CONTRIBUTION TO THE THEORY OF NONLINEAR INTERACTION OF WAVES IN A MAGNETOACTIVE ANISOTROPIC PLASMA

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Submitted to JETP editor February 12, 1965

J. Exptl. Theoret. Phys. (U.S.S.R.) **49**, 797-806 (September, 1965)

General equations that describe nonlinear interaction of waves in a magnetoactive anisotropic plasma are obtained. Explicit expressions are derived for the probabilities of scattering of normal waves by plasma electrons and ions and for the probabilities of the decay processes.

1. INTRODUCTION

THE problems of nonlinear interaction of waves in a plasma have recently attracted much attention, both in connection with the extensive expansion of experiments in which intense oscillations are excited in a plasma by various means (such as noise[1,2]), and in connection with the theoretical problems of finding the stationary or quasi-stationary spectra of the oscillations of a turbulent plasma.[3,4] Nonlinear interactions of waves in a plasma situated in a magnetic field were considered for some particular cases in[5,6] (see[3-14] concerning wave interaction in an isotropic plasma). In the present paper we attempt to use a previously developed procedure[13,14] to obtain general equations describing nonlinear interaction of waves in an isotropic plasma both in the presence and in the absence of external magnetic fields. Unlike in earlier papers[13,9], the results are not confined to the assumption that the plasma is isotropic[1)] even when H = 0, i.e., they are suitable for a description of the interaction of waves in a system of interpenetrating plasmas, in the presence of beams, etc. An important factor is that the results can be used for an analysis of the interaction and nonlinear conversion of non-potential oscillations and waves in a plasma, a fact of interest for the problem of interaction between a plasma and intense high frequency radiation in the radio band and in the optical band (see[14]; concerning non-potential oscillations of an anisotropic plasma see[15,16]).

We confine ourselves, as in[13], to an analysis of weakly damped waves, neglecting damping and introducing the concept of the number of quanta. This makes it possible to analyze in simplest fashion the interaction of waves having random phases. The equations obtained are limited by the assumption of small nonlinearity, i.e., weak turbulence of the plasma[17]. In this approximation, the nonlinear effects describe the interaction between waves that satisfy the dispersion relations of the linear theory.

Let us expand the linear fields in normal waves[18-20], introducing for the wave σ polarization vectors $\mathbf{a}_\sigma$:[2)]

$$(k^2\delta_{ij} - k_ik_j - \omega^2\varepsilon_{ij})a_{\sigma j}(\mathbf{k}) = 0, \quad \mathbf{a}_\sigma^*(\mathbf{k})\mathbf{a}_\sigma(\mathbf{k}) = 1. \quad (1.1)$$

We neglect the antihermitian part of ϵ_{ij}. We introduce ϵ^σ:

$$\varepsilon^\sigma(\omega, \mathbf{k}) = \varepsilon_{ij}(\omega, \mathbf{k})a_{j\sigma}(\mathbf{k})a^*_{i\sigma}(\mathbf{k}) + \omega^{-2}(\mathbf{k}\mathbf{a}_\sigma(\mathbf{k}))(\mathbf{k}\mathbf{a}_\sigma^*(\mathbf{k})). \quad (1.2)$$

If the dispersion equation contains only ω^2, we have two roots, denoted below by $\omega_{\sigma\pm}(\mathbf{k}) = \pm|\omega_\sigma(\mathbf{k})|$; the corresponding normal unit vectors are denoted by $\mathbf{a}_{\sigma\pm}(\mathbf{k})$. It is easy to see that $\mathbf{a}_{\sigma-}(-\mathbf{k}) = \mathbf{a}^*_{\sigma+}(\mathbf{k})$ and consequently, in the expansion of the field in normal waves

$$\mathbf{E}(\mathbf{r}, t) = \sum_{\sigma\pm}\int d\mathbf{k}\mathbf{a}_{\sigma\pm}(\mathbf{k})E_{\sigma\pm}(\mathbf{k})\exp\{i[\mathbf{k}\mathbf{r} - \omega_{\sigma\pm}(\mathbf{k})t]\} \quad (1.3)$$

we have $E_{\sigma-}(-\mathbf{k}) = E^*_{\sigma+}(\mathbf{k})$. From this we get for random phases

$$\langle E_i(\mathbf{k}, \omega)E_j(\mathbf{k}'\omega')\rangle = \sum_{\sigma\pm} a_{i\sigma\pm}(\mathbf{k})a^*_{j\sigma\pm}(\mathbf{k})|E_{\sigma\pm}(\mathbf{k})|^2 \times \delta[\omega - \omega_{\sigma\pm}(\mathbf{k})]\delta(\omega + \omega')\delta(\mathbf{k} + \mathbf{k}'). \quad (1.4)$$

[1)]Many problems involving the interaction of potential waves in an anisotropic plasma are considered in the cited literature, especially in[9].

[2)]It is convenient in what follows to use a gauge $a_4 = 0$.

The number of quanta for the normal wave $N_{\mathbf{k}}^{\sigma}$ is best introduced by comparing the quantum and classical expressions for the energy of the electromagnetic field:

$$|E_{\mathbf{k}\sigma}|^2 = \frac{1}{2\pi^2}\omega_\sigma^2(\mathbf{k})N_{\mathbf{k}}^\sigma \Big/ \left|\frac{\partial}{\partial\omega}\omega^2\varepsilon^\sigma(\omega,\mathbf{k})\right|_{\omega=\omega_{\sigma+}(\mathbf{k})}. \quad (1.5)$$

The introduction of the number of quanta enables us to interrelate the induced and spontaneous processes and makes it possible to obtain, by means of a simple semiquantum analysis, general expressions describing nonlinear wave interaction (Sec. 2). We also show that the sought probabilities, which are contained in these equations, can be expressed in general form, in the small-nonlinearity approximation, in terms of the components of the nonlinear plasma current (Sec. 3) and a tensor describing the oscillations of the charge in the field of the normal waves.

