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CONTRIBUTION OF THE THEORY OF SPATIAL DISPERSION IN A FERROMAGNETIC METAL IN A STRONG MAGNETIC FIELD

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The surface impedance of a ferromagnetic metal is calculated in the case of anomalous skin effect at frequencies near ferromagnetic resonance when spatial dispersion of the magnetic susceptibility must be taken into account in principle. The field distribution within the metal is investigated. It is shown that at resonance the field is a standing wave.

1. INTRODUCTION. FORMULATION OF PROBLEM

INTEREST in effects due to allowance for spatial dispersion has increased recently. This is connected with more concerted studies of wave propagation in a plasma,[1] and especially plasma effects in solids [2]. A natural measure of the role of spatial dispersion in the analysis of electromagnetic properties of a medium is the ratio a/λ, where λ is the electromagnetic wavelength, and the "characteristic length" a is essentially connected with the nature of the excitations interacting with the electromagnetic field.

A ferromagnetic metal contains, besides free carriers, also a spin branch of the energy spectrum. Therefore, two different parameters arise in general when spatial dispersion is taken into account. In the case of a free carrier a is equal to the mean free path l. We note immediately that in a strong magnetic field the effective length is the cyclotron radius $R = l\,|\nu - i\omega|/\omega_c$ (ν = collision frequency, ω = frequency of alternating field, ω_c = cyclotron frequency). When allowance is made for spatial dispersion of the magnetic moment, a must be taken to be the lattice constant. In this respect we note the following. The ratio a/λ is $\lesssim 10^{-4}$ even in the optical band. Since we encounter most frequently the square of this quantity, it is clear that the spatial dispersion connected with spin waves can become significant only at frequencies close to resonance, when the parameter $(a/\lambda)^2$ must be compared not with unity, but with the relative deviation from resonance $\Delta\omega/\omega_r$, the minimum value of which is determined by the line width.

In the radio frequency region ($\lambda \sim 1-10$ cm) the condition $(a/\lambda)^2 \gtrsim \Delta\omega/\omega_r$ cannot of course be satisfied even in the most perfect dielectric samples. In metals, however, this relation is satisfied relatively readily, since the wavelength in the metal is several orders of magnitude smaller than the wavelength in vacuum.

In a ferromagnetic metal there is a particularly favorable possibility for the study of spatial-dispersion effects. Resonance is ensured in a ferromagnetic metal by excitation of a spin wave, and is manifest in a corresponding singularity of the magnetic susceptibility. The conduction electrons not only produce in this case (owing to the skin effect) the magnetic-moment inhomogeneity required to observe the volume effects, but can by themselves be put relatively easily in conditions of strong spatial dispersion. In other words, the wavelength dependence is significant not only in the magnetic permeability μ, but also in the conductivity σ (anomalous skin effect [3]).

This paper is devoted essentially to the calculation of the surface impedance of a metal at frequencies close to ferromagnetic resonance, when the spatial dispersion of the magnetic permeability must be taken into account. The impedance is determined by the wave properties of the medium, and the latter are in turn determined by the dispersion equation, which includes both the magnetic permeability and the dielectric constant (for a metal $\epsilon = 4\pi i\sigma/\omega$):

$$k^2 = \omega^2 c^{-2} \varepsilon(\omega, k)\, \mu(\omega, \mathbf{k}). \tag{1}$$

Here $\mathbf{k}$ is the wave vector. Naturally, a change in the form of the function $\epsilon(\omega, \mathbf{k})$ changes the wave properties of the medium, and consequently also the surface impedance. We shall trace this change by analyzing several cases. The influence of the magnetic permeability on the dispersion properties of the medium is manifest, in particular, in the possible existence of electromagnetic waves at frequencies for which there are no waves in a nonmagnetic medium. For example, in the region of frequencies where $\epsilon = -\omega_0^2/\omega^2$ (ω_0—plasma frequency), it becomes possible for a weakly damped wave to propagate if

$$\mathrm{Re}\,\mu < 0, \qquad |\mathrm{Re}\,\mu| \gg \mathrm{Im}\,\mu.$$

The formulation of the problem is as follows. We consider the reflection of an electromagnetic wave from a metallic ferromagnetic half-space situated in a magnetic field parallel to the surface of the metal. The alternating field propagates normally to the surface. In the resonance region of interest to us, the magnetic permeability can be written in the form

$$\mu(\omega, \mathbf{k}) = (\xi + \beta k^2)^{-1}, \quad \xi = (\omega_r - \omega)/\Omega, \tag{2}$$

where we have for a field parallel to the surface

$$\omega_r = \gamma(BH)^{1/2}, \quad \beta = \alpha/4\pi, \quad \Omega = 2\pi\gamma M(B/H)^{1/2}.$$

Here H is the constant field, $B = H + 4\pi M$ the magnetic induction, M the saturation magnetic moment, $\gamma = ge/2mc$ (g = spectroscopic factor, m = free-electron mass), and $\alpha = \Theta_c a^2/\mu_B M$ is the exchange constant (Θ_c coincides in order of magnitude with the Curie temperature and μ_B is the Bohr magneton.)

Being interested only in the influence of exchange effects on the shape of the resonance line, we do not take into account the dissipation in explicit form, bearing in mind that in the case of necessity (for example, when choosing solutions of the dispersion equation (1) or in the presence of singularities along the integration contour in the corresponding integrals) we can add an infinitesimally small imaginary part to the frequency ω, which enters in the expressions for $\mu(\omega, \mathbf{k})$ and $\sigma(\omega, \mathbf{k})$.

