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THEORY OF REFLECTION OF FAST ELECTRONS FROM CONDUCTING MEDIA

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Reflection of fast electrons with small energy losses from conducting media is considered. The dependence of the electron reflection coefficient on the energy ω lost during reflection from a semiconductor is determined.

1. INTRODUCTION

THE energy spectrum of fast electrons reflected from solids consists of a peak of elastically reflected electrons and a certain number of inelastically reflected electrons. The width of this peak is approximately 1—3 eV for electrons with energy of approximately 1 keV[1)]. The number of electrons that lose an energy larger than the width of the elastic-reflection peak depends essentially on the material from which the electrons are reflected. In the case of dielectrics there are practically no electrons reflected with energy loss up to about 10 eV. To the contrary, in the case of metals one observes a noticeable number of almost-elastically reflected electrons, this number being practically independent of the energy loss if the latter is small. These facts can be explained in a natural manner by taking account of the fact that the electron can lose an energy ω in the interval $\Delta E > \omega > \delta E$ (ΔE is the width of the forbidden band and δE the width of the peak of elastic reflection) only as a result of interaction with the electrons of the conduction band.

We investigate in this paper the influence of the conduction electrons in a solid on the behavior of the coefficient of reflection of electrons as a function of the energy transferred to the target. Besides the interaction of the fast electron with the lattice and with the conduction electrons, we take into account also the interaction between the conduction electrons themselves. The target considered is an electronic semiconductor, for which the electron-electron interaction can be described most consistently. This makes it possible to take the plasma effects into account in natural fashion. In addition, there are grounds for assuming that the spectrum of the electrons reflected with an energy loss of several eV from a metallic target is determined by the interaction between the conduction electrons themselves. For a qualitative understanding of the peculiarities of the reflection process it is therefore desirable in this case to take this interaction into account in a model in which the electron-electron interaction is weak.

1)In an instrument of the spherical-capacitor type. The author is indebted to A. R. Shul'man and Yu. A. Morozov for these data.

2. TRANSITION-PROBABILITY AMPLITUDE IN THE PRESENCE OF ELECTRON-LATTICE AND ELECTRON-ELECTRON INTERACTIONS

The Hamiltonian of the medium is represented in the form

$$H = H_0 + H_1, \quad H_0 = \sum_{p'} \varepsilon_{p'} a_{p'}^+ a_{p'} + \sum_{k} \omega_k b_k^+ b_k.$$

The interaction of the conduction electrons with one another is described by the operator

$$H_1 = \frac{1}{2} \sum_{p', p, q} V_q a_p^+ a_{p'}^+ a_{p'-q} a_{p+q}.$$

We introduce the operators H_l and H_i. The operator H_l describes the interaction between the fast electron and the lattice; the operator H_i the interaction between the fast electron and the conduction electrons:

$$H_i = \sum_{p,q} V_q a_p^+ \alpha_{p-q}^+ \alpha_p a_{p-q}.$$

The operators α pertain to the fast particles. We assume that

$$[\alpha_p, a_p^+] = 0 \quad [a_p, a_{p'}^+]_+ = \delta_{pp'}.$$

The purpose of the investigation is to find the probability of the transition of the fast electron incident on the target to the states of reflected electrons. This probability is proportional to the current of these electrons.

The probability per unit time of a transition accompanied by transfer of a momentum **q** to the medium is ($c = \hbar = m = e = 1$)

$$w_q = 2\pi |T_q|^2 \delta(E_M - E_N + \varepsilon_{p-q} - \varepsilon_p).$$

The indices M and N are characteristics of the state of the medium, while ϵ is the energy of the fast particle. The transition amplitude T_q is

$$T_{\mathbf{q}} = \langle M, \mathbf{p}-\mathbf{q} | H_i + H_l | \Psi^+_{N,\mathbf{p}} \rangle.$$

The function $\Psi^+_{N,p}$ is a solution of the Lippmann-Schwinger equation[1]:

$$\Psi^+_{N,\mathbf{p}} = | N, \mathbf{p}\rangle + \hat{G}(N, \mathbf{p}) [H_i + H_l] \Psi^+_{N,\mathbf{p}},$$

$$\hat{G}(N, \mathbf{p}) = [E_N + \varepsilon_{\mathbf{p}} - H + i\delta]^{-1}.$$

