THE BREAKING UP OF ATOMIC PARTICLES BY AN ELECTRIC FIELD AND BY ELECTRON COLLISIONS

B. M. SMIRNOV and M. I. CHIBISOV

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The problem of the breakup of an atomic particle in a constant homogeneous electric field is considered. An asymptotically exact (in the limit of low field intensity) expression is obtained for the probability of the detachment of an electron per unit time. The result obtained is used to determine the binding energy of an electron in a negative helium ion from the experimentally measured dependence of the lifetime of the ion on the intensity of the external electric field in which it is breaking up. The electron affinity energy of a helium atom is 0.06 ± 0.005 eV. The cross section for the breaking up of a negative ion by electron impact as a result of which an s-electron is liberated has been calculated. For this it is assumed that the break up cross section is large compared to the characteristic dimension of a negative ion and that the incident electron moves in a classical orbit. These assumptions are justified by the result obtained. Two mechanisms of breakup of negative ions are considered: the "squeezing out" of a weakly bound electron of a negative ion by the static field of the incident electron and the breakup resulting from the time variation of the field of the incident electron. It is shown that at high collision velocities the second mechanism gives a result corresponding to the Born approximation. The dependence of the cross section for the breakup of H^- on the velocity of the incident electron is compared to the results of other calculations.

IN this paper we consider the breakup of atoms and of ions in a constant electric field and the disintegration of negative ions by electron collisions. The common feature of these problems is the fact that the effects under consideration are determined by the behavior of the valence electron in an atom at large distances from the nucleus. The removal of an electron from an atom or an ion placed in a constant electric field occurs as a result of the tunnel effect. The difficulty in this problem consists of the fact that the barrier through which the electron penetrates is a three-dimensional one. In order to overcome this difficulty the problem is artificially reduced to a one-dimensional problem [1,2], but the method of introducing the effective potential barrier is not justified and the obtained result is incorrect. In two cases the problem can be solved exactly. In the case of a hydrogen atom in the ground state [3] placed in an electric field the variables in the Schrödinger equation for the wave function of the electron can be separated in parabolic coordinates so that the problem is reduced to a one-dimensional problem. The Schrödinger equation can also be solved for the wave function of an electron situated in a spherically symmetrical field of force of zero range and in a constant electric field [4].

Of practical interest is the case when the intensity of the external electric field is much smaller than the intensity of the characteristic atomic fields. If this condition is satisfied the breakup of the atomic particle occurs slowly compared to the characteristic atomic times and the leaking out of the electron takes place primarily in directions close to the direction of the electric field. Therefore, in order to determine the frequency of the passage of the electron through the barrier it is sufficient to solve the Schrödinger equation near an axis directed along the electric field and passing through the atomic nucleus. This is carried out in the present paper by the usual quasiclassical methods on the basis of the fact that far from the nucleus the fields acting on the electron change by only a small amount when the electron is displaced by a distance of the order of atomic dimensions. As a result we obtain an asymptotically exact (in the limit of low intensity of the external electric field) value for the probability per unit time of the electron passing through the barrier. For such a determination of the transition probability it is sufficient to know the asymptotic form of the electron wave function in the atom for large distances from the nucleus.

In a similar manner one can solve the problem of the disintegration of a negative ion situated in the static field produced by a Coulomb center. The result obtained is utilized to find the cross section for the breakup of a negative ion by an electron collision. This effect which is important in the disintegration of a negative ion by a slow electron was investigated by O. B. Firsov. At large collision velocities the negative ion disintegrates largely due to the variation in time of the field of the incident electron. In the present paper the cross sections for the breaking up of negative hydrogen ions by electrons due to both effects were calculated on the assumption that their value considerably exceeds the effective size of the negative ion. The result obtained justifies the assumptions made.

THE BREAKING UP OF AN ATOMIC PARTICLE IN A CONSTANT ELECTRIC FIELD

For the investigation of this problem we utilize the one-electron approximation. The probability of the transition of an electron into the continuous spectrum is determined by the expression

$$W = \int_S \mathbf{j}\, ds,$$

$$\mathbf{j} = {}^1/_2 i(\Psi\nabla\Psi^* - \Psi^*\nabla\Psi), \qquad (1)$$

where $\mathbf{j}$ is the electron current density; we utilize the system of atomic units $\hbar = m_e = e^2 = 1$. For the surface S it is convenient to choose a plane perpendicular to the direction of the electric field. For a low intensity of the external electric field F the integral (1) converges rapidly near the x axis directed along the field and passing through the atomic nucleus. Therefore, in order to determine the probability of the transition of an electron through the barrier it is sufficient to obtain the wave function of the electron near this axis.