2. GENERAL EQUATIONS

To obtain general equations for the nonlinear interaction of plasma particles and normal waves, we introduce the probabilities of the different processes (the index σ pertains to the type of wave, and α to the particle species): $w^{\sigma\alpha}(\mathbf{p}_\alpha, k_\sigma)$ are the probability that a particle α with momentum $\mathbf{p}_\alpha$ will emit a wave σ with momentum $k_{\sigma'}$; $w^{\sigma\sigma'\alpha}(\mathbf{p}_\alpha, \mathbf{k}_\sigma, \mathbf{k}_{\sigma'})$ is the probability that a wave σ with momentum k_σ will be scattered by a particle α with momentum $\mathbf{p}_\alpha$ and be transformed into a wave σ' with momentum $k_{\sigma'}$; $u^{\sigma\sigma'\sigma''}(k_\sigma, k_{\sigma'}, k_{\sigma''})$ is the probability of decay of a wave σ into σ' and σ'' with corresponding momenta k_σ, $k_{\sigma'}$, and $k_{\sigma''}$ and frequencies ω_σ, $\omega_{\sigma'}$, and $\omega_{\sigma''}$; and $\tilde{w}^{\sigma\sigma'\alpha}(\mathbf{p}_\alpha, \mathbf{k}_\sigma, \mathbf{k}_{\sigma'})$ is the probability of emission of two waves. In the quantum approach, the motion of the particle must be characterized by quantum numbers p_z and n. It must be borne in mind here that the emission probability depends on $\nu = n' - n$.

In considering the scattering process it is convenient to write $n' - n = \nu - \nu'$, assuming that $w^{\sigma\sigma'\alpha}$ depends on ν and ν'. The equations of nonlinear interaction of the waves are balance equations for the number of particles and the number of waves. The derivation of such equations is a simple generalization of [20]. We present expressions for the variation of the number of quanta $N_{\mathbf{k}}^{\sigma}$, assuming that the wave function of the particles $f(\mathbf{p}_\alpha)$ depends only on $p_\perp$ and p_z, which characterize the parameters of particle motion along a helical line. We have

$$\frac{\partial N_{\mathbf{k}}^\sigma}{\partial t} = \sum_{\alpha\nu\nu'}\int w_\nu^{\sigma\alpha}(\mathbf{p}_\alpha, k_\sigma)N_{\mathbf{k}}^\sigma \times\left[\left(k_{\sigma z}\frac{\partial f^\alpha}{\partial p_z^\alpha}\right)+\frac{\nu\varepsilon^\alpha\omega_H^\alpha}{p_\perp^\alpha}\frac{\partial f^\alpha}{\partial p_\perp^\alpha}\right]d\mathbf{p}_\alpha$$

$$+\sum_{\alpha\sigma'\nu'\nu}\int w_{\nu\nu'}^{\sigma\sigma'\alpha}(\mathbf{p}_\alpha, k_\sigma, k_{\sigma'})N_{\mathbf{k}}^\sigma N_{\mathbf{k}}^{\sigma'}\left[(k_{\sigma z}-k_{\sigma' z})\frac{\partial f^\alpha}{\partial p_z^\alpha} + \frac{(\nu'-\nu)\varepsilon^\alpha\omega_H^\alpha}{p_\perp^\alpha}\frac{\partial f^\alpha}{\partial p_\perp^\alpha}\right]d\mathbf{p}_\alpha\, d\mathbf{k}_{\sigma'}$$

$$+\sum_{\alpha\sigma'\nu'\nu}\int \tilde{w}_{\nu\nu'}^{\sigma\sigma'\alpha}(\mathbf{p}_\alpha, k_\sigma, k_{\sigma'})N_{\mathbf{k}}^\sigma N_{\mathbf{k}}^{\sigma'} \times\left[(k_{\sigma z}+k_{\sigma' z})\frac{\partial f^\alpha}{\partial p_z^\alpha} + \frac{(\nu'+\nu)\varepsilon^\alpha\omega_H^\alpha}{p_\perp^\alpha}\frac{\partial f^\alpha}{\partial p_\perp}\right]d\mathbf{p}_\alpha\, d\mathbf{k}_{\sigma'}$$

$$+\sum_{\sigma'\sigma''}\int d\mathbf{k}_{\sigma'}\,d\mathbf{k}_{\sigma''}\,[(N_{\mathbf{k}}^{\sigma'}N_{\mathbf{k}}^{\sigma''}-N_{\mathbf{k}}^{\sigma}N_{\mathbf{k}}^{\sigma'}-N_{\mathbf{k}}^{\sigma}N_{\mathbf{k}}^{\sigma''}) \times u^{\sigma\sigma'\sigma''}(k_\sigma, k_{\sigma'}, k_{\sigma''}) + (N_{\mathbf{k}}^{\sigma'}N_{\mathbf{k}}^{\sigma''}+N_{\mathbf{k}}^{\sigma}N_{\mathbf{k}}^{\sigma'}-N_{\mathbf{k}}^{\sigma}N_{\mathbf{k}}^{\sigma''})$$

$$\times u^{\sigma\sigma'\sigma''}(k_\sigma, -k_{\sigma'}, k_{\sigma''}) + (N_{\mathbf{k}}^{\sigma'}N_{\mathbf{k}}^{\sigma''}-N_{\mathbf{k}}^{\sigma}N_{\mathbf{k}}^{\sigma'}+N_{\mathbf{k}}^{\sigma}N_{\mathbf{k}}^{\sigma''}) \times u^{\sigma\sigma'\sigma''}(k_\sigma, k_{\sigma'}, -k_{\sigma''})], \quad (2.1)$$

where

$$k_\sigma = \{\mathbf{k}_\sigma, \omega_\sigma\}, \qquad \omega_H^\alpha = \frac{|eH|}{\varepsilon^\alpha}.$$

The equations for $f(\mathbf{p}_\alpha)$ are not given here, since they will not be used. In Eq. (2.1) we take into account only induced processes.

