OPERATING FEATURES OF THE RING LASER

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The phase relations between longitudinal modes in a gas ring laser have been investigated. A method is given for determining these phase relations; this method assumes a maximum "equalization" distribution of the optical electric field along the length of the active medium. Results of calculation based on this method have been verified by experiments in which we have measured the depth of modulation of the optical flux in "splitting" (for example, by virtue of rotation) of the frequencies generated in the system. The experiments were carried out with a three-mirror device at a wavelength of 0.63 μ. The effects of small changes in resonator length on the flux have also been analyzed.

A number of papers published in the last two years[1-5] have been concerned with the operation of the helium-neon ring laser. This laser employs a closed resonator along which two travelling waves propagate in opposite directions. The frequencies of these waves are the same if the system is motionless; however, if the system rotates about an axis perpendicular to the plane of the laser the frequencies of the two modes are "split," the frequency difference Δf being proportional to the angular velocity Ω. The frequency difference Δf is observed by the interference in a photoelectric detector of two beams which are transmitted through one of the mirrors in the laser circuit.

In most work on the ring laser known to the present authors the analysis of operation reduces to the determination of the magnitude of Δf, the majority of papers being concerned with design questions and with experimental tests of the dependence of Δf on Ω. Experiments carried out recently with a ring laser have shown, however, that there are a number of other properties of the system which have not been described in the literature and which require more detailed investigation. Some of the results of these experiments are given below and tentative explanations are proposed.

We first describe the apparatus (Fig. 1). Three spherical mirrors A, B, and C form the resonator circuit, which is an equilateral triangle 1.5 meters on a side. The dashed line shows the closed propagation path inside the resonator. The letters X and Y denote helium-neon discharge tubes, which are provided with Brewster end windows: these are dc discharges and the tubes are one meter in length. Radiation is generated at a wavelength of 0.63 μ.

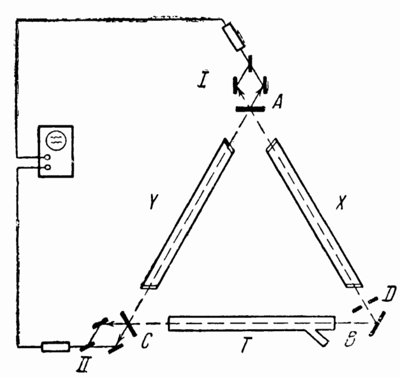

FIG. 1

The experiments were initially carried out with only one tube X; the second tube Y was used in later experiments. Near the mirrors marked A and C there are two mixers, I and II, in which the beams coming from the mirrors interfere; the interference effects are observed in a photodiode whose output goes to a two-beam oscilloscope. In the propagation path there is an aperture D with an adjustable iris which is used to control the power level of the laser and to suppress oscillation in undesired transverse modes; the presence of these modes increases the noise level at the output of the photodiodes and makes it difficult to observe the desired effect, i.e., the beat note at Δf. Frequency splitting is achieved by rotating the entire apparatus at 0.01 rpm or by blowing air through the open tube T, whose axis coincides with the laser beam. In the latter case the splitting is due to the Fresnel-Fizeau effect. The frequency splitting Δf in the present experiments is of the order of hundreds or thousands of cycles.

The question arises as to why it is necessary to use two mixers, I and II, for observation of the interference effects. Anticipating later results, we note at this point that the observations in the two mixers are generally different and that in certain cases this difference can be quite significant.

In an ordinary laser (not a ring laser) the surfaces of the resonator mirrors determine uniquely the distribution in space of the nodes and antinodes of the standing waves generated in the system. In the ring laser the waves travelling in opposite directions also form standing waves (if there is no frequency splitting), but the location of the nodes and antinodes cannot be specified; indeed, the location of the nodes and antinodes is an open question which acquires particular interest if more than one mode is excited in each wave.[1] The frequency splitting can be regarded as a slow displacement (to the right or left) of the standing-wave pattern, resulting in modulation of the optical flux (at a frequency Δf). If only one mode oscillates the depth-of-modulation is 100% in both I and II.[2] However, if a large number of modes are excited the depth of modulation will depend on the location of the antinodes and nodes of the standing waves associated with different modes; under certain conditions the modulation can be small, in which case it is difficult to observe the beat oscillations at frequency Δf. In order to investigate this problem we have carried out a number of experiments whose results are discussed below.