The wave function of an electron making the transition is a solution of the Schrödinger equation

$$[-{}^1/_2\Delta + U(r) - Fx]\Psi = \varepsilon\Psi, \qquad (2)$$

where U(r) is the potential of the interaction of the electron with the atomic core with the potential taken to be spherically symmetric; r is the distance from the electron to the nucleus.

We assume that the electric field is small, so that there exists a range of distances from the nucleus along the x axis where the binding energy of the electron is $-\epsilon = \gamma^2/2 \gg |U(x)|$, $\gamma^2/2 \gg Fx$. In the domain of these and smaller distances one can neglect the term Fx in equation (2), so that the solution of equation (2) will be the atomic wave function:

$$\Psi = \psi(r)\left[\frac{2l+1}{2}\frac{(l-m)!}{(l+m)!}\right]^{1/2} P_l^m(\cos\theta)\frac{e^{\pm im\Phi}}{\sqrt{2\pi}}. \qquad (3)$$

Here r, θ, and Φ are the spherical polar coordinates of the electron, l is the orbital angular momentum of the electron, m is its component along the x axis.

Far from the nucleus the radial wave function of the electron is a solution of the Schrödinger equation with the potential $U = -Z/r$, where Z is the charge of the atomic core, and has the asymptotic form:

$$\psi(r) = Br^{Z/\gamma-1}e^{-\gamma r}. \qquad (4)$$

The constant B which is determined by the behavior of the electron inside the atom we shall take as known.

For $r \gg 1$ the wave function satisfies the Schrödinger equation

$$\left(-\frac{\Delta}{2} - \frac{Z}{r} - Fx\right)\Psi = -\frac{\gamma^2}{2}\Psi \qquad (5)$$

and goes over into (3) and (4) in those regions where the electric field can be neglected. Equation (5) is separable in parabolic coordinates[3,5]:

$$\xi = r + x, \quad \eta = r - x, \quad \Psi = \varphi(\xi)\chi(\eta)/\sqrt{\xi\eta},$$

$$\varphi'' + \left(-\frac{\gamma^2}{4} + \frac{\beta_1}{\xi} + \frac{1}{4\xi^2} + \frac{F}{4}\xi\right)\varphi = 0,$$

$$\chi'' + \left(-\frac{\gamma^2}{4} + \frac{\beta_2}{\eta} + \frac{1}{4\eta^2} - \frac{F}{4}\eta\right)\chi = 0. \qquad (6)$$

The separation constants β_1 and β_2 are related by the equation $\beta_1 + \beta_2 = Z$ and in solving the Schrödinger equation near the x axis ($\eta \ll \xi$) and $x \gg 1$ they are determined by comparing (6) with the asymptotic form (3) and (4) of the wave function of the electron in the atom near the x axis:

$$\Psi = B\left(\frac{\xi}{2}\right)^{Z/\gamma-1-m/2}$$

$$\times\, e^{-\gamma\xi/2}\left[\frac{(2l+1)(l-m)!}{2(l+m)!}\right]^{1/2}\eta^{m/2}e^{-\gamma\eta/2}\frac{e^{\pm im\Phi}}{\sqrt{2\pi}}. \qquad (7)$$

Substituting (7) into (6) we obtain

$$\beta_1 = Z - {}^1/_2\gamma(m+1), \qquad \beta_2 = {}^1/_2\gamma(m+1). \qquad (8)$$

Utilizing the fact that the integral (1) is damped out rapidly near the x axis and selecting for the surface S a plane perpendicular to the x axis we obtain $ds = \rho d\rho d\Phi = {}^1/_2\,\xi d\eta d\Phi$, since

$$\eta = (\rho^2 + x^2)^{1/2} - x \approx \rho^2/2x \approx \rho^2/\xi,$$

where ρ is the distance from the x axis. The electron current is given by

$$j = \frac{i}{2}\left[\Psi\frac{d\Psi^*}{dx} - \Psi^*\frac{d\Psi}{dx}\right] = \frac{i}{2\pi}\frac{\chi^2}{\xi\eta}\left[\varphi\frac{d\varphi^*}{d\xi} - \varphi^*\frac{d\varphi}{d\xi}\right],$$

and since near the x axis the function $\chi = \eta^{m/2}e^{-\gamma\eta/2}$, the probability of transition per unit time is given by

$$W = \frac{i}{\xi}\left(\varphi\frac{d\varphi^*}{d\xi} - \varphi^*\frac{d\varphi}{d\xi}\right)\frac{m!}{\gamma^{m+1}}. \tag{9}$$