To obtain Eqs. (2.1)—(2.4) there is no need of using the complete system of nonlinear plasma equations and averaging the resultant relations (see, for example, [9a]), and we can use the correspondence principle [13,14], assuming, for example, that one of the $N_{\mathbf{k}}^{\sigma}$ is quite small, so that one can confine oneself to an examination of spontaneous processes. The equations for the waves then take the form [3)]

$$\frac{\partial N_{\mathbf{k}}^\sigma}{\partial t} = \sum_{\alpha\sigma'\nu'\nu}\int d\mathbf{p}_\alpha f(\mathbf{p}_\alpha)\,d\mathbf{k}_{\sigma'}N_{\mathbf{k}}^{\sigma'}(\tilde{w}_{\nu\nu'}^{\sigma\sigma'}(\mathbf{p}_\alpha, k_\sigma, k_{\sigma'}) + \tilde{w}_{\nu\nu'}^{\sigma\sigma'}(\mathbf{p}_\alpha, k_\sigma, k_{\sigma'}))$$

$$+\sum_{\sigma\sigma'}\int d\mathbf{k}_{\sigma'}\,d\mathbf{k}_{\sigma''}N_{\mathbf{k}}^{\sigma'}N_{\mathbf{k}}^{\sigma''}[u^{\sigma\sigma'\sigma''}(k_\sigma, k_{\sigma'}, k_{\sigma''}) + u^{\sigma\sigma'\sigma''}(k_\sigma, -k_{\sigma'}, k_{\sigma''}) + u^{\sigma\sigma'\sigma''}(k_\sigma, k_{\sigma'}, -k_{\sigma''})]. \quad (2.2)$$

It is important that to find the sought probabilities it is sufficient to employ (2.2).

3) Unlike in (2.1), the expression for the scattering takes into account the spontaneous scattering.

Let us find the change in the energy of the normal wave σ, due to decay and scattering. This change has, owing to (2.2), the form

$$Q^\sigma = \frac{\partial w^\sigma}{\partial t} = \sum_\alpha \int d\mathbf{p}_\alpha f(\mathbf{p}_\alpha) Q_\alpha^\sigma(\mathbf{p}_\alpha)$$

$$+ \sum_{\sigma\sigma'} \int d\mathbf{k}_{\sigma'} d\mathbf{k}_\sigma N_\mathbf{k}^{\sigma'} N_\mathbf{k}^{\sigma} |\omega_\sigma| (2\pi)^{-3}$$

$$\times [u^{\sigma\sigma'\sigma''}(k_\sigma, k_{\sigma'}, k_{\sigma''}) + u^{\sigma\sigma'\sigma''}(k_\sigma, -k_{\sigma'}, k_{\sigma''})$$

$$+ u^{\sigma\sigma'\sigma''}(k_\sigma, k_{\sigma'}, -k_{\sigma''})]. \qquad (2.3)$$

Here $Q^\sigma(\mathbf{p}_\alpha)$ is the change in energy upon scattering of a single test charge of the plasma in the field of the wave σ:

$$Q^\sigma(\mathbf{p}_\alpha) = \sum_{\sigma' vv'} \int \omega(\mathbf{k}_\sigma) d\mathbf{k}_\sigma d\mathbf{k}_{\sigma'} N_\mathbf{k}^{\sigma'} (w_{vv'}^{\sigma\sigma'}(\mathbf{p}_\alpha, k_\sigma, k_{\sigma'})$$

$$+ \tilde{w}_{vv'}^{\sigma\sigma'}(\mathbf{p}_\alpha, k_\sigma, k_{\sigma'}))(2\pi)^{-3}.$$

The last relation makes it possible to determine the probabilities of scattering by considering the emission of the wave σ by a test charge in the field of the normal waves σ'.

3. CONNECTION BETWEEN DECAY PROBABILITIES AND THE NONLINEAR PLASMA CURRENT COMPONENTS

Let us consider a certain current $\mathbf{j}(\mathbf{k}, \omega)$ in the plasma. It is easy to find the intensity of its radiation from (see[13])

$$Q^\sigma = -\lim_{T\to\infty} \frac{(2\pi)^4}{T} \int d\mathbf{k}\, \mathbf{j}(\mathbf{k}, \omega) \mathbf{E}(\mathbf{k}, \omega) d\omega, \qquad (3.1)$$

where $\mathbf{E}(\mathbf{k}, \omega)$ is the field produced by the current $\mathbf{j}(\mathbf{k}, \omega)$. Expanding $\mathbf{E}(\mathbf{k}, \omega)$ in normal waves and using Maxwell's equations to express $\mathbf{E}(\mathbf{k}, \omega)$ in terms of $\mathbf{j}(\mathbf{k}, \omega)$, we obtain for a weakly absorbing medium

$$Q^\sigma = \lim_{T\to\infty} \frac{(2\pi)^6}{T} \int d\omega\, d\mathbf{k} |\omega| \sum_{\sigma\pm} |\mathbf{a}_{\sigma\pm}^*(\mathbf{k}) \mathbf{j}(\mathbf{k}, \omega)|^2$$

$$\times \delta[\omega - \omega_{\sigma\pm}(\mathbf{k})] \left| \frac{\partial}{\partial\omega} \omega^2 \varepsilon^\sigma(\omega, \mathbf{k}) \right|^{-1}. \qquad (3.2)$$

When the nonlinearity is weak the plasma current can be regarded as a quadratic function of the intensity of the external field:

$$j_i^{(2)}(\mathbf{k}, \omega)$$

$$= \int d\lambda S_{ijs}(\mathbf{k}, \omega, \mathbf{k}_1, \omega_1, \mathbf{k}_2, \omega_2) E_j(\mathbf{k}_1, \omega_1) E_s(\mathbf{k}_2, \omega_2),$$

$$d\lambda = d\mathbf{k}_1 d\mathbf{k}_2 d\omega_1 d\omega_2 \delta(\mathbf{k} - \mathbf{k}_1 - \mathbf{k}_2) \delta(\omega - \omega_1 - \omega_2). \qquad (3.3)$$

It is important that in the employed approach it is sufficient, for the calculation of the probabilities, to confine oneself to the calculation of the nonlinear plasma current $\mathbf{j}^{(2)}$.