We assume that the energy of the fast electron is such that we can confine ourselves to the Born approximation with respect to H_i. However, with respect to the operator H_l such an approximation would be incorrect. The probability of scattering of a fast electron in the lattice field is relatively large, so that it is necessary to take into account possible consecutive collisions with the lattice. We therefore represent T_q in the form:

$$T_{\mathbf{q}} = T_l(\mathbf{q}) + T_1(\mathbf{q}) + T_2(\mathbf{q}),$$

where

$$T_l(\mathbf{q}) = \langle M, \mathbf{p}-\mathbf{q} | H_l | \tilde{N}, \mathbf{p}\rangle,$$

and the function $|\tilde{N}, \tilde{\mathbf{p}}\rangle$ satisfies the equation

$$|\tilde{N}, \tilde{\mathbf{p}}\rangle = |N, \mathbf{p}\rangle + \hat{G}(N, \mathbf{p}) H_l |\tilde{N}, \mathbf{p}\rangle;$$

$$T_1(\mathbf{q}) = \langle M, \mathbf{p}-\mathbf{q} | H_l | \psi^+_{N,\mathbf{p}}\rangle,$$

and $\psi^+_{N,p}$ is defined as

$$\Psi^+_{N,\mathbf{p}} = \psi^+_{N,\mathbf{p}} + |\tilde{N}, \tilde{\mathbf{p}}\rangle$$

and is a linear function in H_i. The function $\psi^+_{N,p}$, satisfies the equation

$$\psi^+_{N,\mathbf{p}} = \hat{G}(N, \mathbf{p}) \{H_i |\tilde{N}, \tilde{\mathbf{p}}\rangle + H_l \psi^+_{N,\mathbf{p}}\};$$

finally,

$$T_2(\mathbf{q}) = \langle M, \mathbf{p}-\mathbf{q} | H_i | \tilde{N}, \tilde{\mathbf{p}}\rangle.$$

By successively substituting in $T_1(\mathbf{q})$ and $T_2(\mathbf{q})$ in place of $\psi^+_{N,p}$ and $|\tilde{N}, \tilde{\mathbf{p}}\rangle$ the right sides of the equations for these functions, we obtain an infinite sequence of matrix elements. We cut off this series by using the following reasoning. The entire series corresponds to the rigorous solution of the problem with respect to H_l. This solution corresponds to the exact single-scattering amplitude and to an account of an infinite number of successive single collisions. If the amplitude of the single-scattering probability can be obtained by perturbation theory with account of a finite number of perturbation-theory terms, then the series can be cut off, since the presence of a finite probability of essentially inelastic scattering (for example, with excitation of deep electronic shells) in the almost-elastic scattering problem causes the electron to experience a finite number of successive collisions until it either loses a considerable energy or is returned with a small energy loss. Therefore T_q can be written in the form

$$T_{\mathbf{q}} = T_l(\mathbf{q}) + T_i(\mathbf{q}) + \langle M, \mathbf{p}-\mathbf{q}$$
$$\times | H_l \hat{G} (1 + H_l\hat{G} + H_l\hat{G}H_l\hat{G} + \ldots) H_i$$
$$+ H_i\hat{G}H_l (1 + \hat{G}H_l + \hat{G}H_l\hat{G}H_l + \ldots) | N, \mathbf{p}\rangle$$
$$+ \langle M, \mathbf{p}-\mathbf{q} | H_l\hat{G}H_l\hat{G} \ldots H_l\hat{G}H_l\hat{G}H_i\hat{G}H_l\hat{G}H_l$$
$$\ldots \hat{G}H_l | N, \mathbf{p}\rangle.$$

Let us write this more compactly in the form

$$T_{\mathbf{q}} = T_l(\mathbf{q}) + T_i(\mathbf{q}) + \langle M, \mathbf{p}-\mathbf{q} | R_l H_i + H_i R_l' | N, \mathbf{p}\rangle$$
$$+ \langle M, \mathbf{p}-\mathbf{q} | Q_l H_i Q_l' | N, \mathbf{p}\rangle,$$

$$T_i(\mathbf{q}) = \langle M, \mathbf{p}-\mathbf{q} | H_i | N, \mathbf{p}\rangle, \tag{1}$$

where R_l, R'_l, Q_l, and Q'_l are operators made up of the operators H_l and H.

3. TOTAL PROBABILITY OF TRANSITION WITH MOMENTUM TRANSFER q

We shall use the indices m, n, and μ, ν to describe the states of the electronic subsystem and of the lattice respectively: $(m, \mu) = M$, $(n, \nu) = N$. Since the amplitude of the electron-electron transition probability

$$T_i(\mathbf{q}) = V_{\mathbf{q}} (\sum_{\mathbf{p}'} a_{\mathbf{p}'}^+ a_{\mathbf{p}'-\mathbf{q}})_{mn}$$

depends only on the momentum transfer and does not depend on the momenta of the initial and final states of the fast electron separately, we can repre-represent $|T_q|^2$ in the form