The quasiclassical solution of Eq. (6) for $\varphi(\xi)$ to the left and to the right of the turning point ξ_0 has the form [3]

$$\varphi = \begin{cases} i\dfrac{C}{\sqrt{p}}\exp\left[i\displaystyle\int_{\xi_0}^{\xi} p\,d\xi - i\frac{\pi}{4}\right], & \xi > \xi_0 \\ \dfrac{C}{\sqrt{p}}\exp\left[-\displaystyle\int_{\xi_0}^{\xi} p\,d\xi\right], & \xi < \xi_0 \end{cases};$$

$$p = \left|\frac{\gamma^2}{4} - \frac{\beta_1}{\xi} - \frac{F}{4}\xi\right|^{1/2}. \tag{10}$$

From (9) we obtain for the probability of decay per unit time

$$W = m!C^2/\gamma^{m+1}.$$

From (10) we have that in the region $Z/\gamma^2 \ll \xi \ll \gamma^2/F$ the wave function φ has the form

$$\varphi = C\sqrt{\frac{2}{\gamma}}\left(\frac{F}{4\gamma^2}\xi\right)^{\beta_1/\gamma} e^{\gamma^3/3F-\gamma\xi/2}.$$

Requiring that in this region the wave function should coincide with the expression (7) we shall obtain the value of the coefficient C. In this case the probability of detaching an electron per unit time turns out to be equal to

$$W = \frac{B^2(2l+1)}{2\gamma^m}\frac{m!(l+m)!}{(l-m)!}\left(\frac{2\gamma^2}{F}\right)^{2Z/\gamma-m-1} e^{-2\gamma^3/3F}. \tag{11}$$

In the case of a hydrogen atom in the ground state ($Z = \gamma = 1$, $m = l = 0$, $B = 2$), we obtain

$$W = 4F^{-1}e^{-2/3F},$$

which coincides with the well known result [3].

A special case in the discussion of the probability of disintegration in an electric field is presented by a hydrogenlike atom. The stationary states of a hydrogenlike atom in a constant electric field are described by the parabolic quantum numbers n_1, n_2, and m, while the Schrödinger equation (2) is everywhere separable. The separation constants in (6) are:

$$\beta_1 = \gamma\left(n_1 + \frac{m+1}{2}\right), \qquad \gamma = \frac{Z}{n};$$

$$\beta_2 = \gamma\left(n_2 + \frac{m+1}{2}\right);$$

where the principal quantum number is $n = n_1 + n_2 + m + 1$, $m > 0$. The probability of decay of a hydrogenlike atom per unit time obtained by the method described above is equal to

$$W = \frac{e^{-2\gamma^3/3F}\gamma^3(4\gamma^3/F)^{2n_1+m+1}}{Z(n_1+m)!n_1!} \tag{12}$$

The criterion for the applicability of relations (11), (12) obtained above is the requirement that there should exist a region ξ, η, in which the atomic wave function already coincides with its asymptotic expression, while the electric field is as yet unimportant. This yields

$$F \ll \gamma^4/2\beta, \qquad \beta = \max(\beta_1, \beta_2). \tag{13}$$

Formula (11) can be utilized to obtain the binding energy of an electron in a negative ion in terms of the experimentally measured dependence of the mean lifetime for the decay of an ion on the intensity of the electric field [4]. Averaging (11) over the component of the angular momentum of a weakly bound electron we obtain for the probability of decay

$$W_l = \sum_m \frac{W_{lm}}{2l+1} = \frac{B^2F}{4\gamma^2}e^{-2\gamma^3/3F}, \tag{14}$$

where this quantity is entirely determined by the decay of the state the component of whose angular momentum on the direction of the electric field is $m = 0$. In the papers of Riviere and Sweetman [6] the frequency of decay of a negative helium ion was measured as a function of the intensity of the electric field. A comparison of the results of these papers with formula (14) shown in Fig. 1 enables us to determine the binding energy of the electron in a negative helium ion which turns out to be equal to 0.06 ± 0.005 eV. The value of the coefficient B utilized in (14) was obtained from the slope of the experimental curve and is equal to ~ 0.18. The value of the electron affinity energy in a helium atom obtained by Holfien and Midtal [7] on the basis of a variational calculation is equal to 0.075 eV.

