PASSAGE OF POLARIZED NEUTRONS THROUGH A SUPERCONDUCTOR IN THE MIXED STATE

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It is shown that, during passage through a superconductor of the second kind in the mixed state, a beam of polarized neutrons is depolarized. In principle the marked dependence of the depolarization on beam direction and the direct relation between the period of this dependence and the structure of the magnetic field can be used for the experimental investigation of the Abrikosov two-dimensional vortex lattice.

THE magnetic field in a superconductor of the second kind in the mixed state has a two-dimensional periodic structure.[1-3] We shall show that a possible method for studying this structure is through transmission of a beam of polarized neutrons through the sample.[1)]

If the initial polarization of the beam had components $\mathbf{p}_{\parallel}$ along and $\mathbf{p}_{\perp}$ perpendicular to the direction of the field **H**, the polarization $\mathbf{p}'$ after passage through the sample is given by the formula

$$\mathbf{p}' = \mathbf{p}_{\parallel} + \mathbf{p}_{\perp}\langle\cos\varphi\rangle + [\mathbf{p}_{\perp}\mathbf{H}/H]\langle\sin\varphi\rangle, \qquad (1)^*$$

where the symbol $\langle\ldots\rangle$ denotes an average over different neutron trajectories, φ is the angle of rotation of the neutron spin along a given trajectory, which is determined by the integral along the section lying within the sample:

$$\varphi = \frac{2\mu}{\hbar v}\int H dl, \qquad (2)$$

μ is the magnetic moment of the neutron, v its velocity. We shall from now on take $\mathbf{p}_{\parallel} = 0$ and the initial polarization to be $\mathbf{p} = \mathbf{p}_{\perp}$. We shall also assume that the beam is monochromatic.

The rotation of the plane of polarization α and the beam depolarization $D = 1 - p'/p$ are given by the formulas

$$\alpha = \operatorname{arc\,tg}\frac{\langle\sin\varphi\rangle}{\langle\cos\varphi\rangle}, \qquad (3)^\dagger$$

$$D = 1 - (\langle\sin\varphi\rangle^2 + \langle\cos\varphi\rangle^2)^{1/2}. \qquad (4)$$

Thus the changes in direction and magnitude of the polarization (α and D) depend on the distribution of field in the sample $H = H(x, y)$, which determines the difference between $\langle\sin\varphi\rangle$ and $\sin\langle\varphi\rangle$ ($\langle\cos\varphi\rangle$ and $\cos\langle\varphi\rangle$); when these quantities are equal, $\alpha \equiv \langle\varphi\rangle$ and $D \equiv 0$.

To obtain formulas suitable for numerical estimates, it is useful to expand the field H in Fourier series:

$$H = \sum_{m,\,n=-\infty}^{+\infty} h_{mn}\exp\left[2\pi i\left(\frac{mx}{L_x} + \frac{ny}{L_y}\right)\right], \qquad (5)$$

where L_x and L_y are the spatial periods along the respective axes, oriented appropriately in the two-dimensional vortex lattice (cf. below).

Suppose that the beam direction Ox′ and the perpendicular direction Oy′ form a frame x′Oy′, turned through an angle β relative to the reference system xOy ($x = x'\cos\beta - y'\sin\beta$, $y = x'\sin\beta + y'\cos\beta$). Then substitution of (5) in (2) and integration over x′ gives for the angle of rotation of the neutron spin $\varphi(\beta, y')$ along the given trajectory (y′ = const) for the given beam direction (β = const) the value

$$\varphi(\beta, y') = \sum_{m,\,n=-\infty}^{+\infty} \varphi_{mn}\exp\left[2\pi i\left(\frac{n\cos\beta}{L_y} - \frac{m\sin\beta}{L_x}\right)y'\right], \qquad (6)$$

$$\varphi_{mn} = \frac{2\mu h_{mn}}{\hbar v}\frac{\sin[\pi d(\beta)(m\cos\beta/L_x + n\sin\beta/L_y)]}{\pi(m\cos\beta/L_x + n\sin\beta/L_y)}, \qquad (7)$$

where $d(\beta)$ is the sample thickness along this direction.