The dependence of the magnetic permeability on the wave vector leads to the appearance of additional solutions of Eq. (1). To determine the additional (resonating) amplitude we use the boundary condition for the magnetic moment, which we write in the form[1)]

$$\partial \mathbf{m}/\partial z + \zeta \mathbf{m} = 0 \tag{3}$$

on the boundary. The role of the parameter ζ, which characterizes the state of the boundary of the magnetic material, is discussed in a paper by Pincus[4].

In two limiting cases ($\zeta = 0$ and $\zeta = \infty$), the analysis becomes much simpler, for when calculating the impedance by the Fourier method we can use in these cases the values of $\epsilon(\omega, \mathbf{k})$ and $\mu(\omega, \mathbf{k})$ for unbounded space[5]. This possibility is based on the "indifference" of the electrons to the boundary conditions when $l/\lambda \gg 1$. According to[5], the surface impedance in the limiting cases is

$$Z_p(\zeta = \infty) = \frac{2}{\pi i}\frac{\omega}{c}\int_0^\infty \frac{\mu(\omega, k)\,dk}{k^2 - \omega^2 c^{-2}\varepsilon(\omega, k)\mu(\omega, k)}, \tag{4}$$

$$Z_n^{-1}(\zeta = 0) = \frac{2}{\pi i}\frac{\omega}{c}\int_0^\infty \frac{\varepsilon(\omega, k)\,dk}{k^2 - \omega^2 c^{-2}\varepsilon(\omega, k)\mu(\omega, k)}. \tag{5}$$

The role of spatial dispersion of the permeability in a weak field under conditions of normal skin effect

$$l \ll R \ll \lambda \tag{6}$$

was investigated by several authors[6-8]. Nonetheless, we present in Sec. 2 several exact formulas which describe the shape of the resonance curve in this case. In Sec. 3 we consider the reflection of an electromagnetic wave from a ferromagnetic dielectric. The expression obtained for the impedance can naturally be applied to a ferromagnetic metal in which helicon waves propagate[9].

In Sec. 4 we investigate the magnetic metal under conditions of anomalous skin effect. Analysis of the resonance curves with the aid of formulas (4)—(5) discloses a sharp dependence on the boundary condition for the magnetic moment. The impedance in the direct vicinity of resonance is therefore calculated with the aid of the general condition (3). A more complete characteristic is the distribution of the electromagnetic field deep in the metal. This question is dealt with at the end of Sec. 4.

In the Appendix we discuss the distribution of the field in a dielectric near the edge of the exciton absorption band. This question, as shown below, is close to that considered in Sec. 4.

2. NORMAL SKIN EFFECT

The phenomenon of ferromagnetic resonance, as is well known, consists in excitation of homogeneous precession of magnetic moments, which

[1)]It can be easily shown that, in the chosen approximation defined by Eq. (2), the use of a more general boundary condition, for example

$$\partial m_i/\partial z + \zeta_{ik} m_k = \gamma_{ik} h_k,$$

is inconsistent.

leads (if dissipation is neglected) to infinite energy absorption at the resonant frequency. This is manifest, in particular, in the surface impedance becoming infinite ($Z \sim (\omega_r - \omega)^{-1/2}$). When exchange interaction is taken into account, inhomogeneous oscillations of the magnetic moment—spin waves—are excited in the conductor. Spin-wave excitation is connected with a finite energy loss. An account of the exchange interaction therefore leads naturally to a finite value of the impedance even in the absence of dissipation.

When inequalities (6) are satisfied we have $\epsilon(\omega, \mathbf{k}) \equiv \epsilon(\omega) = 4\pi i\sigma/\omega$, where σ_0 is the static conductivity in the absence of a magnetic field. For the surface impedance in the case when $\mathbf{m} = 0$ on the boundary we obtain with the aid of formula (4)

$$Z_p = i\frac{c}{4\pi\sigma_0}\frac{k_1k_2}{k_1+k_2}. \tag{7}$$

The values of $k_{1,2}$ are determined from the dispersion equation

$$\beta k^4 + \xi k^2 - ik_0^2|\varepsilon| = 0 \qquad (k_0 = \omega/c) \tag{8}$$

by the condition $\operatorname{Im} k_{1,2} > 0$.

Formula (7) coincides with the results of Yu Lu and one of the authors [8] at $\xi = \infty$. Of course, the result of Ament and Rado [6], obtained under the assumption that $\partial m/\partial z|_{z=0} = 0$, differs formally from expression (7). The dependence of the impedance on the boundary conditions will be discussed later. We wish to emphasize here that allowance for the exchange effects makes the surface impedance finite at all frequencies.

Being interested in exact formulas for the description of the resonance curve at arbitrarily small deviations from resonance, we write (7) in the form

$$\begin{aligned} Z_p = \frac{1-i}{2\beta[2b(a^2+bi)]^{1/2}} \{[a + (a^2 \\ + bi)^{1/2}]^{1/2} + i[-a + (a^2+bi)^{1/2}]^{1/2}\}, \end{aligned} \tag{9}$$

where we introduce the notation $2a = \xi/\beta$ and $b = k_0^2|\epsilon|/\beta$, with the sign of the square root determined from the condition $\operatorname{Re} Z_p > 0$.

