EFFECT OF INTERELECTRON COLLISIONS ON THE OPTICAL PROPERTIES OF METALS

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The electromagnetic energy absorption coefficient due to interelectron interaction is calculated by employing the model of an ideal Fermi gas with an arbitrary dispersion law. The formulas derived are valid in a broad range of frequencies.

It is well known that in addition to direct absorption of photons by electrons, wherein the latter move from one band to another (internal photoeffect), intraband absorption is also possible, wherein the electrons interact with impurities, with phonons, or with one another[1]. The internal photoeffect occurs at frequencies above a certain (threshold) value, and the absorption coefficient due to this mechanism is large far from threshold. Transitions between bands are customarily assumed to be the main cause of absorption of light by pure metals.

The interaction of the electrons with the impurities, with the boundary of the sample, and with phonons leads to a relatively small absorption coefficient, which reaches saturation with increasing frequency[2-3]. On the other hand, absorption due to electron-electron interaction increases with increasing frequency[4,5] and, in principle, can become comparable with the absorption due to the internal photoeffect.

The purpose of the present paper is to analyze the role of interelectron interaction in the optical region of the spectrum, where the energy of the electromagnetic quantum $\hbar\omega$ is comparable with the width of the conduction band.

The absorption coefficient Γ can be readily calculated if one knows the tensor of the specific electric conductivity σ_{ik}. According to[5]

$$\omega^2\sigma_{ik} = \sum_{\mathbf{g}} \iiiint d\mathbf{p}\,d\mathbf{p}'\,d\mathbf{p}_1\,d\mathbf{p}_1' M(\mathbf{p}, \mathbf{p}'; \mathbf{p}_1, \mathbf{p}_1')$$
$$\times (\mathbf{v} + \mathbf{v}' - \mathbf{v}_1 - \mathbf{v}_1')_i (\mathbf{v} + \mathbf{v}' - \mathbf{v}_1 - \mathbf{v}_1')_k$$
$$\times \frac{1}{2\hbar\omega}[\delta(\varepsilon + \varepsilon' - \varepsilon_1 - \varepsilon_1' + \hbar\omega)$$
$$- \delta(\varepsilon + \varepsilon' - \varepsilon_1 - \varepsilon_1' - \hbar\omega)]$$
$$\times [f_0(\varepsilon) f_0(\varepsilon')(1 - f_0(\varepsilon_1))(1 - f_0(\varepsilon_1')) - f_0(\varepsilon_1) f_0(\varepsilon_1')$$
$$\times (1 - f_0(\varepsilon))(1 - f_0(\varepsilon'))]\,\delta(\mathbf{p} + \mathbf{p}' - \mathbf{p}_1 - \mathbf{p}_1' + \mathbf{g}). \quad (1)$$

Here $f_0(\epsilon) = (\exp[(\epsilon - \epsilon_F)/T] + 1)^{-1}$, $M(\mathbf{p}, \mathbf{p}'; \mathbf{p}_1, \mathbf{p}_1')$ is a quantity proportional to the square of the matrix element of the transition, $\mathbf{g} = 2\pi\hbar\mathbf{b}$, $\mathbf{b}$ is the reciprocal lattice vector, $\mathbf{v} = \partial\epsilon/\partial\mathbf{p}$, and ω is the frequency of the field.

Using the symmetry of the matrix element, we can rewrite (1) in the form

$$\omega^2\sigma_{ik} = \frac{1}{\hbar\omega}\iiint d\varepsilon\,d\varepsilon'\,d\varepsilon_1 (A^{(-)}B_{ik}^{(-)} - A^{(+)}B_{ik}^{(+)});$$
$$A^{(-)} = (1 - f_0(\varepsilon))$$
$$\times (1 - f_0(\varepsilon')) f_0(\varepsilon_1) f_0(\varepsilon + \varepsilon' - \varepsilon_1 - \hbar\omega), \quad (2)$$

$$B_{ik}^{(-)} = \sum_{\mathbf{g}} \iiiint \frac{dS}{v}\frac{dS'}{v'}\frac{dS_1}{v_1}\frac{dS_1'^{(-)}}{v_1'^{(-)}} M(\mathbf{p}, \mathbf{p}'; \mathbf{p}_1, \mathbf{p}_1')$$
$$\times (\mathbf{v} + \mathbf{v}' - \mathbf{v}_1 - \mathbf{v}_1')_i$$
$$\times (\mathbf{v} + \mathbf{v}' - \mathbf{v}_1 - \mathbf{v}_1')_k\,\delta(\mathbf{p} + \mathbf{p}' - \mathbf{p}_1 - \mathbf{p}_1' + \mathbf{g}), \quad (3)$$

where $dS_1'^{(-)}$ is an element of the equal-energy surface $\epsilon_1' = \epsilon + \epsilon' - \epsilon_1 - \hbar\omega$. The quantities $A^{(+)}$ and $B_{ik}^{(+)}$ are obtained from $A^{(-)}$ and $B_{ik}^{(-)}$ respectively by reversing the sign of the frequency ω. We shall henceforth reckon the energy from the Fermi level, so that the bottom of the band corresponds to $-\epsilon_F$ and the upper limit of the band corresponds to an energy $E > 0$.

