimum. When $\delta < \delta_c$, the choice of a solution in the form (3.1) becomes unsatisfactory, because this thickness region corresponds to a zeroth-approximation of the form $m_x^0 \sim \operatorname{sn} u$ and $m_z^0 \sim \operatorname{cn} u$, i.e., when $\delta < \delta_c$ a decrease in the crystal thickness leads to an increase in the lengths of the Neel sections and a decrease in the lengths of the Bloch sections. When $\beta' \neq 0$, the picture becomes greatly complicated, but when $\beta' > \beta'_c$ the length of the Bloch sections exceeds that of the Neel sections at arbitrary crystal thickness.

CONCLUSION

The problem of the structure of the domain boundary was solved by perturbation theory with small parameter $4\pi/\beta$. This allows us to investigate, with a minimum of model assumptions, the complete system of the differential equations (1.7)—(1.9) describing the structure of the boundary in a uniaxial ferromagnet. The results of this investigation differ radically from the results of all earlier investigations, since this is the first time that real boundary conditions were used on the crystal surface (1.8), with account taken of the surface anisotropy β'.

Since $4\pi/\beta > 1$ for most ferromagnets, it was reasonable to break down the results of the calculations into two groups: a) results which admit of a formulation not critical to the choice of the calculation method, and consequently remain valid also when $4\pi/\beta > 1$; b) results valid only when $4\pi/\beta \ll 1$ (for example, for MnBi, for which $\beta \sim 50$ etc.). Such an analysis of the results can be carried out approximately by using various models of the boundary structure, similar to those used in all papers on the domain boundary in a thin magnetic film when $\beta/4\pi < 1$ (it is impossible to solve the problem (1.7)—(1.9) by perturbation theory with a small parameter $\beta/4\pi$, since the smallness of this parameter does not lead to a separation of the system (1.7) and (1.9)). The analysis has demonstrated the following.

The main result for a boundary which is longitudinally homogeneous (Fig. 3) remains approximately valid also when $4\pi/\beta > 1$, if $\sigma = (\beta/\alpha)^{1/2}$ is replaced by the quantity $\sigma^* \sim [(\beta + 2\pi)/\alpha]^{1/2}$. Included in group (a) is also the following result: the Bloch boundary narrows towards the surface of the crystal if $\alpha\beta'/d < 2\pi$, and broadens in the case of the opposite inequality; however, the magnitude of this effect and the details of the structure of the boundary at the surface of the crystal, which can be obtained from the solution of (2.4)—(2.6) (in particular, expression (2.12)), pertains to group (b). For a periodic boundary, the group (a) includes expressions (3.7) for the period and for k when $\delta \gg \delta_c$, and the general character of the variation of the periodic boundary as a function of the crystal thickness and of the value of β'; group (b) pertains to the exact values of T and k for small film thicknesses, which can be obtained from the solution of the system (3.6), and the corrections to the shape of the boundary, which can be calculated from (3.2).

The analysis has thus shown that all the results remain qualitatively (and some also quantitatively) correct also when the relation $4\pi/\beta \ll 1$ is not satisfied.

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Translated by J. G. Adashko
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