Putting $Z_p = R_p + iX_p$ and separating the real and imaginary parts, we obtain for the real part of the impedance

a) $\omega < \omega_r$

$$\begin{aligned} R_p = \frac{\mathrm{ch}^{-1}t}{2\sqrt{2}\,\beta\sqrt{ab}}\Big\{ e^{-t/2}\Big[\mathrm{ch}^{1/2}\frac{t}{4}\Big(\mathrm{ch}\frac{t}{4}+\mathrm{ch}^{1/2}\frac{t}{2}\Big)^{1/2} \\ - \mathrm{sh}^{1/2}\frac{t}{4}\Big(-\mathrm{sh}\frac{t}{4}+\mathrm{ch}^{1/2}\frac{t}{2}\Big)^{1/2}\Big] \\ + e^{t/2}\Big[\mathrm{ch}^{1/2}\frac{t}{4}\Big(-\mathrm{ch}\frac{t}{4}+\mathrm{ch}^{1/2}\frac{t}{2}\Big)^{1/2} \\ + \mathrm{sh}^{1/2}\frac{t}{4}\Big(\mathrm{sh}\frac{t}{4}+\mathrm{ch}^{1/2}\frac{t}{2}\Big)^{1/2}\Big]\Big\}; \end{aligned} \tag{10a*}$$

b) $\omega > \omega_r$

$$\begin{aligned} R_p = \frac{\mathrm{ch}^{-1}t}{2\sqrt{2}\,\beta\sqrt{ab}}\Big\{ e^{t/2}\Big[\mathrm{ch}^{1/2}\frac{t}{4}\Big(\mathrm{ch}\frac{t}{4}+\mathrm{ch}^{1/2}\frac{t}{2}\Big)^{1/2} \\ + \mathrm{sh}^{1/2}\frac{t}{4}\Big(-\mathrm{sh}\frac{t}{4}+\mathrm{ch}^{1/2}\frac{t}{2}\Big)^{1/2}\Big] \\ - e^{-t/2}\Big[\mathrm{ch}^{1/2}\frac{t}{4}\Big(-\mathrm{ch}\frac{t}{4}+\mathrm{ch}^{1/2}\frac{t}{2}\Big)^{1/2} \\ - \mathrm{sh}^{1/2}\frac{t}{4}\Big(\mathrm{sh}\frac{t}{4}+\mathrm{ch}^{1/2}\frac{t}{2}\Big)^{1/2}\Big]\Big\}. \end{aligned} \tag{10b}$$

Here $\sinh t = b/a^2$. An analogous expression can be written for X_p.

The frequency dependence of the real and imaginary parts of the impedance is shown schematically in Fig. 1. The resonant frequency, i.e., the frequency of which R_p has a maximum, turns out to be

$$\omega \approx \omega_r - 0.57\Omega\Big(\frac{\alpha}{\delta_0^2}\frac{M}{H}\Big)^{1/2} \qquad \Big(\delta_0^2 = \frac{c^2}{2\pi\sigma_0\omega}\Big),$$

and differs little from the results of Ament and Rado [6] (where it is assumed that $\partial m/\partial z|_{z=0} = 0$). In addition, as shown by comparison of the resonance curves in cases (4) and (5), the entire shape of the resonance curve is not very sensitive to the boundary conditions for the magnetic moment. The impedance under the general boundary condition (3) was investigated earlier [7,8]. The presence of the additional parameter ξ makes it possible to obtain numerical agreement with experiment [10], although it leads to no qualitative changes in the line shape.

3. DIELECTRIC

Allowance for the exchange interaction in a dielectric also makes the impedance finite in the resonance region, regardless of the boundary condition for the magnetic moment. When

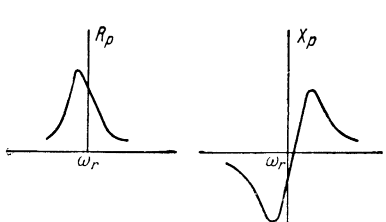

FIG. 1

*sh ≡ sinh, ch ≡ cosh.

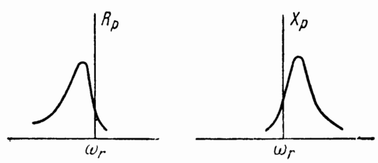

FIG. 2

$\epsilon(\omega, k) \equiv \epsilon(\omega) > 0$ it follows from (4) that

$$R_p = \frac{(\beta k_0^2)^{-1/4}\,\varepsilon^{-3/4}\sqrt{2}}{(\chi^2+4)^{1/2}[(\chi^2+4)^{1/2}-\chi]^{1/2}},$$

$$X_p = \frac{(\beta k_0^2)^{-1/4}\,\varepsilon^{-3/4}\sqrt{2}}{(\chi^2+4)^{1/2}[(\chi^2+4)^{1/2}+\chi]^{1/2}}, \qquad (11)$$

where $\chi = \xi/(\beta k_0^2\epsilon)^{1/2}$. In this case the resonance curve is shown in Fig. 2. The resonance shift is

$$\omega_r - \omega \approx 0\;3\,\Omega\,(\beta k_0^2\varepsilon)^{1/2}.$$

As in the normal skin effect, the resonance curve is little sensitive to the boundary conditions.