The result (3.2) can be used to calculate both the probabilities of the decay processes and the scattering probabilities. Starting from (3.2) and (3.3), assuming that $\mathbf{j}^{(2)}(\mathbf{k}, \omega)$ is the nonlinear current which is produced in the plasma when normal waves are present in it, we can relate the decay probabilities with S_{ijs}. Expanding the fields in normal waves (1.3), substituting (3.3) in (3.2), and using the fact that the average values of the products of four E_σ can be broken up with the required accuracy into paired products, we obtain

$$Q^\sigma = 2(2\pi)^2 \int d\mathbf{k}\, d\mathbf{k}_1 d\mathbf{k}_2 \sum_{\sigma\pm,\, \sigma'\pm,\, \sigma''\pm} |\omega_{\sigma\pm}(\mathbf{k})| \left| \frac{\partial}{\partial\omega} \omega^2 \varepsilon^\sigma \right|^{-1}_{\omega=\omega_\sigma(\mathbf{k})}$$

$$\times |S_{\sigma\pm,\sigma'\pm,\sigma''\pm}(\omega_{\sigma\pm}(\mathbf{k}), \mathbf{k}, \omega_{\sigma'\pm}(\mathbf{k}_1), \mathbf{k}_1, \omega_{\sigma''\pm}(\mathbf{k}_2), \mathbf{k}_2)|^2$$

$$\times \delta(\mathbf{k} - \mathbf{k}_1 - \mathbf{k}_2) \delta[\omega_\sigma(\mathbf{k})$$

$$- \omega_{\sigma'}(\mathbf{k}_1) - \omega_{\sigma''}(\mathbf{k}_2)] |E_{k_1\sigma'}|^2 |E_{k_2\sigma''}|^2, \qquad (3.4)$$

$$S_{\sigma\sigma'\sigma''}(\mathbf{k}, \omega, \mathbf{k}_1, \omega_1, \mathbf{k}_2, \omega_2)$$

$$= a_{\sigma i}^*(\mathbf{k}) S_{ijs}(\mathbf{k}, \omega, \mathbf{k}_1, \omega_1, \mathbf{k}_2, \omega_2) a_{\sigma' j}(\mathbf{k}_1) a_{\sigma'' s}(\mathbf{k}_2). \qquad (3.5)$$

In the derivation of (3.4) we used the fact that

$$S_{ijs}(\mathbf{k}, \omega, \mathbf{k}_1, \omega_1, \mathbf{k}_2, \omega_2) = S_{isj}(\mathbf{k}, \omega, \mathbf{k}_2, \omega_2, \mathbf{k}_1, \omega_1). \qquad (3.6)$$

This symmetry condition can always be satisfied, since (3.3) contains only the part of S_{ijs} satisfying this condition.

To obtain the decay probabilities it is sufficient to express $|E_{k\sigma}|^2$ in terms of the number of quanta and to compare (3.5) with (2.4)[4]:

$$u^{\sigma\sigma'\sigma''}(k_\sigma, k_{\sigma'}, k_{\sigma''}) = 16\pi\, \delta(\mathbf{k}_\sigma - \mathbf{k}_{\sigma'} - \mathbf{k}_{\sigma''}) \delta[\omega(\mathbf{k}_\sigma) - \omega(\mathbf{k}_{\sigma'})$$

$$- \omega(\mathbf{k}_{\sigma''})] \omega^2(\mathbf{k}_{\sigma'}) \omega^2(\mathbf{k}_{\sigma''})$$

$$\times |S_{\sigma\sigma'\sigma''}(\mathbf{k}_\sigma, \omega(\mathbf{k}_\sigma), \mathbf{k}_{\sigma'}, \omega(\mathbf{k}_{\sigma'}), \mathbf{k}_{\sigma''}, \omega(\mathbf{k}_{\sigma''}))|^2$$

$$\times \left| \frac{\partial}{\partial\omega} \omega^2 \varepsilon^\sigma \right|^{-1}_{\omega=\omega(\mathbf{k}_\sigma)} \left| \frac{\partial}{\partial\omega} \omega^2 \varepsilon^{\sigma'} \right|^{-1}_{\omega=\omega(\mathbf{k}_{\sigma'})}$$

$$\times \left| \frac{\partial}{\partial\omega} \omega^2 \varepsilon^{\sigma''} \right|^{-1}_{\omega=\omega(\mathbf{k}_{\sigma''})}; \qquad (3.7)$$

$$\omega_\sigma = |\omega_{\sigma\pm}|, \qquad S_{\sigma\sigma'\sigma''} = S_{\sigma+,\sigma'+,\sigma''+}.$$

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[4]In the derivation of (3.7) we used the following relations, which follow from the fact that the nonlinear current is real:

$$S_{ijs}(-\mathbf{k}, -\omega, -\mathbf{k}_1, -\omega_1, -\mathbf{k}_2, -\omega_2) = S^*_{jis}(\mathbf{k}, \omega, \mathbf{k}_1, \omega_1, \mathbf{k}_2, \omega_2),$$

$$S^*_{\sigma-,\sigma'-,\sigma''-}(-\mathbf{k}, -\omega, -\mathbf{k}_1, -\omega_1, -\mathbf{k}_2, -\omega)$$

$$= S_{\sigma+,\sigma'+,\sigma''+}(\mathbf{k}, \omega, \mathbf{k}_1, \omega_1, \mathbf{k}_2, \omega_2).$$

Further, in order to write out the probabilities of the decays $u^{\sigma-\sigma'\sigma''}(k_\sigma, -k_{\sigma'}, k_{\sigma''})$ and $u^{\sigma\sigma'-\sigma''}(k_\sigma, k_{\sigma'}, -k_{\sigma''})$, which correspond either to absorption of $k_{\sigma'}$ or to the absorption of $k_{\sigma''}$, it is necessary to interchange in (3.7) the signs of the corresponding four-momenta of the quanta, which reduces to a reversal of the signs in the conservation laws and in the arguments of S_{ijs}; it is also necessary to recognize that

$$S_{\sigma,\,-\sigma',\,\sigma''} = S_{\sigma+,\,\sigma'-,\,\sigma''+}, \qquad S_{\sigma,\,\sigma',\,-\sigma''} = S_{\sigma+,\,\sigma'+,\,\sigma''-}. \quad (3.8)$$