In the interests of simplicity, we assume that the mode amplitudes in both waves are the same, being equal to unity. If the number of waves in each mode is n the standing-wave pattern in the resonator can be described in terms of a time-averaged electric field p which is a function of the coordinate x, the distance measured along the propagation path:

$$p = n + \sum_{i=1}^{n} \cos\left[\frac{4\pi(N+i)}{L}x - \theta_i\right] = n + \Psi(x), \quad (1)$$

where L is the length of the closed propagation path, N + i is a whole number of wavelengths of the i-th mode which fit into the path L, and θ_i is the phase, which specifies the positions of the nodes and antinodes of the i-th mode. Here we are neglecting the fact that the dielectric constant is not actually constant over the entire path L; taking this feature into account it is not important for the analysis given here. If frequency splitting occurs it is necessary to modify the argument of all cosine terms by adding a term $2\pi\Delta ft$; it then follows that the values of θ_i determine the amplitude of the oscillations at frequency Δf since the output of the photo detector is proportional to p for the value $x = x_{int}$, which corresponds to the location of the detection point of the interfering beams.

The quantity θ_i is determined in the following way: along the active medium whose nonlinear properties determine the mode of operation of the laser, the quantity p is distributed (at least, to a first approximation) in accordance with (1). Since external conditions do not affect the defined values of the phase θ_i we may assume that stable modes of operation of the system correspond to those values of θ_i which provide the most uniform distribution of p along the entire segment occupied by the active medium. This statement can be formulated mathematically by requiring that the integral $D = \int[\Psi(x)]^2 dx$ taken over the limits of the active medium (i.e., the helium-neon discharge) be a minimum.[3]

The requirement given above has not been derived from a rigorous analysis and it is therefore important to compare its consequences with experimental results. Let us first consider the case of a laser with the one tube X, whose center coincides with the center of the segment AB. The coordinate x is measured from the center of the tube (in the anticlockwise direction) and the length of the discharge is denoted by l. We then require that the quantity

$$D = \int_{-l/2}^{l/2} [\Psi(x)]^2 dx$$

be a minimum.

Since l is much longer than the wavelength, the expression for D is as follows:

when n = 2

$$D_2 = l + \frac{L}{2\pi}\sin\frac{2\pi l}{L}\cos(\theta_2 - \theta_1), \quad (2)$$

when n = 3

$$D_3 = \frac{3}{2}l + \frac{L}{2\pi}\sin\frac{2\pi l}{L}[\cos(\theta_2 - \theta_1) + \cos(\theta_3 - \theta_2)]$$

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[1] It follows from the analysis in the text that we are considering longitudinal modes only (TEM_{00q}).

[2] We assume that the intensities of the interfering beams are the same and neglect any divergence effects.

[3] We assume that the active medium is uniformly distributed over the length of the discharge. We may also note that the requirement on D can be formulated on the basis of other considerations: for instance, let the active medium be regarded as a distributed source of negative conductivity whose modulus is $g = \alpha - \beta p$ where α and β are constants. The work done by the source is proportional to $\int gpdx$. Using (1) we find that this quantity will be a maximum when the value of the integral D is a minimum.

$$+\frac{L}{4\pi}\sin\frac{4\pi l}{L}\cos(\theta_3-\theta_1). \qquad (3)$$

From (2) we find immediately that for the indicated values of L and l, $\theta_2 - \theta_1 = \pi$; substituting this result in (1) we find that the flux and frequency splitting have a constant component I_0 and a variable component at frequency Δf characterized by an amplitude I. The quantities I_0 and I at x = L/6 (the interference point for I) are respectively 2 and 2 cos $\pi/6 \approx 1.7$ (to within certain numerical coefficients). Thus the depth of modulation is $m_I \approx 85\%$. At the same time, substituting in (1) the value x = L/2 (the interference point for II), we obtain the same value for I_0 but find that I vanishes, that is to say, the flux at the output of II is not modulated (at frequency Δf), $m_{II} = 0$.