It is natural to apply formulas (11) to a ferromagnetic conductor placed in a strong magnetic field ($\omega_c\tau \gg 1$), under the condition $kR \ll 1$. It is well known [11,12,9] that when the foregoing inequalities are satisfied helicon waves can propagate in the conductor if the angle Φ between the wave vector and the magnetic field is not too close to $\pi/2$ ($\cot\Phi > R/l$). In this case two waves are excited in the conductor, one of which is weakly damped, and the other is totally reflected. The existence of the weakly damped wave makes the effective dielectric constant real. In the simplest case of parallel **k** and **H** it is convenient to go over to circularly polarized waves $e_\pm = e_x \pm ie_y$. As a result, the impedance element Z_+ corresponding to the resonating wave turns out to be connected in simple fashion with the impedance Z_p defined by expression (11):

$$Z_+ = -iZ_p. \qquad (12$$

In this case it is necessary to put in (11) $\xi = (\omega_r - \omega)/4\pi\gamma M$ and $\omega_r = \gamma H$ ($H \parallel Oz$ in this case), and the role of the effective dielectric constant is assumed by the quantity $\epsilon = 4\pi Nec/\omega H$, where N is the difference of the carrier densities.

We note that in an ordinary metal the element Z_+ is a real quantity (if the number of electrons exceeds the number of holes) [11], whereas in a ferromagnet Z_+ is complex because of the strong coupling between the helicon wave and the damped spin wave. The widths of the $R_p(\omega)$ and $X_p(\omega)$ curves are of the order of $(\beta k_0^2\epsilon)^{1/2}$, and naturally vanish as $\beta \to 0$. The values of R_p and X_p tend in this case to infinity as $\omega \to \omega_r$.

4. ANOMALOUS SKIN EFFECT

1. Let us discuss ferromagnetic resonance in a metal under conditions of anomalous skin effect in a strong magnetic field. We shall assume the following inequalities to be satisfied

$$kR \gg 1, \qquad \omega_c\tau \gg 1. \qquad (13)$$

In the preceding paper [5] we investigated the surface impedance of a ferromagnetic metal under anomalous skin effect without taking account of the spatial dispersion of the magnetic permeability, which, as already mentioned, is significant only in the direct vicinity of the resonance. As shown earlier, [13,5] in the case of the anomalous skin effect the singularity of the impedance is of the order of $Z \sim (\omega_r - \omega)^{-2/3}$. We are interested here in the qualitative changes in the frequency dependence of the surface impedance which result from an account of the exchange interaction.

Let us consider first the dispersion equation (1). The electric conductivity tensor of the metal $\sigma_{ik}(\omega, k)$ was calculated by Kaner and Skobov [14] under the assumption that the carrier energy has an isotropic dependence on the momentum. Under conditions (13), the tensor σ_{ik}, as shown in [14], can be regarded as diagonal (accurate to terms of order $(kR)^{-1}$). The element $\epsilon_{xx}(\omega, k)$, which determines the dispersion properties, is of the form

$$\varepsilon(\omega, k) = -\pi\frac{\omega_0^2}{\omega k v}\operatorname{ctg}\pi\frac{\omega}{\omega_c}, \qquad (14)^*$$

where $\omega_0^2 = 3\pi Ne^2/m$ is of the order of the square of the plasma frequency and v is the carrier velocity on the Fermi boundary.

As seen from (14), the sign of $\epsilon(\omega, k)$ is determined by the ratio of ω to ω_c. The case $\cot(\pi\omega/\omega_c) < 0$ will be discussed in part 4 of the present section. Of greater physical interest is the region $\cot(\pi\omega/\omega_c) > 0$. For simplicity we assume that the ferromagnetic-resonance frequency is small compared with the cyclotron frequency ($\omega \ll \omega_c$). Accordingly, $\epsilon(\omega, k)$ takes the form

$$\varepsilon(\omega, k) = -\omega_0^2/\omega^2 kR. \qquad (15)$$

Using (15) and (2), we write the dispersion equation (1) in the form

$$p(\eta, x) \equiv x^5 + \eta x^3 + 1 = 0. \qquad (16)$$

We have gone over to the dimensionless variable $x = k/q$, $q^5 = \omega_0^2/\beta c^2 R$. The parameter $\eta = \xi/\beta q^2$ characterizes the closeness to resonance.

*ctg ≡ cot.

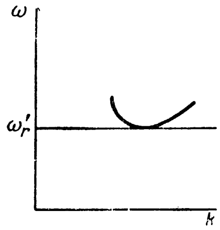

FIG. 3

Equation (16) corresponds to the dispersion curve shown in Fig. 3. We can readily see that Eq. (16) can have real positive solutions. This means the possibility of existence in the metal of weakly damped electromagnetic waves. At large negative values of η, we obtain from (16) for the "tail" of the right branch of the curve of Fig. 3 the dispersion equation of the spin wave

$$\omega = \omega_r + \Omega\beta k^2 \tag{17}$$

and for the "tail" of the left branch the wave with anomalous dispersion—the equation

$$\omega = \omega_r + \Omega\beta q^5 / k^3. \tag{18}$$

A ferromagnetic metal in a strong magnetic field is a natural example of a medium in which a wave with negative group velocity can propagate.