$$|T_{\mathbf{q}}|^2 = |T_i(\mathbf{q}, m, n)|^2 \Gamma^2(\mathbf{q}) + |T_l(\mathbf{q}, \mu, \nu)|^2 (1 + 2\,\mathrm{Re}\, T_i(0))$$
$$+ |\sum_{\mathbf{q}_e} T_i(\mathbf{q}_e, m, n) B_{\mu\nu}(\mathbf{q}, \mathbf{q}_e)|^2,$$

$$B_{\mu\nu}(\mathbf{q}, \mathbf{q}_e) = \sum_{\mathbf{q}', \eta} \langle \mu, \mathbf{p}-\mathbf{q} | Q_l | \eta, \mathbf{p}-\mathbf{q}'-\mathbf{q}_e\rangle$$
$$\times \langle \eta, \mathbf{p}-\mathbf{q}' | Q_l' | \nu, \mathbf{p}\rangle + \langle \mu, \mathbf{p}-\mathbf{q} | R_l | \nu, \mathbf{p}-\mathbf{q}_e\rangle$$
$$+ \langle \mu, \mathbf{p}-\mathbf{q}+\mathbf{q}_e | R_l' | \nu, \mathbf{p}\rangle,$$

$$\Gamma^2(\mathbf{q}) = 1 + 2\,\mathrm{Re}\,[\sum_{\eta, \mathbf{q}'} \langle \nu, \mathbf{p} | Q_l | \eta, \mathbf{p}-\mathbf{q}'-\mathbf{q}\rangle$$
$$\times \langle \eta, \mathbf{p}-\mathbf{q}' | Q_l' | \nu, \mathbf{p}\rangle + \langle \nu, \mathbf{p}-\mathbf{q} | R_l | \nu, \mathbf{p}-\mathbf{q}\rangle$$
$$+ \langle \nu, \mathbf{p} | R_l' | \nu, \mathbf{p}\rangle], \tag{2}$$

where $T_i(0)$ is the electron-electron zero-angle scattering amplitude. The first term in (2) is connected with the electron-electron transition with account of renormalization of the vertex part due to the electron-lattice interaction. The second is connected with the electron-lattice transition with a vortex renormalized by electron-electron interaction. The third term describes a mixed transition, when momentum $\mathbf{q}$ is transferred to both the lattice and the conduction electrons, the momentum transferred to the electrons being $\mathbf{q}_e$, and that to the lattice $\mathbf{q} - \mathbf{q}_e$.

The probability w must be summed in usual fashion over all the final states and averaged statistically over the initial states (with statistical matrices $\rho^{(e)}$ and $\rho^{(l)}$ of the electrons and the lattice):

$$W(\mathbf{q}) = \sum_{m,n,\mu,\nu} w_{\mathbf{q}}(M, N)\rho^{(e)}\rho^{(l)} = 2\pi V_{\mathbf{q}}^2 [\sum_\nu \Gamma^2(\nu, \mathbf{p}, \mathbf{q})\rho_\nu^{(l)}] \times \Phi_1(\mathbf{q}, \omega) + W_l(\mathbf{q})\Phi_2(\beta) + 2\pi \sum_{m,n,\mu,\nu} \rho_n^{(e)}\rho_\nu^{(l)} \sum_{\mathbf{q}_e} V_{\mathbf{q}_e}^2 \times |(\sum_{\mathbf{p}'} a_{\mathbf{p}'}^+ a_{\mathbf{p}'-\mathbf{q}_e})_{mn}|^2 |B_{\mu\nu}(\mathbf{p}, \mathbf{q}_e)|^2 \delta(E_M - E_N - \omega). \quad (3)$$

The functions Φ_1 and Φ_2 are defined as follows:

$$\Phi_1(\mathbf{q}, \omega) = \sum_{m,n} \rho_n^{(e)}(\beta) |(\sum_{\mathbf{p}'} a_{\mathbf{p}'}^+ a_{\mathbf{p}'-\mathbf{q}})_{mn}|^2 \times \delta(E_m - E_n + \varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{p}-\mathbf{q}}), \quad (4)$$

$$\Phi_2(\beta) = \sum_n \rho_n^{(e)}(\beta)[1 + 2\,\mathrm{Re}\,T_i(0)]_n, \quad (5)$$

and $\omega = \omega_p - \epsilon_{p-q}$ is the transferred energy.