THE BREAKING UP OF A NEGATIVE ION BY ELECTRON COLLISIONS

1. The process of the breaking up of negative hydrogen ions colliding with electrons is of interest in astrophysics. In the case of large cross sections for the breaking up of an ion this process

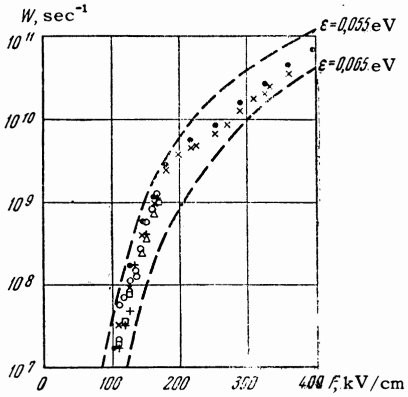

FIG. 1. Dependence of the probability of the breaking-up of He^- in an electric field on the intensity of the field: points—experimental values[6], dotted curves—calculations by means of formula (14).

can alter the thermodynamic equilibrium of negative hydrogen ions in stellar atmospheres[8,9]. For the evaluation of the cross section for the breaking up of H^- by electron collisions Geltman[10] and Rudge[11] in their papers used the Born approximation together with the introduction of some additional assumptions which enable one to cut off the cross section near the threshold. However, these assumptions are not well justified so that the cross sections obtained in these papers do not correspond to the actual cross sections. A sensible approximation to be used for obtaining the cross section for the breaking up of H^- by electron impact was proposed by McDowell and Williamson[12]. But also in this case assumptions are made the accuracy of which cannot be estimated. The method presented in this paper for the calculation of the cross section for the breaking up of a negative ion in a collision with an electron is based on the assumption that the breaking up cross section exceeds the effective dimensions of the negative ion. Moreover, it is assumed that the principal contribution to the cross section is made by collisions with impact parameters ρ for which the classical description of the incident electron is valid

$$l \equiv \rho v \sim v\sqrt{\sigma} \gg 1. \qquad (15)$$

The validity of this condition can be checked after obtaining the final result.

2. The breaking up of a negative ion in a collision with an electron can take place in two ways. One of them is related to the "squeezing out" of a weakly bound electron from the negative ion by the Coulomb field of the free electron, while the other leads to the breaking up of the negative ion as a result of the variation in time of the field of the incident electron. Let us start by considering the first method of the breaking up of a negative ion.

If a negative Coulomb charge is placed not far from a negative ion in such a way that the potential of interaction of the charge with the negative ion would be greater than the binding energy of the electron, then it becomes possible for the electron to leak through the barrier, and this leads to the decay of the negative ion. The probability of decay of a negative ion per unit time as a result of the action of the Coulomb field due to the stationary center can be determined by the same method as in the case of the presence of a homogeneous electric field. The probability of breaking up per unit time is given by relation (11), so that in order to obtain it it is necessary to find the wave function of a weakly bound electron in the negative ion. The wave function of such an electron satisfies the Schrödinger equation

$$\left(-\frac{\Delta}{2}+V(\mathbf{r})+\frac{1}{|\mathbf{R}-\mathbf{r}|}\right)\Psi=\left(-\frac{\gamma^2}{2}+\frac{1}{R}\right)\Psi, \qquad (16)$$

where $V(\mathbf{r})$ is the effective potential for the interaction between the electron and the atom, R is the distance from the atomic nucleus to the Coulomb center of a unit negative charge (free electron), $\gamma^2/2$ is the binding energy of the electron in the negative ion, r is the electron coordinate, and the atomic nucleus is chosen as the origin of the system of coordinates.