As seen from Fig. 3, the dispersion curve $\omega(k)$ has an extremum at $\omega = \omega_r'$, which is the end point of the spectrum. At frequencies larger than ω_r', weakly damped waves (17) and (18) are excited in the metal, and the energy does not penetrate into the metal at lower frequencies. The frequency ω_r' must be identified with the shifted resonant frequency. The shift of the resonance can be readily obtained from the conditions $p(\eta_0, x_0) = 0$ and $p_x'(\eta_0, x_0) = 0$. Hence $x_0^5 = 3/2$, $\eta_0 = -(5/3)(3/2)^{2/5} \approx -2$, and the shifted frequency turns out to be equal to

$$\omega_r' = \omega_r - \eta_0\Omega\beta q^2 \approx \omega_r + 2\Omega\beta q^2. \tag{19}$$

When $H \sim 10^4$ Oe, $\Theta_c \sim 10^{-13}$ erg, and $\delta_0 \sim 10^{-5}$ cm, the shift of the resonance has the following order of magnitude:

$$\frac{\omega_r' - \omega_r}{\omega_r} \sim \frac{\Omega}{\omega_r}\beta q^2 \sim \left(\frac{\Theta_c}{\mu_B M}\right)^{3/5}\left(\frac{a^3}{\delta_0^2 R}\right)^{2/5} \approx 10^{-2}.$$

2. The foregoing dispersion properties should become manifest in observations of surface impedance. Let us investigate first the surface impedance when $m(0) = 0$. We write expression (4) for Z_p, using (2) and (15), in the form

$$Z_p = \frac{2}{\pi i}\frac{\omega}{c\beta q^3}J_p(\eta), \qquad J_p(\eta) = \int_0^\infty \frac{x\,dx}{x^5 + \eta x^3 + 1}. \tag{20}$$

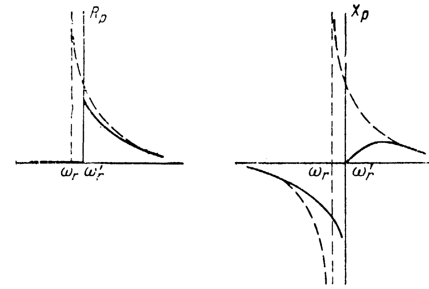

FIG. 4

We put $\eta = \eta_0 + \delta$ and $x_{1,2} = x_0 \pm \Delta$, where δ and Δ are small compared with η_0 and x_0. From (16) we obtain, taking account of the smallness of δ and Δ,

$$\Delta^2 = -\delta x_0^2(3\eta_0 + 10x_0^2)^{-1}. \tag{21}$$

When $\delta < 0$ both roots $x_{1,2}$ are close to the real axis. We represent $p(\eta, x)$ in the form

$$p(\eta, x) = p_3(\eta_0, x)(x - x_1)(x - x_2),$$
$$p_3(\eta_0, x) = x^3 + 2x_0x^2 + {}^4\!/_3 x_0^2 x + {}^2\!/_3 x_0^3.$$

Integrating in (20) with the aid of the formula

$$[x - x_i \pm i\varepsilon]^{-1} = \mathrm{P}\frac{1}{x - x_i} \mp \pi i\delta(x - x_i),$$

we obtain

$$J_p(\eta) = \pi i x_0 / p_3(\eta_0, x_0)\Delta,$$

or, putting $x_0 \approx 1$ and $\eta_0 \approx -2$,

$$J_p(\eta) \approx 2\pi i / 5|\delta|^{1/2}.$$

The roots of the polynomial $p_3(\eta_0, x)$ lie in the complex plane. Their contribution can be readily verified to be negligibly small. When $\delta > 0$, the roots $x_{1,2}$ move away from x_0 in the complex plane, and there are no singularities on the integration contour. $J_p(\eta)$ is then equal to $2\pi/5\delta^{1/2}$.

Far from resonance, at $|\eta| \gg |\eta_0|$, the results of the preceding paper [5] apply. The impedance in this region is equal to

$$Z_p = \frac{4}{3\sqrt{3}}|\xi|^{-2/5}\frac{\omega}{c}\left(\frac{c^2R}{\omega_0^2}\right)^{1/2}\exp\left\{i\pi\left[\frac{2}{3}\theta(\omega - \omega_r) - \frac{1}{2}\right]\right\},$$
$$\theta(\omega - \omega_r) = \begin{cases}0, & \omega < \omega_r \\ 1, & \omega > \omega_r\end{cases}. \tag{22}$$

Thus, an account of the spatial dispersion of the magnetic permeability leads to a root singularity of the impedance, in contrast with the singularity $Z \sim (\omega_r - \omega)^{-2/3}$ without account of the exchange interaction. The resonance curve for the case $m(0) = 0$ is shown in Fig. 4[2)]. When the exchange

[2)]The dashed line here and in Fig. 5 denotes the behavior of the impedance without account of the exchange effects.

constant tends to zero, the maximum value of X_p at $\omega > \omega_r'$ increases, the shift of the resonance tends to zero, and the curve approaches its limiting form.

We investigate in similar fashion the surface impedance in the opposite limiting case $\partial m/\partial z|_{z=0} = 0$. Going over to a new variable, we rewrite (5) in the form

$$Z_n^{-1} = \frac{2}{\pi i}\frac{c}{\omega}\beta^{2/5}\left(\frac{\omega_0^2}{c^2R}\right)^{3/5} J_n(\eta),$$

$$J_n(\eta) = \int_0^\infty \frac{(x^2+\eta)\,dx}{x^5+\eta x^3+1}. \qquad (23)$$

Let us consider the integral in (23) when $\eta > \eta_0$. In this case the integral contains no singularities on the integration path. We can readily see that $dJ_n/d\eta$ is positive in this region. For large values of η

$$J_n(\eta) = 2\pi\eta^{2/3}/3\sqrt{3} > 0.$$

Near the end point of the spectrum $\eta = \eta_0 + \delta$, $\delta > 0$, and the integral is equal to

$$J_n(\eta_0+\delta) \approx -(2\pi/5)\delta^{1/2} < 0.$$

Thus, there exists a value $\eta > \eta_0$ at which $J_n(\eta)$ vanishes. Estimates show that this value of η lies between zero and minus unity. The existence of a zero in the function $J_n(\eta)$ leads to an infinite imaginary part of the impedance X_n.