The nonlinear current in a plasma situated in a magnetic field can be determined by solving the system of nonlinear equations obtained by expanding the kinetic equation in the field amplitudes:

$$S_{ijs}^{(\alpha)}(\mathbf{k}, \omega, \mathbf{k}_1, \omega_1, \mathbf{k}_2, \omega_2) = -\sum_{\mu\mu'\nu\nu'} 2\pi e^3 \int p_\perp\, dp_\perp\, dp_z\, \delta_{\mu'+\nu',\,\mu+\nu}$$
$$\times \exp[i(\mu'-\mu)\varphi_0 + i(\nu'-\nu)\varphi_2](\omega - k_z v_z - \mu\omega_H{}^\alpha)^{-1}$$
$$\times A_i \hat{B}_j (\omega_2 - k_{2z} v_z - \nu'\omega_H{}^\alpha)^{-1} \hat{\Gamma}_s f_0^{(\alpha)}(p_\perp, p_z), \quad (3.9)$$

where

$$\hat{B}_1 = \frac{1}{2} J_{\mu'+1} e^{i\varphi_0}(\hat{D}+L) + \frac{1}{2} J_{\mu'-1} e^{-i\varphi_0}(\hat{D}+L)$$
$$- i\frac{(\nu'-\nu)k_{1y}}{m_\alpha\omega_1} J_{\mu'}, \quad \hat{B}_2 = -i\left[\frac{1}{2} J_{\mu'+1} e^{i\varphi_0}(\hat{D}-L)\right.$$
$$\left. + \frac{1}{2} J_{\mu'-1} e^{-i\varphi_0}(\hat{D}-L) - \frac{(\nu'-\nu)k_{1x}}{m_\alpha\omega_1} J_{\mu'}\right],$$
$$\hat{B}_3 = \frac{k_{1x} - ik_{1y}}{2\omega_1}\,\frac{\hat{D}'+L'}{2}\, e^{i\varphi_0} J_{\mu'+1}$$
$$+ \frac{k_{1x}+ik_{1y}}{2\omega_1}\,\frac{\hat{D}'-L'}{2}\, e^{-i\varphi_0} J_{\mu'-1} + J_{\mu'}\frac{\partial}{\partial p_z},$$
$$A_{1,2} = v_\perp (J_{\mu-1}e^{i\varphi_0} \pm J_{\mu+1}e^{-i\varphi_0}), \quad A_3 = v_z J_\mu, \quad v_\perp = \frac{p_\perp}{\varepsilon},$$
$$v_z = \frac{p_z}{\varepsilon}, \quad \hat{D} = \frac{\partial}{\partial p_\perp} - \frac{k_{1z}}{\omega_1}\hat{D}', \quad \hat{D}' = v_z\frac{\partial}{\partial p_\perp} - v_\perp\frac{\partial}{\partial p_z},$$
$$L = \frac{\nu'-\nu}{p_\perp}\left(1 - \frac{k_{1z}v_z}{\omega_1}\right), \quad L' = \frac{(\nu'-\nu)v_z}{p_\perp},$$
$$\hat{\Gamma}_1 = \frac{\nu J_{\nu'} J_\nu}{z_2}\hat{D}_2, \quad \hat{\Gamma}_2 = -iJ_\nu\frac{\partial J_\nu}{\partial z_2}\hat{D},$$
$$\hat{\Gamma}_3 = \frac{\nu' J_{\nu'} J_\nu}{z_2}\hat{D} - J_\nu J_{\nu'}\frac{\partial}{\partial p_z}, \quad \hat{D}_2 = \frac{\partial}{\partial p_\perp} - \frac{k_{2z}}{\omega_2}\hat{D},$$
$$\sin\varphi_0 = \frac{k_y}{|k_\perp|}, \quad \sin\varphi_2 = \frac{k_{2y}}{|k_{2\perp}|}.$$

The argument of the Bessel functions J_ν and $J_{\nu'}$ is $z_2 = k_{2\perp}v_\perp^\alpha/\omega_H^\alpha$, and that of the functions J_μ and $J_{\mu'}$ is $z = k_\perp v_\perp^\alpha/\omega_H^\alpha$; the distribution $f_0(p_\perp, p_z)$ is not assumed Maxwellian anywhere.

If, in particular, the distributions of the ions and of the electrons are Maxwellian, and the magnetic field is large so that $k_\perp v_\perp^\alpha \ll \omega_H^\alpha$ for all waves, then the tensor S_{ijs} becomes much simpler. Let us consider some cases of decays of transverse waves into transverse waves, transitions forbidden in the isotropic case by the conservation laws. If the waves propagate strictly along the field, the decay is forbidden by the angular momentum conservation law. Let us find the probability of the decay of an extraordinary high-frequency wave ($\omega_1 \gg \omega_{He}$) into an extraordinary high-frequency wave ($\omega_2 \gg \omega_{He}$) and into an extraordinary wave of frequency $\omega \sim \omega_{He}$, with all the foregoing waves propagating transverse to the field. Substituting (3.9) in (3.7), we get

$$u^{ttt} = \frac{e^2\omega_{0e}{}^4}{8\pi m_e{}^2\omega_{He}{}^3}\,\frac{k^2}{\omega_1\omega_2}\left[1 + \sum_\alpha \frac{\omega_{0\alpha}{}^2}{\omega_{H\alpha}{}^2(1-\omega_{H\alpha}/\omega)^2}\right]^{-1}$$
$$\times \delta(\omega - \omega_1 + \omega_2)\,\delta(\mathbf{k} - \mathbf{k}_1 + \mathbf{k}_2). \quad (3.10)$$

The decay of a high frequency extraordinary wave into a high frequency extraordinary and magnetic-sound wave with propagation transverse to the field is described by the formula

$$v_A = \omega_{Hi}/\omega_{0i}$$

$$u^{tMt} = \sum_\alpha \frac{e^2\omega_{0e}{}^4 v_A k_M{}^2}{8\pi m_\alpha{}^2\omega_1\omega_2\omega_{He}}\,\delta(\omega_M - \omega_1 + \omega_2)\,\delta(\mathbf{k}_M - \mathbf{k}_1 + \mathbf{k}_2). \quad (3.11)$$