We shall assume that the intensity of the field due to the Coulomb center is not great in the region where the weakly bound electron is situated, so that the wave function of the electron in the region where the atomic field is effective is the same as in the absence of the Coulomb center. In this case one can neglect the corrections to the electron energy due to the action of the homogeneous field of the Coulomb center, i.e.,

$$\alpha / 2R^4 \ll \gamma^2 / 2, \quad 1 / R. \qquad (17)$$

Here α is the polarizability of the negative ion, where in the case of an s-electron[4] $\alpha = A^2/4\gamma^4$, and where the coefficient A is defined in expression (19).

Condition (17) defines the criterion for the applicability of the method utilized. If this condition is satisfied beyond the range of action of the atomic field, then the variables in equation (16) are separable in elliptic coordinates[3,13]:

$$\xi=(r_1+r_2)/R, \quad \eta=(r_1-r_2)/R, \quad \Psi=X(\xi)Y(\eta);$$

$$\frac{d}{d\xi}\left[(\xi^2-1)\frac{dX}{d\xi}\right]+\left[-R\xi-\frac{R^2\gamma^2}{4}\xi^2+\frac{R\xi^2}{2}+D\right]X=0,$$

$$\frac{d}{d\eta}\left[(1-\eta^2)\frac{dY}{d\eta}\right]+\left[-R\eta+\frac{R^2\gamma^2}{4}\eta^2-\frac{R\eta^2}{2}-D\right]Y=0. \qquad (18)$$

Here $r_{1,2}$ are the distances from the electron to the atomic nucleus and to the Coulomb center, D is the separation constant.

3. We shall make use of the fact that when (17) is satisfied the electron goes through the barrier primarily in the directions $\eta = -1$, and we shall solve the Schrödinger equation near this axis. Let us consider the process of detachment of an s-electron from the negative ion. The asymptotic form of the wave function for an s-electron in a negative ion can be written in the form (4)

$$\Psi = A\sqrt{\frac{\gamma}{2\pi}}\,\frac{e^{-\gamma r}}{r}. \qquad (19)$$

We shall obtain the constant D in (18) from the requirement that for $r_1 \ll R$, $r_1\gamma \gg 1$ the wave function (18) near the $\eta = -1$ axis should go over into (19). This yields

$$D = R^2\gamma^2/4 + R/2 - R\gamma.$$

Further, utilizing the same method for solving equations (18) which was used in investigating the breakup of an atomic particle in a constant electric field, we shall obtain in the case under consideration for the probability of breakup of the ion per unit time

$$W = \frac{B^2}{2\gamma R^2}\exp\left\{-2\gamma R f\left(\frac{R\gamma^2}{2}\right)\right\},$$
$$f(x) = \arcsin x^{1/2}/[x(1-x)]^{1/2} - 1. \qquad (20)$$

In the limit $R\gamma^2/2 \ll 1$ this yields

$$W = \frac{B^2 F}{2\gamma}e^{-2\gamma^3/3F}, \qquad (21)$$

and this coincides with formula (14) ($A = B\sqrt{2\gamma}$). Here $F = 1/R^2$ is the intensity of the field of the Coulomb center at the center of the negative ion.

4. The probability $P(\rho, t)$ that in colliding with an electron of impact parameter ρ a negative ion has disintegrated by the time t satisfies the equation

$$\frac{dP(\rho, t)}{dt} = W[1 - P(\rho, t)].$$

From this the probability of squeezing out an electron from a negative ion in the case of an impact parameter ρ is given by the formula

$$P(\rho) = 1 - \exp\left[-\int_{-\infty}^{+\infty} W(R)\,dt\right].$$

The trajectory for the motion of an incident classical electron is determined by the Coulomb potential of its interaction with the negative ion, so that the expression for the relative motion of the electron and the negative ion $R(t)$ given in parametric form is described by the relations [14]

$$R = a(\varepsilon\,\mathrm{ch}\,\xi + 1), \quad t = b(\varepsilon\,\mathrm{sh}\,\xi + \xi), \quad a = v^{-2},$$
$$b = v^{-3}, \quad \varepsilon = (1 + \rho^2 v^4)^{1/2}, \quad -\infty < \xi < +\infty, \qquad (22)^*$$

where v is the relative velocity in the collision. Using this law of motion we shall obtain the following expression for the cross section for the decay of a negative ion colliding with an electron as a result of the squeezing out of a weekly bound electron:

$$\sigma = 2\pi\int_0^\infty \rho[1 - e^{-w}]\,d\rho,$$

$$w = \frac{A^2 v}{2\gamma}\int_{-\infty}^{+\infty}\frac{d\xi}{(\varepsilon\,\mathrm{ch}\,\xi + 1)}$$
$$\times\exp\left\{-\frac{2\gamma(\varepsilon\,\mathrm{ch}\,\xi + 1)}{v^2} f\left[\frac{\gamma^2}{2v^2}(\varepsilon\,\mathrm{ch}\,\xi + 1)\right]\right\}. \qquad (23)$$

