ON A CERTAIN POSSIBILITY OF MEASURING THE MAGNETIC MOMENT OF THE Σ^+ HYPERON

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The possibility of measuring the magnetic moment of the Σ^+ hyperon on the basis of the depolarization of Σ^+ hyperons from the $\pi^+ + p \to \Sigma^+ + K^+$ reaction[1)] in a photographic emulsion is discussed. A formula is obtained, which describes the behavior of the polarization of Σ^+ hyperons in matter, with account taken of the possible presence of an external magnetic field.

MEASUREMENT of the polarization of the Σ^+ hyperon in matter (especially in emulsion) can apparently be one of the possible methods of determining its magnetic moment. It is known that in the reaction

$$\pi^+ + p \to \Sigma^+ + K^+$$

the Σ^+ hyperons are produced polarized and a strong correlation is observed experimentally between the direction of emission of the pion from the hyperon decay and the vector $\mathbf{n} = \mathbf{P}_\pi \times \mathbf{P}_{\Sigma^+}$, which determines the production plane.

In the hyperon c.m.s., the angular distribution of the pions from decay of Σ^+ hyperons about the vector $\mathbf{n}$ is of the form $1 + a\cos\vartheta$. The asymmetry coefficient $a = a_0 P$ is determined by measuring the angular distribution of the decay pions in matter. Here P is the relative polarization of the Σ^+ hyperon at the instant of decay, i.e., the ratio of polarization in matter to polarization in vacuo (where $P = 1$). The constant a_0 (the asymmetry coefficient in vacuum) is determined by measuring the angular distribution in vacuum.

The Σ^+ hyperon is the heaviest positively-charged particle, and its slowing down in matter is similar to that of μ^+ mesons[1]. It is therefore natural to expect, just as in the case of the μ^+ meson, that the Σ^+ meson is depolarized by the matter, and that the main depolarization mechanism is production of a bound system comprising the Σ^+ hyperon and an electron (sigmonium). The presence for such a mechanism for the Σ^+ hyperon can be verified by measuring the behavior of the polarization in a magnetic field[2].

The sigmonium spin density matrix ρ, which determines uniquely the polarization of the Σ^+ hyperon, can be represented in the form

$$\rho = \sum_{\varkappa, k=0}^{3} \rho_{\varkappa k} U_\varkappa U_k, \qquad U_0 = \frac{\chi}{\sqrt{2}}, \qquad \mathbf{U} = \frac{\boldsymbol{\sigma}}{\sqrt{2}},$$

where χ is a unit operator and $\boldsymbol{\sigma}$ is the Pauli operator. The first subscript of $\rho_{\kappa k}$ pertains to the Σ^+ hyperon and the second to the electron.

The system of equations for the sigmonium density matrix, describing the simultaneous influence of the contact interaction in sigmonium, the external magnetic field, and the medium on the spin states of the Σ^+ hyperon, is of the form[1]

$$d\rho_{\varkappa 0} / dt = -{}^1\!/_2\omega_0 e_{\varkappa\lambda l}\rho_{\lambda l}(t) + \zeta e_{\varkappa\lambda\mu}\omega_\mu{}'\rho_{\lambda 0}(t),$$

$$d\rho_{0k} / dt = {}^1\!/_2\omega_0 e_{k\lambda l}\rho_{\lambda l}(t) + e_{kml}\omega_m{}'\rho_{0l}(t) - 2\nu\rho_{0k}(t),$$

$$d\rho_{\varkappa k} / dt = {}^1\!/_2\omega_0[e_{\varkappa k\lambda}\rho_{\lambda 0}(t) - e_{\varkappa kl}\rho_{0l}(t))]$$
$$- \zeta e_{\varkappa\mu\lambda}\omega_\mu{}'\rho_{\lambda k}(t) + e_{kml}\omega_m{}'\rho_{\varkappa l}(t) - 2\nu\rho_{\varkappa k}(t). \qquad (1)$$

Here κ, $k = 1, 2, 3$, $\boldsymbol{\omega}'$ is a vector in the direction of the magnetic field $\mathbf{H}$ with magnitude $\omega' = 2\beta H/\hbar$, $\zeta = \beta_{\Sigma^+}/\beta$ is the ratio of the magnetic moments of the Σ^+ hyperon and the electron, $\omega_0 = 32\beta\beta_{\Sigma^+}/3a_0^3\hbar$ is the frequency of the hyperfine splitting of the ground-state level of sigmonium, and ν is the average number of the sigmonium electron spin flips per second (i.e., the number of reversals of the spin direction).

Nosov and the author[1] have considered for the μ^+ meson the following two limiting cases: (1) $\nu \gg (\omega_0^2 + \omega'^2)^{1/2}$ (fast relaxation of the electronic spin of muonium), and (2) $\nu \ll (\omega_0^2 + \omega'^2)^{1/2}$ (slow relaxation of the electron spin). The second limiting case is realized in emulsion ($\nu \ll \omega_0^{(\mu^+)} = 2.78 \times 10^{10}\ \mathrm{sec}^{-1}$).