It is easy to see that when $d \gg L_x, L_y$, those coefficients for which

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*$[\mathbf{p}_{\perp}\mathbf{H}] \equiv \mathbf{p}_{\perp}\times\mathbf{H}$.

†arctg ≡ tan⁻¹.

$$\frac{m}{n} = -\frac{L_x}{L_y}\,\mathrm{tg}\,\beta \tag{8}^*$$

are large. Since, furthermore, the coefficients h_{mn} decrease with increasing m and n and satisfy the conditions $h_{mn} = h_{m,-n} = h_{-m,n} = h_{-m,-n}$ (cf. below), in all the specific cases considered below the series (6) can be approximated by the two terms:

$$\varphi(\beta, y') = \varphi_{00} + 2\varphi_{MN}(\beta)\cos[A_{MN}(\beta)y'], \tag{9}$$

where M and N are the indices for which the coefficients φ_{MN} (h_{MN}) are larger than all others, except φ_{00} (h_{00}), satisfying the condition (8) for the given β; A_{MN} is determined by the corresponding value of the exponential in (6). Then the integration over y′ which is necessary for determining the trajectory gives

$$\langle\sin\varphi\rangle = J_0(2\varphi_{MN})\sin\varphi_{00} \tag{10}$$

and an analogous formula for the cosine, where J_0 is the Bessel function of the first kind.

Thus the rotation of the plane of polarization α is determined by the average field (induction) $B = h_{00}$ and depends on the beam direction only in a trivial way—through $d(\beta)$:

$$\alpha = 2\mu Bd(\beta)\,/\,\hbar v. \tag{11}$$

On the other hand the depolarization of the beam is strongly dependent on the character of the field distribution in the sample. In first approximation it is given by the formula

$$D(\beta) = 1 - |J_0(4\mu h_{MN}d(\beta)\,/\,\hbar v)| \tag{12}$$

and depends on the beam direction, since the indices M and N are in general different for different β.

We now proceed to consider specific examples. The field distribution in the sample is known[1] for the cases where $H_0 \lesssim H_{c2}$ and $H_{c1} < H_0 \ll H_{c2}$, where H_0 is the external field, while H_{c1} and H_{c2} are the critical fields limiting the region of the mixed state. In the first case, applying formula (10) of Abrikosov's paper[1] and formulas (5) and (6) of the paper of Kleiner et al.[2] we get:

a) for a square lattice ($L_x = L_y = L = \sqrt{2\pi\delta/\kappa}$, where δ is the London penetration depth and the coordinate axes are along the minimum separations between vortices)

$$h_{mn} = \frac{\sqrt{2}\,H_{cm}}{2\varkappa}(-1)^{mn}|c_0|^2\exp\left[-\frac{\pi}{2}(m^2+n^2)\right] \tag{13}$$

(with m = 0, ±1, ±2, . . . ; n = 0, ±1, ±2, . . . ; m = n = 0 excluded),

$$h_{00} = H_0 - \frac{\sqrt{2}\,H_{cm}}{2\varkappa}|c_0|^2, \tag{14}$$

$$|c_0|^2 = \frac{2\varkappa(\varkappa - H_0/\sqrt{2}\,H_{cm})}{1.18(2\varkappa^2-1)}, \tag{15}$$

where κ is the parameter of the Ginzburg-Landau theory, and H_{cm} is the thermodynamic critical field;

b) for a triangular lattice[2)] ($L_x = \sqrt{3}L_y = 2\sqrt{\pi\delta}/3^{1/4}\kappa$, axis Oy along the direction of minimum separation of vortices)

$$h_{mn} = h_{2m'+n,\,n} \equiv h(m', n) = \frac{\sqrt{2}\,H_{cm}}{2\sqrt{3}\,\varkappa}(\pm 1)^n \times (-1)^{mn}|c_0'|^2\exp\left[-\frac{\pi}{\sqrt{3}}(m'^2 + m'n + n^2)\right] \tag{16}$$

(here m′ = 0, ±1, ±2, . . . ; n = 0, ±1, ±2, . . . ; m′ = n = 0 excluded),

$$h_{00} = H_0 - \frac{\sqrt{2}\,H_{cm}}{2\sqrt{3}\,\varkappa}|c_0'|^2, \tag{17}$$

$$|c_0'|^2 = \frac{2\sqrt{3}\,\varkappa(\varkappa - H_0/\sqrt{2}\,H_{cm})}{1.16(2\varkappa^2-1)}. \tag{18}$$