Now consider the three-mode case. Analysis of (3) shows that the minimum value of D_3 holds (for the indicated values of L and l) when $\theta_3 - \theta_2 = \theta_2 - \theta_1 = \pi$. Substituting this value in (1) we now find a shift between the values of π in the neighboring modes: $m_I = 43.5\%$ and $m_{II} = 0$.

These results have been obtained under the assumption that all mode amplitudes are the same and obviously this assumption does not correspond to the experimental situation. If the mode amplitudes are not equal the depth of modulation m changes but the relations between the values of the various θ_i do not change; the relation between the values of m_I are not affected greatly, the effect being more noticeable in m_{II}. The value $m_{II} = 0$ has been obtained for the two-mode case; if we now assume that the mode intensities differ by a factor of 2 or 3 the value of m_{II} can become 30—50%. In the four-mode case we found $m_{II} = 0$; a difference in mode intensity of a factor of 2 or 3 can increase m_{II} to 10 or 20%. In any case the significant difference in the depth of modulation of the fluxes in I and II still holds when the mode amplitudes are not equal.

In order to verify these results we have carried out experiments in which the modulation patterns for the two fluxes were observed simultaneously on a two-beam oscilloscope. To make a quantitative determination of the depth of modulation in the laser circuit we introduce a rotating disc with a slit (not shown in the figure); this chopper periodically (50—70 cps) switches on the laser for a time 6—9 msec. The number of modes is controlled by the iris in the aperture D and is observed by means of a scanning Fabry-Perot interferometer which drives a photomultiplier whose output is displayed on an oscilloscope (not shown in the figure).[4] Unfortunately the stability of the entire laser system was low, making it difficult to determine the mode intensity ratios (at times, even the number of modes); when more than one mode is excited the values of m_I do not vary greatly (in a relative sense) while the values of m_{II} are highly irregular and can change from 0 to 20 or 30% in the two-mode case.

The patterns on the screen have been photographed (with a short exposure time) and analysis shows that the considerations given above are verified completely. For example, near threshold, where only one mode is excited, the depths of modulation of both optical fluxes are approximately equal, being 75—90%. The low value of m as compared with the theoretical value of 100% is undoubtedly due to the divergence of the interfering beams and other experimental factors. When three modes are excited m_I lies between 60 and 70%, m_{II} lies between 22 and 35%. In the four mode case we find m_I = 32—52%, and m_{II} = 2—15% respectively.

The result given above, i.e., that the value of θ in modes characterized by successive mode numbers differs by π[5] can be verified in another way. If the photodetector receives only one of the beams from the resonator, when more than one mode is excited the photocurrent spectrum exhibits a component (resulting from beats between neighboring modes) at frequency c/L where c is the velocity of light (in the present case this frequency is approximately 66 Mc). Now consider two photodetectors, each of which provides separate detection of both laser beams; the beat frequency Δf at each detector can be observed separately and the phase difference between the two beats can be determined by means of an appropriate circuit. This phase difference can be predicted by a simple calculation; it depends on the exit point of the rays which are detected. In particular, if we take x = L/2 (that is, rays from the mirror C) the calculation shows that this phase difference is π (using the relation given above between different θ_i). The experimental results verify this conclusion completely.

The results given above pertain to the case in which the laser circuit contains only one discharge tube X. Now let us consider the situation when the

[4]The operation of such a system is described, for example, by Fork et al.[6] Measures were also taken to prevent the feedback effects of the scanning interferometer from affecting the laser.