The form of $f(x)$ is defined by formula (20). The coefficient A can be obtained by means of joining the asymptotic expression for the wave function (19) to the wave function for the negative ion constructed on the basis of the variational method. For a negative hydrogen ion this yields $A_2 = 2.65$ [15,16].

5. In evaluating the cross section for the decay of a negative ion as a result of the motion of the electron we shall as before assume that the incident electron moves along a classical orbit and the decay takes place primarily at distances between the electron and the negative ion considerably in excess of the dimensions of the negative ion:

$$R \gg 1/\gamma. \qquad (24)$$

If, moreover, we can neglect the curvature of the trajectory of the incident electron, i.e., describe the incident electron by a plane wave and assume that the principal contribution to the cross section is made by the range of impact parameters for which perturbation theory is applicable, then we shall obtain the Born approximation which corresponds to the cross section for the decay of an atomic particle ([3], p. 664)

$$\sigma = \frac{8\pi}{v^2}(z^2)_{00}\ln\left(\frac{\rho_{max}}{\rho_{min}}\right). \qquad (25)$$

Here ρ_{max} is the characteristic impact parameter for which scattering occurs into such small angles that the quantum nature of the scattering must be taken into account; ρ_{min} is the characteristic impact parameter for which perturbation theory no longer holds.

*sh ≡ sinh, ch ≡ cosh.

Let us evaluate more accurately the expression in the argument of the logarithm in formula (25) in the case of the breaking up of a negative ion by electron impact, and let us establish the collision velocities for which the curvature of the trajectory of the incident electron is significant, so that formula (25) no longer holds. Utilizing relation (24) we expand the operator for the interaction $V = |R - r|^{-1}$ between a weakly bound electron in a negative ion and the incident Coulomb charge in a power series and restrict ourselves to the first few terms of the expansion:

$$V = \frac{1}{|\mathbf{R}-\mathbf{r}|} \approx \frac{1}{R} + \frac{\mathbf{Rr}}{R^3}. \tag{26}$$

If we use perturbation theory to obtain the probability amplitude for the transition and if we assume that the expression for the relative motion of the negative ion and the incident Coulomb center corresponds to free motion ($R^2 = \rho^2 + v^2t^2$), we shall obtain for the probability for the electron to make a transition to the state k in the case of a collision with impact parameter ρ the expression

$$P_k(\rho) = \frac{4\omega^2 z_{0k}^2}{v^4}\left[K_0^2\left(\frac{\omega\rho}{v}\right) + K_1^2\left(\frac{\omega\rho}{v}\right)\right], \tag{27}$$

where ω is the excitation energy of the electron, K_p are Macdonald functions; the matrix element is evaluated between the initial (0) and the final (k) states of a weakly bound electron. The first term in formula (27) corresponds to a transition into the state in which the component of the angular momentum of the electron along the direction of motion is m = 0, and the second term to a transition into the state with the component m = ±1.

The contribution to the cross section for the transition of the electron to the state k made by the range of impact parameters $\rho > \rho_0$ for which perturbation theory is valid is equal to

$$\sigma_k^{>} = 2\pi\int_{\rho_0}^{\infty}\rho P_k(\rho)\,d\rho = 8\pi z_{0k}^2\frac{\omega\rho_0}{v^3}K_0\left(\frac{\omega\rho_0}{v}\right)K_1\left(\frac{\omega\rho_0}{v}\right). \tag{28}$$