From a comparison of theory with experiment it was found for the μ^+ meson in emulsion that $\nu\tau = 80$, where τ is the time required for the muonium to enter in the chemical reaction, and

1)The idea of setting up such an experiment is due to M. I. Podgoretskiĭ.

consequently, the time of cessation of the μ^+-meson depolarization. At the same time, experiments with electronics[2] have shown that no depolarization of the μ^+ meson takes place after a time $\sim 10^{-7}-10^{-6}$ sec, i.e., this process is completed earlier, within a shorter time. We therefore obtain an upper estimate for τ, $< 10^{-7}$ sec, and consequently also for ν. We thus have

$$8\cdot 10^8 < \nu \ll 2.78\cdot 10^{10}\ \text{sec}^{-1}.$$

It is natural to assume that in the region of thermal velocities the value of ν for the Σ^+ hyperons is of the same order of magnitude for the μ^+ meson. Although some of the Σ^+ hyperons decay without even reaching the Bohr region[2)] ($v \sim 10^8$ cm/sec), they can be readily separated and excluded from consideration by taking into account Σ^+ hyperon decays at the end of track only. On the other hand, an estimate of the time necessary to slow down from $v \approx 10^8$ cm/sec to $v \approx 10^6-10^5$ cm/sec yields $t \approx 10^{-11}$ sec, which is much shorter than the Σ^+-hyperon lifetime $\tau_{\Sigma^+} \approx 10^{-10}$ sec. Such a result is to some degree promising and gives grounds for assuming that during their deceleration (from $v \approx 10^8$ to $v \approx 10^6$ cm/sec), the Σ^+ hyperons have no time to decay and that the decay takes place already at thermal velocities, i.e., during the muonium states, when $\nu_{\Sigma^+} \sim \nu_{\mu^+}$. For sigmonium in the thermal-velocity region, ν and $\omega_0^{\Sigma^+}$ turn out to be of the same order of magnitude. Therefore the formulas obtained in[1] for the polarization cannot be used in general, and a rigorous solution of the system (1) is necessary. The problem can be simplified by assuming that the direction of the external magnetic field coincides with the direction of the polarization of the Σ^+ hyperon[3)]. In this simpler case the system (1) takes the form

$$\begin{aligned}
d\rho_{30}/dt &= -\tfrac{1}{2}\omega_0[\rho_{12}(t) - \rho_{21}(t)],\\
d\rho_{03}/dt &= \tfrac{1}{2}\omega_0[\rho_{12}(t) - \rho_{21}(t)] - 2\nu\rho_{03}(t),\\
d\rho_{11}/dt &= -\omega'\rho_{12}(t) - 2\nu\rho_{11}(t),\\
d\rho_{22}/dt &= \omega'\rho_{21}(t) - 2\nu\rho_{22}(t),\\
d\rho_{12}/dt &= \tfrac{1}{2}\omega_0[\rho_{30}(t) - \rho_{03}(t)] + \omega'\rho_{11}(t)\\
&- 2\nu\rho_{12}(t), \qquad d\rho_{21}/dt = -\tfrac{1}{2}\omega_0[\rho_{30}(t) - \rho_{03}(t)]\\
&- \omega'\rho_{22}(t) - 2\nu\rho_{21}(t).
\end{aligned} \tag{2}$$

The quantity ζ is assumed to be equal to zero, in view of the smallness of the magnetic moment of the Σ^+ hyperon. The direction of the axis z = 3 coincides with the directions of polarization and of the magnetic field **H**.

The initial conditions, which reflect the fact that the sigmonium electron is not polarized at the instant of capture and that there is no polarization correlation, take the form

$$\rho_{30}(0) = \rho_{30}^{(0)}, \qquad \rho_{03}(0) = 0, \qquad \rho_{11} = \rho_{22} = 0,$$
$$\rho_{12} = \rho_{21} = 0. \tag{3}$$

The quantity $\rho_{30}(t)$, obtained by solving (2) with initial conditions (3), determines the time dependences of the Σ^+-hyperon polarization P(t)

$$P(t) = \rho_{30}(t) = \rho_{30}^{(0)}[ae^{-r_1 t} + e^{-r_2 t} + \{(b+ic)e^{ir_3 t} + \text{c.c.}\}];$$
$$r_1 = \tfrac{4}{3}\nu - v_+ - v_-, \quad r_2 = \tfrac{4}{3}\nu + \tfrac{1}{2}v_+ + \tfrac{1}{2}v_-,$$
$$r_3 = \tfrac{1}{2}\sqrt{3}(v_+ - v_-);$$
$$v_\pm = [\tfrac{2}{27}\nu(4\nu^2 + \tfrac{9}{4}\omega_0^2 + 9\omega'^2) \pm \tfrac{1}{54}\{27[4(\omega_0^2+\omega'^2)^3$$
$$+ 32\nu^4(\omega_0^2 + 2\omega'^2) - 13\omega_0^4\nu^2 + 32\omega'^4\nu^2 - 8\omega'^2\omega_0^2\nu^2\}^{1/2}]^{1/3};$$
$$a = \beta/\alpha, \qquad b = (\alpha-\beta)/2\alpha, \qquad c = \gamma/2\alpha r_3;$$
$$\alpha = (n^2+\omega'^2)[n(r_2^2+r_3^2) + r_1(r_3^2+l^2)] + 2\nu\omega_0^2 kn;$$
$$\beta = (n^2+\omega'^2)(r_2^2+r_3^2)n, \qquad \gamma = r_2(\beta-\alpha)$$
$$+ \omega_0^2 nk[r_2^2 + r_3^2]; \quad n = \tfrac{2}{3}\nu + v_+ + v_-,$$
$$l = \tfrac{2}{3}\nu - \tfrac{1}{2}v_+ - \tfrac{1}{2}v_-, \qquad k = \tfrac{1}{3}\nu - v_+ - v_-.$$