[5]This is obviously true if one measures x from the center of the discharge.

laser resonator contains two discharge tubes, X and Y, which are assumed to be identical (both in length as well as discharge "activity"). To find the phase angles θ_i we use the considerations developed above. Now, however, the integral $\int[\Psi(x)]^2 dx$ must be regarded as the sum of two integrals; the limits of these integrals correspond to the limits of the discharge spaces.

We now measure x from the surface of mirror A. The mixer marked I in this case corresponds to the value x = 0 while II corresponds to x = L/3. The analysis of the problem leads to lengthy but elementary calculations and we shall only present the results.

We assume first that the boundaries of the discharges in both tubes are at equal distances from the mirror A. Then, if the distance from A to the boundary of the discharge x_d is less than $1/8(L-4l)$, the values of θ_i in neighboring modes are shifted by π and the values of θ_i coincide for large x_p.[6)] The magnitude of $1/8(L-4l)$ is 6.25 cm in the present case; for technical reasons it is not possible to use a much smaller value of x_d and consequently the shift between different values in θ_i does not appear in the present experiments.

Let us now consider what the depth of modulation of the photocurrent will be for a frequency splitting Δf. From (1) we know that the depth of modulation in I for any number of modes must be $m_I = 100\%$; in II the value of m_{II} for one mode is 100%, for two modes 50%, for three modes 0 and for four modes 25%. Obviously nonuniformity in the amplitudes of individual modes will effect these results: the modulation percentage of 100% will be reduced to some extent and in place of the zero modulation one expects (for mode intensity differences of a factor of two or three) modulation percentages up to 10 or 20%.

We have assumed that the distances from the tubes to the mirror A are the same. Any nonuniformity in these distances will evidently mean that there will be an additional shift $(2\pi/L)\Delta x_d$ between values of θ_i, where Δx_d is the difference in the indicated distances. Qualitatively the results are the same as for nonuniformity in the mode amplitudes.

Experiments carried out to check this point have

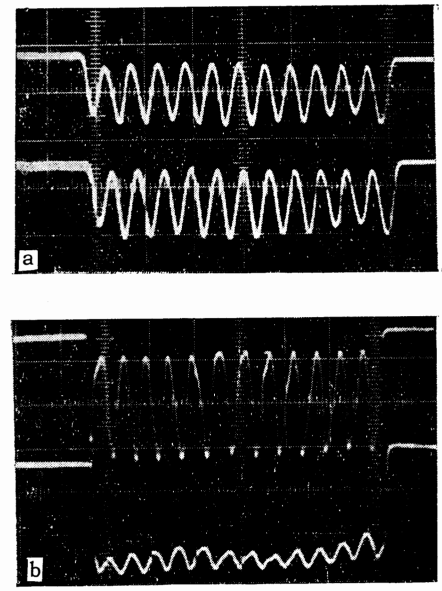

FIG. 2

also verified the considerations given above. The tubes were located in the center of the resonator arms AB and AC. As an example, in Figs. 2a and 2b we show oscillograms of both photocurrents (using the chopper described above and $\Delta f \approx 2$ kc). Fig. 2a corresponds to single-mode operation of the laser close to threshold; in this case both optical fluxes exhibit a modulation depth of 80 or 85%. Fig. 2b corresponds to three-mode operation in which case we find $m_I \approx 75\%$ and $m_{II} \approx 10\%$.

We now consider the effect of small changes in resonator length L. In the literature one frequently encounters the statement that since the change in L is the same for both beams the frequency splitting Δf is essentially unaffected by such variations. Actually, however, the situation is rather complicated and requires further analysis.