4. SCATTERING PROBABILITIES

We now proceed to the calculation of the scattering cross sections. To obtain these cross sections we must regard $\mathbf{j}(\mathbf{k}, \omega)$ in (3.2) as the current produced in the plasma by a charge $\mathbf{p}_\alpha$ with account of the perturbations of the motion of the charge by the normal waves of the plasma. The part of $\mathbf{j}(\mathbf{k}, \omega)$ independent of the wave field gives the probabilities $w_\nu^\sigma(\mathbf{p}_\alpha)$ for the emission of waves by a charge. On the other hand, the probabilities of scattering are described by the part of the current $\mathbf{j}(\mathbf{k}, \omega)$ which is proportional to the first power of the electric field of the waves:

$$j_i^{(\alpha)}(\mathbf{k}, \omega) = \int d\mathbf{k}_1\, d\omega_1 \Lambda_{ij}^{(\alpha)}(\mathbf{k}, \omega, \mathbf{k}_1, \omega_1) E_j(\mathbf{k}_1, \omega_1). \quad (4.1)$$

It is necessary to take into account here the fact that $\Lambda_{ij}^{(\alpha)}$ consists of two parts, which describe two physically different scattering mechanisms:

$$\Lambda_{ij}^{(\alpha)}(\mathbf{k}, \omega, \mathbf{k}_1, \omega_1) = \Lambda_{ij}^{\alpha(1)}(\mathbf{k}, \omega, \mathbf{k}_1, \omega_1)$$
$$+ \Lambda_{ij}^{\alpha(2)}(\mathbf{k}, \omega, \mathbf{k}_1, \omega_1). \quad (4.2)$$

The first, $\Lambda^{(1)}$, is connected with the oscillations of the charge in the field of the waves, while the second, $\Lambda^{(2)}$ is connected with polarization effects which arise in the plasma and are interpreted in particular cases as emissions of the type of transition radiation from the plasma inhomogeneities produced by the plasma waves[13]. An important factor is that in the case of electrons these currents can cancel each other, greatly reducing the scattering cross section. For ions, owing to their large mass, only $\Lambda^{(2)}$ is of importance for high frequency waves; therefore scattering by ions, if allowed by the conservation laws, yields nonlinear-interaction effects which exceed greatly the effects of induced scattering by the electrons[13].

Expanding in terms of normal waves in (4.1), and then substituting (4.1) in (2.4), we obtain after averaging over the phases

$$\sum_{\sigma\pm} Q^\sigma(\mathbf{p}_\alpha) = \lim \frac{(2\pi)^6}{T} \sum_{\sigma\pm,\,\sigma'\pm} \int d\mathbf{k}\, d\mathbf{k}_1\, \omega_\sigma(\mathbf{k})\,\omega_{\sigma'}{}^2(\mathbf{k}_1)$$
$$\times \left|\frac{\partial}{\partial\omega}\,\omega^2\varepsilon^\sigma\right|^{-1}_{\omega=\omega_\sigma(\mathbf{k})} \left|\frac{\partial}{\partial\omega}\,\omega^2\varepsilon^{\sigma'}\right|^{-1}_{\omega=\omega_{\sigma'}(\mathbf{k}_1)}$$
$$\times \left|\Lambda^{\sigma\pm,\,\sigma'\pm}_{p_\alpha}(\mathbf{k}, \omega_{\sigma\pm}(\mathbf{k}), \mathbf{k}_1, \omega_{\sigma'\pm}(\mathbf{k}_1))\right|^2,$$
$$\Lambda^{\sigma\pm,\,\sigma'\pm}_{p_\alpha}(\mathbf{k}, \omega_{\sigma\pm}(\mathbf{k}), \mathbf{k}_1, \omega_{\sigma'\pm}(\mathbf{k}_1))$$
$$= a^*_{i\sigma\pm}(\mathbf{k})\Lambda_{ij}(\mathbf{k}, \omega, \mathbf{k}_1, \omega_1)\, a_{j\sigma'\pm}(\mathbf{k}_1). \qquad (4.3)$$

Substituting in (3.3) $\mathbf{E}$ in the form of the sum of the wave field and the field produced by the charge moving on a helical line, we can readily express $\Lambda^{(2)}$ in terms of S_{ijs}:

$$\Lambda_{ij}{}^{(2)}(\mathbf{k}, \omega, \mathbf{k}_1, \omega_1) = [S_{ijs}(\mathbf{k}, \omega, \mathbf{k}-\mathbf{k}_1, \omega-\omega_1, \mathbf{k}_1, \omega_1)$$
$$+ S_{isj}(\mathbf{k}, \omega, \mathbf{k}_1, \omega_1, \mathbf{k}-\mathbf{k}_1, \omega-\omega_1)]$$
$$\times E_j{}^{(\alpha)}(\mathbf{k}-\mathbf{k}_1, \omega-\omega_1). \qquad (4.4)$$

We see from (4.3) and (4.4) that the frequency of the charge field $\omega - \omega_1 = \omega_{\sigma\pm} - \omega_{\sigma'\pm}$ is generally speaking not close to the frequency of the normal field oscillations. It is clear, however, that the inverse Maxwellian operator has poles when $\omega = \omega_{\sigma\pm}(\mathbf{k})$

$$\Pi_{ij}(\omega, \mathbf{k}) = \sum_{\sigma''} \frac{\Pi_{ij}{}^{\sigma''}(\omega, \mathbf{k})}{k^2/\omega^2 - \varepsilon^{\sigma''}(\omega, \mathbf{k})},$$
$$(k^2\delta_{ij} - k_ik_j - \omega^2\varepsilon_{ij})\Pi_{jq} = \delta_{iq},$$
$$k^2/\omega_{\sigma''}{}^2 - \varepsilon^{\sigma''}(\mathbf{k}, \omega_{\sigma''}) = 0. \qquad (4.5)$$