When $\eta < \eta_0$, the denominator in (22) has real roots. Carrying out calculations similar to those made above, we obtain near η_0

$$J_n \approx -(2\pi i/5)|\delta|^{1/2}.$$

When $|\eta| \gg |\eta_0|$, as shown earlier [5], Z_n differs from Z_p only by a real factor of the order of unity.

Finally, the resonance curve for $m'(0) = 0$ is shown in Fig. 5. We note that in this case the real part of the impedance vanishes at resonance

3. In view of such an appreciable dependence of the resonance curve on the boundary condition for the magnetic moment, it is of interest to calculate the impedance under the general condition (3). It is necessary for this purpose to consider the complete system of equations consisting of Maxwell's equations and the Landau-Lifshitz equation for the magnetic moment:

$$\frac{\partial h_y}{\partial z} = -\frac{4\pi}{c}j_x, \quad \frac{\partial e_x}{\partial z} = \frac{i\omega}{c}(h_y + 4\pi m_y);$$

$$\omega_a\beta\frac{\partial^2 m_y}{\partial z^2} - (\omega_r^2-\omega^2)m_y = -\gamma M\omega_a h_y,$$

$$\omega_a = \omega_r + 4\pi\gamma M. \qquad (24)$$

In the derivation of the linearized equation for the magnetic moment we made use of the fact that the resonant behavior is displayed only by the y-component of the magnetic moment. In the equation for m_z we can neglect the exchange term, after which m_z can be readily eliminated.

It is necessary to go over in (24) to Fourier components. It is convenient to continue the field e to the half-space $z < 0$ in even fashion, and continue h and m in odd fashion. This leaves in the equations that relate the Fourier components the values $h_y(0)$ and $m_y(0)$ of the field and of the magnetic moment on the boundary. To eliminate $m_y(0)$ with the aid of the boundary condition (3), it is necessary to determine $m_y'(0)$. This is done by differentiating the last equation of (24) and then taking the Fourier components. The final expression for the impedance, under the general boundary condition, is

$$Z_\zeta = \frac{2}{\pi i}\frac{\omega}{c}\left\{\int_0^\infty \frac{\mu_k}{D_k}dk + \frac{\pi}{2}\beta\frac{A^2}{\xi B - \zeta}\right\}, \qquad (25)$$

where

$$A = \frac{2}{\pi}\int_0^\infty \frac{\mu_k k^2 dk}{D_k}, \quad B = \frac{2}{\pi}\int_0^\infty \frac{\mu_k g_k dk}{D_k},$$

$$D_k = k^2 - \omega^2\varepsilon_k\mu_k/c^2, \quad g_k = k^2 - \omega^2\varepsilon_k\mu_0/c^2,$$

μ_0 is the permeability without account of the exchange interaction. When the parameter ζ has values infinity or zero, this expression goes over into (4) and (5), respectively. In particular, when $\zeta = 0$, the impedance Z_ζ at the point $\xi = 0$ remains finite, because g_k contains ξ^{-1}.

We are interested in the behavior of Z_ζ near the shifted resonance frequency ω_r'. By calculations similar to those carried out above we find that Z_ζ is equal to ($\eta = \eta_0 - \delta$, $\delta > 0$)

$$Z_\zeta = c_1\zeta/(ic_2 + \zeta\delta^{1/2}), \qquad (26)$$

where the constants c_1 and c_2 are respectively equal to $4\omega_r/5c\beta q^3$ and $4q/5$. We see from (26)

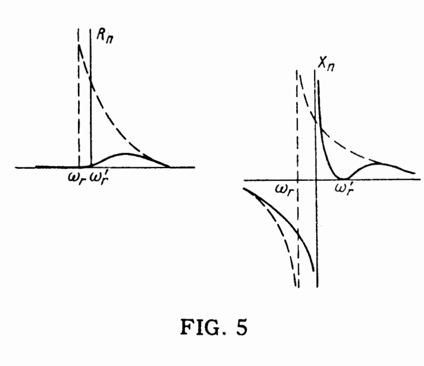

FIG. 5

that the impedance in the resonance region remains finite when the mixed condition (3) is satisfied on the boundary. Formula (26) allows us to determine the parameter ζ from experiment.

We note that the derivative $dR_\zeta/d\eta$ has a singularity at resonance. Expanding in (26), for finite ζ but small δ, we obtain

$$\frac{dR_\zeta}{d\eta}=\frac{5}{8}\frac{\omega_r\zeta^2}{c\beta q^5}\delta^{-1/2} \tag{27}$$

At frequencies below resonance, i.e., for $\eta = \eta_0 + \delta$ and $\delta > 0$, we get from (25)

$$Z_\zeta = -ic_1\zeta/(c_2+\zeta\delta^{1/2}). \tag{28}$$

The derivative $dX_\zeta/d\eta$ becomes infinite when resonance is approached from the low-frequency side.