The analysis given above yields relations giving the relative location of nodes or antinodes of the individual modes and the experiments verify the results of this analysis. However, the problem of locating the actual position of a node of a given mode, i.e., the value of θ for a given mode (say θ_1) is still not solved. This point does not arise in the analysis given above i.e., determination of the depth of modulation of the optical flux. However, if one is interested in determining the effect of a change in L on the optical flux the quantity θ_1 is important, more precisely, the change in this quantity for small changes in L. Say that L increases by $\delta < \lambda$ where λ is the wavelength and that θ_1 changes by an amount ϵ. Assuming for simplicity that $x = x_{int}$ (the location of the mixer) is not changed when L varies by this amount we find from (1) that

[6)]A rigorous analysis was carried out for the two-mode and three-mode cases but the analysis is evidently valid for any number of modes. The shift of π for the value of x_d can be understood easily; if the distance between the discharges is small the situation is approximately the same as that for a single discharge for which we have already obtained a phase shift of π.

in the single-mode case $\delta \neq 0$ means a change in optical flux Δi proportional to

$$\cos\left[\frac{4\pi(N+1)}{L}x_{\text{int}}-\frac{4\pi(N+1)}{L}\frac{x_{\text{int}}}{L}\delta-\theta_1-\varepsilon\right]$$
$$-\cos\left[\frac{4\pi(N+1)}{L}x_{\text{int}}-\theta_1\right]. \quad (4)$$

It is then obvious that whatever the values of θ_1 and ϵ, a change in x_{int} amounting to a fraction of a wavelength will cause a considerable change in Δi.[7)] Evidently the readings of I and II will in general be very sensitive to small changes in length L. If now we write $\delta = \delta_0 \cos 2\pi\nu t$ where ν is a low frequency, both optical fluxes will be modulated periodically with modulation that will generally be different.[8)] When $\delta_0 \approx \lambda/4$ the depth of modulation in one or the other of the optical mixers will be approximately 100%. The result we have obtained pertains to the case in which there is no frequency splitting in the system. However, if there is a frequency splitting $\nu \ll \Delta f$ it is evident that the oscillations at frequency Δf will be modulated in phase (frequency) at frequency ν; the nature of this modulation (both quantitatively and qualitatively) will be different at the outputs of I and II.

Similar results are obtained when several modes are excited; in this case the outputs of the two mixers are again different. We find that when $\delta_0 \approx \lambda/4$ the optical flux is modulated at the same depth as for the desired effect, with a phase modulation of the frequency Δf at frequency ν in the presence of splitting. In particular, for those values of x_{int} and a number of modes n for which the desired effect does not give modulation when $\delta \neq 0$ there is no variation in the optical flux. Obviously, nonuniformity in the mode amplitudes will destroy this compensation effect to some extent.

In order to verify this analysis we have carried out the following experiment: a glass plate 0.5 mm thick fastened to a piezoelectric element and oriented at the Brewster angle is introduced in the system. When a potential is applied the plate changes its inclination with respect to the propagation direction continuously, thereby providing a means of changing the optical path length L. If an alternating voltage is applied to the piezoelectric element L is changed periodically by an amount of the order of a fraction of a wavelength (this is usually done at a frequency of 130 cps). The results of this experiment are entirely in accord with the effects predicted above: the patterns of variation of the optical fluxes at the outputs of the two devices are very different and when the laser frequencies are split the oscillations at frequency Δf are found to be modulated in phase. Moreover, the nature of the modulation is different in the two optical fluxes.

Although the parameters of the system are maintained at fixed values so far as is possible, the modulation varies in time with a correlation time of the order of 10 seconds. This means that the parameters of the system, both L and x_{int} (and possibly θ_1 and ϵ) remain fixed only for time intervals of the order of 10 seconds.

The magnitude of the quantity θ_1 (and variations in this quantity for small changes in L) still remains unclear. It may be assumed, however, that θ_1 is determined by the various small inhomogeneities in the plasma in the discharge tubes.

We would like to thank V. I. Leonov and P. V. Shobyrev for help in building the apparatus and carrying out the experiments.

[7)]Small changes in x_{int} do not change the values of m considered above.

[8)]Here we have in mind, for example, the case in which one of the optical fluxes varies almost sinusoidally (at a frequency ν) while the other exhibits a large second-harmonic content or is completely second harmonic.

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