Thus in the case of the square lattice the direction $\beta = 0$, which corresponds to M = 0, N = 1, is singled out; it has the maximum depolarization

$$D_\square(0) = 1 - |J_0(a(0)|c_0|^2e^{-\pi/2})|, \tag{19}$$

$$a(\beta) = 2\sqrt{2}\mu H_{cm}d(\beta)\,/\,\hbar\varkappa v. \tag{20}$$

The next largest depolarization occurs when $\beta = \pi/4$, M = 1, N = 1:

$$D_\square(\pi/4) = 1 - |J_0(a(\pi/4)|c_0|^2e^{-\pi})|. \tag{21}$$

The function $D_\square(\beta)$ is of course periodic with period $\pi/2$, so long as this periodicity is not violated in a trivial way by the inequality $d(\beta) \neq d(\beta + \pi/2)$.

In the case of the triangular lattice the first two distinguished directions are $\beta = \pi/6$ (M = 1, N = −1) and $\beta = 0$ (M = 0, N = 2), when

$$D_\Delta\left(\frac{\pi}{6}\right) = 1 - \left|J_0\left(\frac{a(\pi/6)}{\sqrt{3}}|c_0'|^2e^{-\pi/\sqrt{3}}\right)\right|, \tag{22}$$

$$D_\Delta(0) = 1 - \left|J_0\left(\frac{a(0)}{\sqrt{3}}|c_0'|^2e^{-\pi\sqrt{3}}\right)\right| \tag{23}$$

The period of the function $D_\Delta(\beta)$ in this case is $\pi/3$ (with the same comment as for $D_\square(\beta)$).

*tg ≡ tan.

2)In this case, in the nonvanishing terms of (5) m and n have the same parity.

Similar results are also obtained when $H_{c1} < H_0 \ll H_{c2}$, when the coefficients in (5) can be obtained by using formula (39) or (38) of[1]. This gives for the square and triangular lattices, respectively,

$$\text{a) } h_{mn} = \frac{2\sqrt{2}\pi H_{cm}}{\varkappa} \frac{\delta^2}{(2\pi\delta)^2(m^2+n^2)+L^2}, \tag{24}$$

$$\text{b) } h_{mn} = h_{2m'+n,\, n} \equiv h(m', n)$$

$$= \frac{2\sqrt{2}\pi H_{cm}}{\varkappa} \frac{(2\delta)^2}{(4\pi\delta)^2(m'^2+m'n+n^2)+(\sqrt{3}L_y)^2} \tag{25}$$

from which

$$D_\square(0) = 1 - \left| J_0\left(a(0)\frac{\pi(2\delta)^2}{(2\pi\delta)^2+L^2}\right)\right|, \tag{26}$$

$$D_\square\left(\frac{\pi}{4}\right) = 1 - \left| J_0\left(a\left(\frac{\pi}{4}\right)\frac{\pi(2\delta)^2}{2(2\pi\delta)^2+L^2}\right)\right|; \tag{27}$$

$$D_\triangle\left(\frac{\pi}{6}\right) = 1 - \left| J_0\left(a\left(\frac{\pi}{6}\right)\frac{2\pi(4\delta)^2}{(4\pi\delta)^2+(\sqrt{3}L_y)^2}\right)\right|, \tag{28}$$

$$D_\triangle(0) = 1 - \left| J_0\left(a(0)\frac{2\pi(4\delta)^2}{3(4\pi\delta)^2+(\sqrt{3}L_y)^2}\right)\right|. \tag{29}$$

It should be noted that the approximations leading to (26)—(29) are valid only for L_x, $L_y \ll \delta$, i.e., when the inequality $H_0 > H_{c1}$ is strong ($H_{c1} \ll H_0 \ll H_{c2}$). Thus in the immediate neighborhood of the first critical field ($H_0 \gtrsim H_{c1}$) it is more convenient to use a different method for getting the formula for $D(\beta)$, namely directly substituting formula (39) of[1] in Eq. (2). We obtain the expression

$$\varphi(\beta, y') = \frac{2\sqrt{2}\pi\mu H_{cm}\delta}{\hbar\varkappa v} \sum_v \exp\left[-\frac{d_v(\beta, y')}{\delta}\right] \tag{30}$$

which is a series over all vortices, the distances to which from the particular trajectory are denoted by $d_v(\beta, y')$.