The region of applicability of the formulas obtained above is determined by the inequalities (13). Putting $k = qx_0$ ($x_0 \sim 1$), we obtain

$$kR\sim\left[\left(\frac{R^2}{\delta_0 a}\right)^2\frac{4\pi M\mu_B}{\Theta_c}\right]^{1/5},$$

which amounts to 10^2 in fields $H \sim 10^4$ Oe. If the condition $kR \gg 1$ is regarded as a condition imposed on the magnetic field, then we can see that it is satisfied for all real values of the field.

We note that in ordinary metals the condition $kR \gg 1$ is valid only in very strong fields. It is readily realized in our case because the effective magnetic permeability, determined by the magnitude of the exchange effects, becomes anomalously large in the direct vicinity of resonance. The condition under which the skin effect becomes anomalous, $kl > 1$, is satisfied automatically by virtue of the second inequality in (13). The requirement that the parameter ka, which is connected with the spatial dispersion of the magnetic permeability, be small likewise does not impose any limitations on the magnitude of the field. When $H \sim 10^4$ Oe we have

$$ka\sim\left[\frac{a^3}{\delta_0^2 R}\frac{4\pi M\mu_B}{\Theta_c}\right]^{1/5}\sim 10^{-3}.$$

The most difficult to attain in ferromagnetic metals is apparently the second condition of (13), $R \ll l$, because of the large residual resistance. In recent experiments of Reed and Fawcett [15], however, in which the purest samples of iron were used, the ratio of the mean free path at helium temperatures to the mean free path at room temperature amounted to $l_{He}/l_r \sim 200-300$. In a field $H \sim 10^5$ Oe (the field used in [15]), this apparently makes the inequality $R \ll l$ realizable. In addition, to observe the investigated effects it is necessary, of course, that the proper line width of the ferromagnetic resonance be considerably smaller than the exchange shift (19). We note in this connection that Rado and Weertman [10] observed exchange broadening under the conditions of normal skin effect.

4. The existence of the weakly damped waves (17) and (18) near the resonant frequency causes the metal to become transparent in the corresponding frequency region. Let us consider in this connection the field distribution inside the metal. For simplicity we confine ourselves to the case $m(0) = 0$. The electric field in the metal is described by the expression

$$e(z)=\frac{2}{\pi i}\frac{\omega}{c}h(0)\int_0^\infty\frac{\mu(\omega,k)\cos kz}{k^2-\omega^2c^{-2}\varepsilon(\omega,k)\mu(\omega,k)}dk. \tag{29}$$

We transform this expression into

$$e(z)=\frac{h(0)}{\pi i}\frac{\omega}{c\beta q^3}\int_0^\infty\frac{x(e^{iqzx}+e^{-iqzx})}{x^5+\eta x^3+1}dx. \tag{30}$$

When $\eta < \eta_0$ the denominator of the integrand in (13), as already mentioned, has two roots located respectively in the upper and lower half-planes near the real axis. Closing the contour of integration of the first integral in the upper half-plane, and of the second integral in the lower half-plane, we obtain for the field at infinity, $z > 0$, in the frequency region $|\eta| \gg |\eta_0|$ ($\mathrm{Re}\,\eta < 0$),

$$e(z)=2h(0)\frac{\omega}{c\beta q^3}\left[\frac{\exp(-i|\eta|^{-1/3}qz)}{3|\eta|^{2/3}}+\frac{\exp(i|\eta|^{1/2}qz)}{2|\eta|^{3/2}}\right]. \tag{31}$$

The first term in (31) characterizes the propagation of a wave with anomalous dispersion (18). In this wave the phase grows towards the boundary, whereas the energy decreases in the direction of positive z. The second term in (31) describes a deeply damped spin wave (17).

Near the end point of the spectrum, the real positive roots of the dispersion equation coalesce. There are no damped waves at frequencies lower than ω_r'. Let us consider the field distribution in this case [3]. Let us transform (29) by expressing the field in terms of the amplitude E_0 of the incident wave:

$$e(z)=\frac{4}{\pi i}\frac{\omega}{c}\frac{E_0}{1+Z_p}\int_0^\infty\frac{\mu(\omega,k)\cos kz\,dk}{k^2-\omega^2c^{-2}\varepsilon(\omega,k)\mu(\omega,k)}. \tag{32}$$

[3]The authors are grateful to S. I. Pekar who called their attention to this question, which is in close analogy with the distribution of the field near the edge of the exciton absorption band in the dielectric [16] (see the appendix).

As shown above, the impedance Z_p has a root singularity near the shifted resonance frequency ω_r'. The same singularity is possessed by the integral in (32). As a result, the field is equal to

$$e(z) = 2E_0 \cos qzx_0, \tag{33}$$

i.e., it has the form of a standing wave. The incident wave experiences total reflection and, as mentioned above, no energy penetrates into the metal.

Under the more general boundary condition (3), the field inside the metal forms, as before, a standing wave at resonance, and the parameter ζ determines the phase of the field. This is the cause of the essential dependence of the impedance at resonance on the boundary condition for the magnetic moment. It can be shown that an analogous situation takes place also at frequencies below resonance. In this case the field has a more complicated structure, but there is no energy flux deep inside the metal.