Let us make use of the expression for the matrix element z_{0k}:

$$z_{0k}^2 = \frac{32}{\pi}\frac{B^2}{6\gamma^3}\frac{x^4}{(1+x^2)^4}, \qquad x = \frac{k}{\gamma}, \tag{29}$$

which holds for transitions to states with not very large values of electron momenta which give the principal contribution to the cross section for the transition. It is obtained if for the wave function of the bound electron one utilizes the function (19), while for the electron in the continuous spectrum one utilizes a plane wave. On the basis of formulas (28) and (29) we shall obtain for that part of the breakup cross section which is determined by the range of impact parameters $\rho > \rho_0$ for which perturbation theory is valid the formula

$$\sigma^{>} = \frac{8\pi(z^2)_{00}}{v^2}F_1\left(\frac{\omega_0\rho_0}{v}\right);$$
$$F_1(\alpha) = \alpha\int_0^{\infty}\frac{32}{\pi}\frac{x^4dx}{(1+x^2)^3}K_0[\alpha(1+x^2)]K_1[\alpha(1+x^2)],$$
$$F_1(\alpha) \to \ln\frac{0.46}{\alpha} \quad \text{as } \alpha\to 0. \tag{30}$$

In determining the probability of transition for $\rho < \rho_0$ one can assume that it is determined by formula (27) if the total probability of the decay of the ion calculated by means of this formula is less than unity, and that the total probability of the decay of the ion is equal to unity in the opposite case. Utilizing this we obtain that the cross section for the breaking up in the case $\omega\rho_{pert}/v \ll 1$ (ρ_{pert} is the impact parameter of the collision for which $\Sigma_k P_k(\rho) = 1$) is equal to

$$\sigma = \frac{8\pi(z^2)_{00}}{v^2}\ln\left(\frac{0.63v}{\omega_0\rho_{pert}}\right),$$

where, as follows from (27) and (29), for $\omega\rho_{pert}/v \ll 1$ we have

$$\rho_{pert} = \frac{2A}{v\gamma\sqrt{6}} \cong \frac{2(z^2)_{00}^{1/2}}{v}.$$

A more accurate evaluation than the one given above of the probability of decay for $\rho < \rho_0$ yields for the cross section for the breakup of a negative ion

$$\sigma = \frac{8\pi(z^2)_{00}}{v^2}\ln\left(\frac{1.55v^2}{A\gamma}\right). \tag{31}$$

Relation (31) determines the cross section for the breaking-up of a negative ion by electron impact. It agrees with the general formula for the cross section for inelastic scattering in the Born approximation (25) in which the expression in the argument of the logarithm has been evaluated.

6. Formula (31) is valid in the case when one can neglect the curvature of the trajectory, so that

$$a/\rho_{max} = \omega/v^2 \ll 1. \tag{32}$$

Here $\rho_{max} \approx v/\omega$ is the characteristic impact parameter such that for greater values of ρ the probability amplitude for the transition is $C_k \approx \exp(-\rho/\rho_{max})$, $a = v^{-2}$ is the radius of curvature. The curvature of the trajectory leads to the fact that in such collisions distances at which transitions effectively take place are not attained, and this leads to a lowering of the value of the cross section for the breaking up of a negative ion.

Assuming that Eq. (32) is satisfied we determine the correction due to the curvature and establish the collision velocities at which one can neglect the curvature. Utilizing the law of motion in the Coulomb field (22) and perturbation theory for the transition amplitude we obtain for the transition probability in place of (27) the formula

$$P_k(\rho)=\frac{4\omega^2 z_{0k}^2}{v^4}\left[K_0^2\left(\frac{\omega\rho}{v}\right)+K_1^2\left(\frac{\omega\rho}{v}\right)\right]\left(1-\pi\frac{\omega}{v^3}\right),$$
$$\frac{\omega}{v^3}\ll 1.$$

Since the principal contribution to the cross section (31) is made by transitions with $\omega = (2-4)\omega_0$, the curvature of the trajectories significantly lowers the cross section for the breakup of a negative ion even in the range of velocities of the order of atomic velocities.

We determine the total cross section for the breaking-up of a negative ion by electron impact. Formula (23) for the cross section for "squeezing out" a weakly bound electron by the static field of an incident electron is valid if the characteristic frequency of the variation of the field of the incident electron $v\rho/R_{min}^2$ is much smaller than the characteristic atomic frequency $\omega_0 = \gamma^2/2$. Here $R_{min} = (1+\epsilon)/v^2$ is the minimum distance of approach of the incident electron and the negative ion at impact parameters $\rho \sim \sqrt{\sigma}$ which make the principal contribution to the cross section. This yields for $\sigma v^4 \lesssim 1$:

$$\sigma_{sq} < \gamma^2/v^5.$$

Thus, at low collision velocities the breaking up of a negative ion in a collision with an electron occurs as a result of the "squeezing out" of the electron by the static field of the incident electron, while the Born cross section in this range of velocities is small due to the curvature of the trajectory. At high velocities when the curvature of the trajectory becomes unimportant, the "squeezing out" of the electron no longer occurs due to the rapid variation of the field of the incident electron. Joining together the cross section for "squeezing out" and the Born cross section in the intermediate range of velocities we shall obtain the total cross section for the breaking up of a negative ion by electron impact. As a result we determine correctly the cross section for breaking up at large and at not very small energies while in the intermediate energy range where the cross section has its maximum we cannot guarantee the accuracy of the result. The results of the calculation of the cross section for the breaking up of a negative hydrogen ion are shown in Fig. 2 and compared to the results of other calculations.

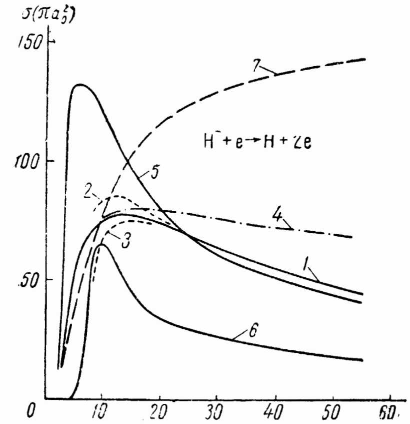

FIG. 2. Cross section for the breaking-up of H^- by an electron: 1 – total cross section, 2 – Born cross section (31), 3 – Born cross section taking the curvature of the trajectory into account, 4 – squeezing-out cross section (23), 5 – the result of McDowell and Williamson[12], 6 – the same using Geltman's corrections, 7 – Geltman's result[10] reduced by a factor 10.

Of practical interest is the quantity $\langle \sigma v \rangle$ where the averaging is made with the aid of the electron distribution function. If the electron distribution is assumed to be Maxwellian then this characteristic turns out to be equal to 1.3×10^{-8} cm^3/sec at a temperature of 6000°. On the basis of this one can conclude that if in solar and stellar atmospheres Maxwellian velocity distribution holds for electrons then the thermodynamic equilibrium of H^- is not violated in those regions.

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[1] S. N. Kaplan, G. A. Paulikas, and R. V. Pule, Phys. Rev. **131**, 2574 (1963).

[2] G. Darewich and S. Neamtan, Nucl. Instr. and Meth. **21**, 241 (1963).

[3] L. D. Landau and E. M. Lifshitz, Kvantovaya mekhanika (Quantum Mechanics), Fizmatgiz, 1963.

[4] Yu. N. Demkov and G. F. Drukarev, JETP **47**, 918 (1964), Soviet Phys. JETP **20**, 614 (1965).

[5] H. Bethe and E. Salpeter, Hand. Physik. Vol. 35, Springer, 1957. (Russ. Transl. IIL, 1960, Sec. 6).

[6] A. C. Riviere and D. R. Sweetman, Phys. Rev. Letters **5**, 560 (1960); Proc. V Intern. Conf. on

Ionic Phenomena in Gases, Munich, 1961, North Holland Publ. Co., Amsterdam, 1962, 2, p. 1236.

[7] E. Holfien and J. Midtal, Proc. Phys. Soc. 68, 815 (1955).

[8] B. E. J. Pagel, Monthly Not. Roy. Astron. Soc. **116**, 608 (1956).

[9] B. E. J. Pagel, Monthly Not. Roy. Astron. Soc. **119**, 609 (1959).

[10] S. Geltman, Proc. Phys. Soc. **75**, 67 (1960).

[11] M. R. H. Rudge, Proc. Phys. Soc. **83**, 419 (1964).

[12] M. R. C. McDowell and J. H. Williamson, Phys. Letters 4, 159 (1963).

[13] S. S. Gershteĭn and V. D. Krivchenkov, JETP **40**, 1491 (1961), Soviet Phys. JETP **13**, 1044 (1961).

[14] L. D. Landau and E. M. Lifshitz, Mekhanika (Mechanics), Fizmatgiz, 1958, p. 51.

[15] B. H. Armstrong, Phys. Rev. **131**, 1132 (1963).

[16] T. Ohmura and H. Ohmura, Phys. Rev. **118**, 154 (1960).

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