In the case of the μ^+ meson, its depolarization in the medium ceases because the muonium enters into a chemical reaction. However, in the case of sigmonium, the situation is apparently different. The lifetime of the Σ^+ hyperon, $\tau_{\Sigma^+} = 0.8\times 10^{-10}$ sec, is much shorter than that of the μ^+ meson, $\tau_{\mu^+} = 2\times 10^{-6}$ sec, so that there is little probability that the sigmonium has time to react chemically with the medium. Consequently, the depolarization of the Σ^+ hyperon ceases because of its decay, and P(t) must be integrated over the distribution $e^{-t/\tau}dt/\tau$ of the acts of Σ^+-hyperon depolarization cessation, replacing τ by τ_{Σ^+}—the lifetime of the Σ^+ hyperon[4)].

For the observed value of the polarization we obtain[5)]

$$\bar{P} = P_0\left\{\frac{a}{1+r_1\tau} + \frac{2[(1+r_2\tau)b - \tau r_3 c]}{(1+r_2\tau)^2 + r_3^2\tau^2}\right\}.$$

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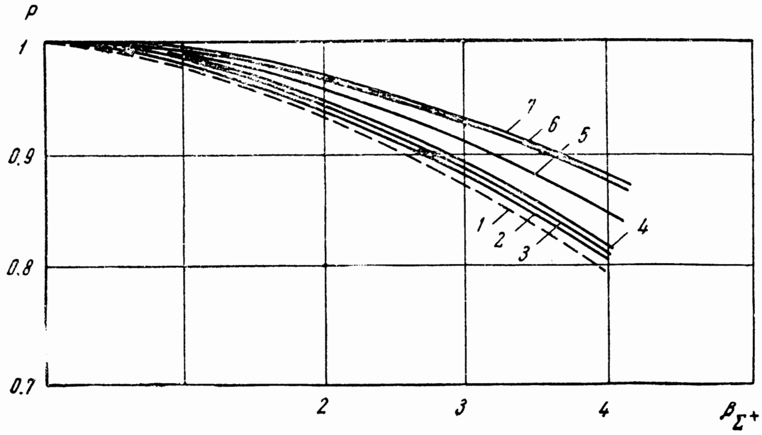
2)The time necessary for the Σ^+ hyperon to slow down to Bohr velocities is $t \sim 10^{-10}$ sec.

3)In a magnetic field of arbitrary orientation the transverse component of polarization vanishes upon averaging[3].

4)$1/\tau = 1/\tau_{\text{chem}} + 1/\tau^{(\Sigma^+)}$, where τ_{chem} is the time necessary for the sigmonium to enter in a chemical reaction.

5)This formula can be used also to investigate the behavior of μ^+ -meson polarization in substances for which ν and $\omega_{0(\mu^+)}$ have the same order of magnitude. In this case $\tau \sim \tau_{\text{chem}}$, where τ_{chem} is the time necessary for the muonium to enter into the chemical reaction.



P vs. the magnetic moment of the Σ^+ hyperon for different ν: 1 – $\nu = 0$, 2 – $\nu = 10^9$, 3 – $\nu = 1.6 \times 10^9$, 4 – $\nu = 3 \times 10^9$, 5 – $\nu = 5 \times 10^9$, 6 – $\nu = 8 \times 10^9$, 7 – $\nu = 10^{10}$. The magnetic moment is plotted in units of $\beta_{\Sigma^+} = e\hbar/2m_{\Sigma^+}c$.

The relative polarization is

$$P = \left\{\frac{a}{1 + r_1\tau} + \frac{2[(1 + r_2\tau)b - \tau r_3 c]}{(1 + r_2\tau)^2 + r_3^2\tau^2}\right\}.$$

In the case when ν of the substance in which the experiment is performed is known, we obtain a unique dependence of the relative polarization P on the magnetic moment β_{Σ^+} of the Σ^+ hyperon.

Thus, we can determine the magnetic moment of the Σ^+ hyperon by determining P from measurements of the angular distribution of the pions from the Σ^+ hyperon decay in a substance with known ν.

Usually such experiments are performed in emulsions. From the considerations presented above, we have for emulsions $\nu \approx 10^9$—10^{10} sec^{-1}. The figure shows the dependence of P on the magnetic moment β_{Σ^+} of the Σ^+ hyperon, at different values of ν in this interval. It has been assumed in the calculation that there is no magnetic field (H = 0), since this corresponds to the largest variation of P in matter, compared with P = 1 in vacuum.

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