In conclusion we note again the circumstance, referred to at the beginning of the section. The effective dielectric constant of the metal depends essentially on the ratio of ω to ω_c (see formula (14)). If $\cot(\pi\omega/\omega_c) < 0$, then the dispersion relation near the resonance has the following form:

$$\omega = \omega_r - \frac{\omega_0{}^2\Omega}{c^2Rk^3}\left|\operatorname{ctg}\pi\frac{\omega_r}{\omega_c}\right| + \Omega\beta k^2 \tag{34}$$

and is shown in Fig. 6. We see from this that in this case there is no end point of the spectrum, and the field inside the metal should constitute a traveling wave. Of course, in this case the impedance has no singularities.

APPENDIX

DISTRIBUTION OF FIELD IN A DIELECTRIC NEAR THE EXCITON-ABSORPTION BAND EDGE

The complete system is made up of the equations

$$\frac{i\omega}{c}(e + 4\pi P) = \frac{\partial h}{\partial z}, \qquad \frac{i\omega}{c}h = \frac{\partial e}{\partial z},$$

$$(\omega_0{}^2 - \omega^2)P - \alpha\frac{\partial^2 P}{\partial z^2} - i\lambda\omega P = \gamma e, \tag{A.1}$$

where P is the polarization, ω_0 is the exciton absorption frequency, γ is proportional to the oscillator strength, and λ is the damping constant ($\lambda > 0$). The term that takes damping into account is essential for a correct choice of the solution.

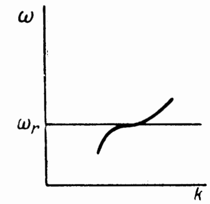

FIG. 6

In fact, the case of reflection of electromagnetic waves from the boundary of a dielectric near the exciton-absorption line differs from the reflection considered above from a ferromagnetic dielectric (Sec. 2) only in the sign of the effective mass of the excitation. In a ferromagnet the effective mass of the spin wave always is larger than zero, and the additional waves can exist only as a result of spatial dispersion of the conductivity. In a dielectric the exciton effective mass can be either positive or negative. Its sign coincides with the sign of the parameter α. We are, naturally, interested only in the case of negative effective mass, when additional waves can propagate in the medium.

The dispersion equation of the system (A.1) has for $\alpha < 0$ the form

$$k^2 = \frac{\omega^2}{c^2}\frac{p}{s - k^2};$$

$$p = 4\pi\gamma / |\alpha|, \quad s = (\omega_0{}^2 - \omega^2 - i\lambda\omega) / |\alpha|. \tag{A.2}$$

It is easy to see (Fig. 7) that Eq. (A.2) has at the point $s = s_0 = 2\omega_0 p^{1/2}/c$ multiple roots with respect to k^2 when $\lambda = 0$.

We put $s = s_0 + \delta$, $\delta = \delta' + i\delta''$, where $\delta' > 0$. From the definition of s it follows that $\delta'' < 0$. We assume that δ'' is small compared with δ'. The solution of (A.2) is

$$k_{1,2,3,4} = \pm\sqrt{s_0/2}\left[1 \pm \sqrt{\delta'/2s_0}(1 + i\delta''/\delta')\right]. \tag{A.3}$$

It is convenient to introduce the notation

$$\varkappa = \sqrt{s_0/2}, \qquad \Delta = {}^1/_2\sqrt{\delta'}, \qquad \tau = {}^1/_2\delta''/\sqrt{\delta'}.$$

Solutions satisfying the condition at infinity ($\operatorname{Im} k > 0$) are

$$k_1 = \varkappa - \Delta - i\tau, \qquad k_2 = -\varkappa - \Delta - i\tau. \tag{A.4}$$

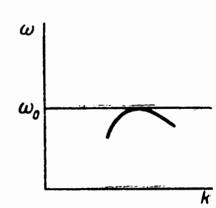

FIG. 7

The field e is written in the form

$$e(z) = Ae^{ik_1z} + Be^{ik_2z}, \qquad (A.5)$$

after which we obtain A and B from the boundary conditions

$$E_0 + E_{\text{ref}} = e(0), \quad H_0 + H_{\text{ref}} = h(0),$$

$$dP/dz + \zeta P|_0 = 0. \qquad (A.6)$$

At resonance

$$A_0 = \frac{k_0}{\varkappa}\frac{\zeta - i\varkappa}{\zeta - ik_0}, \qquad B_0 = -\frac{\zeta + i\varkappa}{\zeta - i\varkappa}A_0,$$

where $k_0 = \omega_0/c$, and expression (A.5) for the field takes the form

$$e(z) = \frac{k_0}{\varkappa}\frac{E_0}{\zeta - ik_0}\{(\zeta - i\varkappa)e^{i\varkappa z} - (\zeta + i\varkappa)e^{-i\varkappa z}\}. \qquad (A.7)$$

We finally write (A.7) in the form

$$e(z) = 2iE_0\frac{k_0}{\varkappa}\left|\frac{\zeta - i\varkappa}{\zeta - ik_0}\right| e^{i\psi}\sin(\varkappa z - \varphi),$$

$$\psi = \tan^{-1}(k_0/\zeta), \qquad \varphi = \tan^{-1}(\varkappa/\zeta). \qquad (A.8)$$

Thus, at resonance the field forms a standing wave and there is no energy flux into the dielectric. The boundary parameter ζ is connected with the phase of both the standing wave in the dielectric and the reflected wave in the vacuum. The surface impedance is then imaginary:

$$Z = e(0)/h(0) = -ik_0/\zeta, \qquad (A.9)$$

corresponding to a unity reflection coefficient.

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