If $\hbar\omega \ll \epsilon_F$ (the temperature is always assumed to be small, $T \ll \epsilon_F$, E), then we get from (2) the well known result[5]:

$$\omega^2\sigma_{ik} = {}^1/_6[(\hbar\omega)^2 + (2\pi T)^2]B_{ik}^{(0)}, \quad (4)$$

where the index (0) indicates that the quantities B_{ik} are taken on the Fermi surface: $\epsilon = \epsilon' = \epsilon_1 = \epsilon_1' = 0$.

We now proceed to consider higher frequencies, at which the energy of the quantum $\hbar\omega$ is comparable with the electron energies ϵ_F and E. We note first that when $\hbar\omega \gg T$ the quantity $A^{(+)}$ is exponentially small and can be discarded. On the other hand, $A^{(-)}$ differs from zero and is equal to unity (at $T = 0$) if the energy satisfies the conditions

$$0 < \varepsilon,\ \varepsilon' < E, \qquad -\varepsilon_F < \varepsilon_1 < 0,$$
$$-\varepsilon_F < \varepsilon + \varepsilon' - \varepsilon_1 - \hbar\omega < 0.$$

Thus, the problem reduces to an integration of $B_{ik}^{(-1)}$ over a three-dimensional region in the space of the energies ϵ, ϵ', and ϵ_1. This region is part of a right parallelepiped with edges E, E, and ϵ_F, bounded by the planes

$$\varepsilon + \varepsilon' - \varepsilon_1 = \hbar\omega, \qquad \varepsilon + \varepsilon' - \varepsilon_1 = \hbar\omega - \varepsilon_F.$$

It is clear that a change in frequency leads to motion of these planes through the parallelepiped. When the frequency is low, the region of integration has the form of a trihedral prism with vertex at the origin and with altitude $\hbar\omega/\sqrt{3}$. The volume of this prism is equal to $(\hbar\omega)^3/6$, in full agreement with [3]. With increasing frequency, the planes will pass through the vertices of the parallelepiped and the analytic dependence of the volume of the integration region will change. This will lead to singularities in the frequency dependence of the absorption at

$$\hbar\omega = \varepsilon_F,\ E,\ \varepsilon_F + E,\ 2\varepsilon_F,\ 2E,\ \varepsilon_F + 2E,\ 2\varepsilon_F + E.$$

We can show, however, that these singularities are quite weak. The most essential of them occur at the points $\hbar\omega = \epsilon_F$ and $\hbar\omega = E$, where the fourth (!) derivative the absorption coefficient becomes infinite.

When $\hbar\omega \geq 2(\epsilon_F + E)$ the absorption is equal to zero. It is easy to show that this vanishing is like

$$\Gamma \sim \Delta^{9/2}, \qquad \Delta = 2(\varepsilon_F + E) - \hbar\omega.$$

The $9/2$ power is due to the following causes. The volume of the region of integration is $\Delta^3/6$. In addition, we must take into account the fact that B_{ik} also vanishes together with Δ, for as $\Delta \rightarrow 0$ the surface integral in (3) is taken near the points of minimum or maximum energy. An analysis shows that $B_{ik}^{(-)} \sim \Delta^{3/2}$.

All the foregoing results pertain not only to normal collisions ($g = 0$), but also to an equal degree to collisions which are accompanied by umklapp ($g \neq 0$).

The foregoing analysis is based on a model of an ideal Fermi gas, in which no account is taken in consistent fashion of the interaction between the electrons. As is well known, this approximation is justified for electrons lying near the Fermi limit. Therefore the results obtained above have a qualitative character and possibly can serve as a check on the correctness of the single-electron approximation for electrons with arbitrary energy.

The absorption coefficient can be expressed in simple fashion in terms of the complex dielectric constant of the medium [1]. In the frequency region of interest to us

$$\varepsilon_{ik}(\omega) = \varepsilon_{ik}'(\omega) + 4\pi i \sigma_{ik}(\omega) / \omega,$$

and the real part has the following characteristic frequency dependence [6]: $\epsilon'_{ik}(\omega) = \delta_{ik} - (\Omega^2)_{ik}/\omega^2$.

In the case of an isotropic dispersion law we have

$$(\Omega^2)_{ik} = \Omega^2 \delta_{ik}, \qquad \Omega^2 = 4\pi n e^2 / m.$$

It must be taken into account that the real part of the dielectric constant $\epsilon'(\omega)$ can reverse sign on going over to the high-frequency region. This, of course, will influence the value of the absorption coefficient and the character of its frequency dependence.

The influence of the interelectron interaction in the near infrared region of the spectrum ($\Gamma \sim \omega$) has been observed recently in a large number of metals. The most reliable are the results pertaining to the noble metals [7-10], in which the boundary of the internal photoeffect lies in the optical region of the spectrum, so that the quadratic dependence of $\Gamma(\omega)$ can be separated with assurance. The experimentally known relatively large absorption in the region of high frequencies is usually related to the internal photoeffect. However, from the results of this paper we see that, owing to the interelectron interaction, the intraband transitions can make an appreciable contribution to the absorption.

The separation of intraband absorption can apparently be aided by the specific character of the frequency dependence of the optical characteristics of the metal.

It is quite unlikely that the deviation from the $\Gamma \sim \omega^2$ law is connected solely with the internal photoeffect for all metals.

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