The individual terms of this series describe the process of scattering via a virtual polarization wave σ'', while the individual terms entering in the operator are the Green's functions for the corresponding waves. We obtain

$$E_s{}^{(\alpha)} = \Pi_{se}(\omega, \mathbf{k})\frac{e}{(2\pi)^3}\Gamma_e{}^\nu\delta[\omega - k_zv_z - \nu\omega_H{}^\alpha], \qquad (4.6)$$

$$\Gamma_1{}^\nu = v_\perp\frac{\nu J_\nu(z)}{z}, \quad \Gamma_2{}^\nu = -iv_\perp\frac{\partial J_\nu(z)}{\partial z},$$
$$\Gamma_3 = v_zJ_\nu(z), \quad z = \frac{k_\perp v_\perp{}^\alpha}{\omega_H{}^\alpha}. \qquad (4.7)$$

Substituting (4.4)—(4.7) in (4.3) and comparing with (2.5), we obtain the scattering probability in the form

$$w_\nu{}^{\sigma\sigma'}(\mathbf{p}_\alpha, k_\sigma, k_{\sigma'}) = 2\omega_{\sigma'}{}^2(k_{\sigma'})\,\delta[\omega_\sigma(\mathbf{k}_\sigma) - \omega_{\sigma'}(\mathbf{k}_{\sigma'})$$
$$-(k_{\sigma z} - k_{\sigma' z})v_z - \nu\omega_H{}^\alpha]\left|\frac{\partial}{\partial\omega}\,\omega^2\varepsilon^\sigma\right|^{-1}_{\omega=\omega_\sigma(\mathbf{k}_\sigma)}$$
$$\times\left|\frac{\partial}{\partial\omega}\,\omega^2\varepsilon^{\sigma'}\right|^{-1}_{\omega=\omega_{\sigma'}(\mathbf{k}_{\sigma'})} \times \left|\Lambda^{(1)\sigma\pm\sigma'\pm}_{p_\alpha} + a_{i\sigma+}(\mathbf{k}_\sigma)\right.$$
$$\times [S_{isj}(\mathbf{k}_\sigma, \omega_{\sigma+}(\mathbf{k}_\sigma), \mathbf{k}_{\sigma'}, \omega_{\sigma'+}(\mathbf{k}_{\sigma'}), \mathbf{k}_2, \omega_2)$$
$$+ S_{ijs}(\mathbf{k}_\sigma, \omega_{\sigma+}(\mathbf{k}_\sigma), \mathbf{k}_2, \omega_2, \mathbf{k}_{\sigma'}, \omega_{\sigma'}(\mathbf{k}_{\sigma'})]\, a_{j\sigma'+}(\mathbf{k}_{\sigma'})$$
$$\left.\times \sum_{\sigma''}\frac{\Pi_{st}{}^{\sigma''}(\mathbf{k}_2, \omega_2)}{k_2{}^2/\omega_2{}^2 - \varepsilon^{\sigma''}(\mathbf{k}_2, \omega_2)}\Gamma_e{}^\nu\right|^2. \qquad (4.8)$$

$$\mathbf{k}_2 = \mathbf{k}_\sigma - \mathbf{k}_{\sigma'}, \qquad \omega_2 = \omega_\sigma(\mathbf{k}_\sigma) - \omega_{\sigma'}(\mathbf{k}_{\sigma'}).$$

The tensor Λ_{ij} can be obtained by solving by perturbation theory the equation of motion of the charge in the field of the normal waves and in an external magnetic field.

For arbitrary charge velocities we obtain

$$\Lambda_{ij}{}^{(\alpha)}(\mathbf{k}, \omega, \mathbf{k}_1, \omega_1) = \frac{e}{m}\sqrt{1-v^2}\, e^{i(\nu-\mu)\varphi}R_{ij}$$
$$\times\,\delta[\omega(\mathbf{k}) - \omega(\mathbf{k}_1) - (k_z - k_{1z}')v_z - (\mu-\nu)\omega_H{}^\alpha], \quad (4.9)$$

where

$$R_{ij} = P_j{}^i + \Gamma_iP_j{}^sk_s; \quad P_2{}^1 = \frac{i\omega_{H\alpha}}{\Omega^2 - \omega_{H\alpha}{}^2}\Big[pJ_\nu - qk_{1x}$$
$$\times\left(B_+ + \frac{\Omega}{\omega_{H\alpha}}B_-\right) + \frac{\Omega}{\omega_{H\alpha}}\frac{v_\perp{}^2}{2}(J_{\nu-2}e^{2i\varphi} - J_{\nu+2}e^{-2i\varphi})\Big],$$
$$P_2{}^2 = \frac{i\Omega}{\Omega^2 - \omega_{H\alpha}{}^2}\Big[pJ_\nu - qk_{1x}\left(B_+ + \frac{\omega_{H\alpha}}{\Omega}B_-\right)$$
$$-\frac{v_\perp{}^2}{2}(2J_\nu - J_{\nu-2}e^{2i\varphi} + J_{\nu+2}e^{-2i\varphi})\Big],$$
$$P_1{}^2 = \frac{\omega_{H\alpha}}{\Omega^2 - \omega_{H\alpha}{}^2}\Big[pJ_\nu + iqk_{1y}\left(B_- + \frac{\Omega}{\omega_{H\alpha}}B_+\right)$$
$$-\frac{\Omega}{\omega_{H\alpha}}\frac{v_\perp{}^2}{2}(J_{\nu-2}e^{2i\varphi} - J_{\nu+2}e^{-2i\varphi})\Big],$$
$$P_1{}^1 = \frac{\Omega}{\Omega^2 - \omega_{H\alpha}{}^2}\Big[pJ_\nu + iqk_{1y}\left(B_- + \frac{\omega_{H\alpha}}{\Omega}B_+\right)$$
$$-\frac{v_\perp{}^2}{2}(2J_\nu + J_{\nu-2}e^{2i\varphi} + J_{\nu+2}e^{-2i\varphi})\Big],$$

$$P_3{}^1 = \frac{\Omega}{\Omega^2 - \omega_{H\alpha}{}^2}\left[\frac{v_z}{\omega_1}\left(k_{1x} + \frac{i\omega_{H\alpha}}{\Omega}k_{1y}\right)J_\nu - v_\perp v_z B_+\right],$$

$$P_3{}^2 = \frac{\omega_{H\alpha}}{\Omega^2 - \omega_{H\alpha}{}^2}\left[\frac{v_z}{\omega_1}\left(k_{1x} + i\frac{\Omega}{\omega_{H\alpha}}k_{1y}\right)J_\nu - v_\perp v_z B_-\right],$$