Under the conditions considered ($H_0 \gtrsim H_{c1}$, L_x, $L_y \gg \delta$), the main terms in (29) are those containing the distances to the vortices nearest to the trajectory. In particular when the condition (8) is satisfied, we can restrict ourselves to the approximation

$$\sum \exp\left(-\frac{d_v}{\delta}\right) \approx d(\beta)\exp\left(-\frac{y'}{\delta}\right)\Big/ L_1(\beta),$$

where $L_1(\beta)$ is the distance between vortices along the direction β, and y' is restricted by the inequality $0 \le y' \le L_2(\beta)$; $L_2(\beta)$ is half the distance between rows of vortices parallel to the given trajectory (on one of which the axis Ox' is fixed). Then the averaging over y' gives

$$\langle \sin\varphi \rangle = \frac{\delta}{L_2(\beta)}$$

$$\times \left\{ \operatorname{si}\left[\frac{\pi a(\beta)\delta}{L_1(\beta)}\right] - \operatorname{si}\left[\frac{\pi a(\beta)\delta}{L_1(\beta)}\exp\left(-\frac{L_2(\beta)}{\delta}\right)\right]\right\}. \tag{31}$$

A similar formula with si replaced by ci is obtained for cos (si and ci are the integral sine and cosine).

Substituting (31) in (4), we get an expression for $D(\beta)$ which can provide a specific formula if we make use of the fact that, with the coordinate systems oriented as indicated above, a) for the square lattice

$$L_1(0) = L, \quad L_2(0) = L/2,$$

$$L_1(\pi/4) = \sqrt{2}L, \quad L_2(\pi/4) = \sqrt{2}L/2$$

and b), for the triangular lattice

$$L_1(\pi/6) = L_y, \quad L_2(\pi/6) = \sqrt{3}L_y/4,$$

$$L_1(0) = \sqrt{3}L_y, \quad L_2(0) = L_y/4.$$

In other words, after turning of the beam through a half period $D(\beta)$, the quantities $L_1(\beta)$, $L_2(\beta)$ are multiplied by a factor of $\sqrt{2}$ in case a) and $\sqrt{3}$ in case b).

A common feature of all the cases considered is the presence of a maximum of the depolarization when $\beta = 0$, $\beta = \tan^{-1}(L_y/L_x)$ and for other beam directions determined by the condition (8), where the function $D(\beta)$ has the period $\pi/2$ or $\pi/3$ depending on whether the symmetry of the vortex lattice is square or triangular.[3] This last point provides a simple experimental possibility for determining the symmetry of the lattice (independent of the value of the external field H_0).

We should mention that the maxima of the depolarization which should be observed when the condition (8) is satisfied, are extremely narrow and restricted to a range of angles of order $\Delta\beta = L/d$, outside of which the depolarization is practically zero. This presents difficulties for the proposed experiment, in which strict requirements must be imposed on the collimation of the beam. But since we are talking about investigations of the transmitted beam (and not the scattered beam, as is the case for other methods proposed for studying superconductors of the second kind by using neutrons ([4-6] and private communication from F. L. Shapiro)), we may hope that sufficient intensity will still be available with the better collimation. It appears that the main difficulty of the experiment with polarized neutrons is the necessity for using a sample with a "single crystal" lattice of vor-

[3]As already remarked above, this statement is valid only in the absence of a dependence of d on β, or at least if $d(0) = d(\pi/2)$ (or $d(0) = d(\pi/6)$ in the triangular case).

tices, which is more important in the present case than in experiments on neutron diffraction. (The depolarization of the beam is assured by the fact that different trajectories lie in regions with different magnetic fields. In directions slightly different from those given by condition (8), and in any direction in a nonideal lattice, this is not the case, and the average field along the trajectory is practically identical for all trajectories.)

Numerical estimates based on the formulas obtained here have shown that depolarizations of the order of tens of percent are entirely attainable.

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