$$P_1{}^3 = iqB_+(k_{1z}/\omega_1 - v_z), \quad P_2{}^3 = -qB_-(k_{1z}/\omega_1 - v_z),$$

$$P_3{}^3 = i\Omega^{-1}[(1 - v_z{}^2)J_\nu - q(k_{1x}B_+ - ik_{1y}B_-)],$$

$$B_\pm = (J_{\nu-1}e^{i\varphi} \pm J_{\nu+1}e^{-i\varphi}); \quad \Gamma_1 = q(J_{\mu+1}e^{i\varphi} + J_{\mu-1}e^{-i\varphi}),$$

$$\Gamma_2 = -iq(J_{\mu+1}e^{i\varphi} - J_{\mu-1}e^{-i\varphi}), \quad \Gamma_3 = \frac{v_z}{\Omega}J_\mu,$$

$$(p = 1 - k_{1z}v_z/\omega_1, \quad q = v_\perp/2\Omega,$$

$$\Omega = \omega(\mathbf{k}_1) - k_{1z}v_z - \nu\omega_H{}^\alpha). \qquad (4.10)$$

In a plasma with a Maxwellian velocity distribution situated in a strong magnetic field ($k_\perp v_\perp^\alpha \ll \omega_H^\alpha$), $\mu - \nu$ runs only through the values 0 and ± 1. When $\mu - \nu = 0$ the scattering is both from electrons and ions if

$$(k_z - k_z')v_{Ti} \gg (k_z - k_z')v_{Te} \gg \omega_2,$$

and from ions only if

$$(k_z - k_z')v_{Ti} \gg \omega_2 \gg (k_z - k_z')v_{Te}.$$

When $\mu - \nu = \pm 1$, for scattering by ions it is necessary to have

$$(k_z - k_z')v_{Ti} \gg \omega_2 \pm \omega_{Hi}.$$

The condition for the electrons is similar.

Assuming the spatial dispersion of the ω and ω' waves to be weak, we simplify S_{ijs} for the case of scattering by ions. Let us write by way of an example an equation describing the interaction of plasma waves with frequencies ω and $\omega' \sim \omega_{He}$, in a narrow cone along the field, when $T_i \gg T_e$. When

$$\frac{1}{(k_z - k_z')^2 r_{Di}^2} \gg \frac{|\mathbf{k} - \mathbf{k}'|^2}{(\omega - \omega')^2} \gg 1,$$

$$1 \gg \frac{|k_z - k_z'|^2}{|\mathbf{k} - \mathbf{k}'|^2} \gg \frac{T_e}{T_i}$$

this equation takes the form

$$\frac{d\overline{W}_s(\omega, \theta)}{dt} = \frac{\pi\sqrt{\pi}}{32\sqrt{2}}\frac{\overline{W}_s(\omega, \theta)}{nm_i v_{Ti}^2}\int d\omega'\,d\theta'\frac{\sin\theta'}{\omega'}\overline{W}_{s'}(\omega', \theta')$$
$$\times(\omega^2 + \omega'^2) \times \frac{|k_z - k_z'|v_{Ti}}{\omega' - \omega}\frac{(\omega' - \omega)^4}{(\mathbf{k} - \mathbf{k}')^4}|\varepsilon_z{}^l - 1|^2. \qquad (4.11)$$

Here $\mu - \nu = 1$, $\epsilon_z^l - 1 = (k_z - k_z')^{-2}r_{De}^{-2}$, $\overline{W}_s(\omega, \theta)$, and $\overline{W}_{s'}(\omega', \theta')$ are the energy densities of the s and s′ waves, averaged over the angle φ in the plane perpendicular to the field, while ω, ω' and θ, θ' are their respective frequencies and wave-vector angles with the external magnetic field.

The scattering by electrons is usually small compared with the scattering by ions. It must be noted that in a strong magnetic field $k_\perp v_\perp^\alpha \ll \omega_H^\alpha$, in scattering by electrons, the compensation of the two scattering mechanisms (4.2), unlike the isotropic case[13], can occur only in particular cases ($|k_z - k_z'|^2/|\mathbf{k} - \mathbf{k}'|^2 \sim 1$, $T_e \gg T_i$). But even in the absence of compensation, the scattering by the electrons is usually small compared with scattering by ions. Thus, for example, for the interaction of two plasma waves with frequencies ω, $\omega' \sim (\omega_{He}\omega_{Hi})^{1/2}$, we obtain when $\mu - \nu = 0$

$$\frac{d\overline{W}_s(\omega, \theta)}{dt} = \frac{\pi\sqrt{\pi}}{16\sqrt{2}}\frac{\overline{W}_s(\omega, \theta)}{nm_e v_{Te}^2}\frac{1}{v_{Te}}\int d\omega'\,d\theta'\frac{\sin\theta'}{\omega'}\overline{W}_{s'}(\omega', \theta')$$
$$\times\left|\frac{\varepsilon_z{}^l - 1}{\varepsilon_z}\right|^2\frac{\omega^2 + \omega'^2}{|k_z + k_z'|^2}\frac{(\omega' - \omega)^3}{|k_z - k_z'|^3}$$
$$\times\frac{k_\perp{}^4 + k_\perp'^4 + 7k_\perp{}^2k_\perp'^2}{\omega_{He}{}^4\cos^2\theta\cos^2\theta'}\frac{k_\perp{}^2k_\perp'^2}{k^2k'^2},$$

$$\varepsilon_z = \frac{1}{(k_z - k_z')^2}\left(\frac{1}{r_{De}^2} + \frac{1}{r_{Di}^2}\right) + 1. \qquad (4.12)$$

In this plasma-wave frequency region the interaction turns out to be stronger for waves propagating transverse to the field. A detailed analysis of the equations which follow from (3.9) and (4.8) shows that in the frequency region $\omega \sim \omega_{He}$ the plasma waves interact most strongly in the angle region θ, $\theta' \ll 1$.

As can be seen from these examples, the nonlinear wave interaction effects described above can find application in problems of turbulent heating of plasma, in astrophysics, etc.

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Translated by J. G. Adashko
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