The function $\Phi_1(\mathbf{q}, \omega)$ was introduced by Larkin[2]. He showed that this function is connected with the temperature of the two-particle Green's function $K(\mathbf{q}, \omega)$ by the relation

$$\Phi_1(\mathbf{q}, \omega) = \mathrm{Im}\,\tilde{K}(\mathbf{q}, \omega) / \pi(1 + e^{-\beta\omega}), \quad (6)$$

$\tilde{K}(\mathbf{q}, \omega)$ being the function K analytically continued in the upper half-plane of ω, and related with the polarization operator $\Pi(\mathbf{q}, \omega)$ as follows:

$$\tilde{K}(\mathbf{q}, \omega) = \Pi(\mathbf{q}, \omega)[1 - V(\mathbf{q})\Pi(\mathbf{q}, \omega)]^{-1}. \quad (7)$$

The three terms of which (3) consists will be denoted by $W_1(\mathbf{q})$, $W_2(\mathbf{q})$, and $W_3(\mathbf{q})$, respectively.

4. CONNECTION BETWEEN $W_3(\mathbf{q})$ AND THE POLARIZATION OPERATOR

We can neglect in the energy δ-function of (3) the energy transferred by the fast electron to the lattice, compared with the energy transferred to the conduction electrons. This can be done because of the relative elasticity of the electron-lattice scattering and the finite probability of essentially inelastic scattering of the electron. Then $W_3(\mathbf{q})$ can be written in the form

$$W_3(\mathbf{q}) = 2\pi \sum_{\mathbf{q}_e} V_{\mathbf{q}_e}^2 L(\mathbf{q}_e)\Phi_1(\mathbf{q}_e, \omega),$$
$$L(\mathbf{q}_e) = \sum_{\mu,\nu} \rho_\nu^{(l)} |B_{\mu\nu}(\mathbf{q}, \mathbf{q}_e, \mathbf{p})|^2. \quad (8)$$

Taking (6) and (7) into account, we represent (8) in the form

$$W_3(\mathbf{q}) = 2\sum_{\mathbf{q}_e} V_{\mathbf{q}_e}^2 L(\mathbf{q}_e) \frac{1}{1 - e^{-\beta\omega}} \mathrm{Im} \frac{\Pi(\mathbf{q}_e, \omega)}{1 - V_{\mathbf{q}_e}\Pi(\mathbf{q}_e, \omega)}. \quad (9)$$

5. PROBABILITY OF REFLECTION WITH ENERGY LOSS ω

The probability $W(\omega)$ of electron reflection with transfer of an energy ω to the medium is

$$W(\omega) = \int_{(\Omega)} W(\mathbf{p}, \mathbf{p}') \frac{d^3\mathbf{p}'}{(2\pi)^3}.$$

The integration is over the states of the electrons that are reflected and lose an energy ω. The term W_2 is connected with pure elastic reflection, and will not be considered. Let us examine the expression for $W_3(\omega)$:

$$W_3(\omega) = 2\pi \sum_{\mathbf{q}_e} V_{\mathbf{q}_e}^2 \Phi_1(\mathbf{q}_e, \omega) S(\mathbf{p}, q_e),$$
$$S(\mathbf{p}, q_e) = \int_{(\Omega)} \frac{d\mathbf{p}'}{(2\pi)^3} L(\mathbf{p}, \mathbf{q}_e, \mathbf{p}'). \quad (10)$$

We go over in (10) from summation to integration. The limits of integration with respect to $\mathbf{q}_e$ can be established by going over to integration in the space of the momenta $\mathbf{p}'_e$ of the final states after electron-electron interaction. In this space the integral is taken between the limits $+1 > \cos\theta_{p'} > -1$ with $|\mathbf{p}'_e| = (\mathbf{p}^2 - 2\omega)^{1/2}$. Going back to the variables $\mathbf{q}_e$, we have (leaving out the factor preceding the integral)

$$\int_0^\infty dq_e q_e \int_{-pq_e - q_e^2/2}^{+pq_e - q_e^2/2} d\omega_e W(\mathbf{q}_e, \omega_e)\delta(\omega_e - \omega),$$

so that

$$W_3(\omega) \sim \int_{\omega/p}^{2P} dq_e q_e V_{qe}^2 \Phi_1(\mathbf{q}_e, \omega) S(q_e). \quad (11)$$

We break this integral up into two parts in the usual manner—for small and large values of momentum transfer. In the region of small q_e, when $\omega/p < q_e \ll q_1$, Eq. (11) takes the form

$$\frac{1}{1 - e^{-\beta\omega}} \int_{\omega/p}^{q_1} dq_e \frac{S(q_e)}{q_e^3} \mathrm{Im} \frac{\Pi(q_e, \omega)}{1 - 4\pi q_e^{-2}\Pi(q_e, \omega)}. \quad (12)$$

Here

$$\omega \gg q_1^2 \gg \varkappa^2, \quad \varkappa^2 = 4\pi n_e e^2 \beta^{-1},$$

$$\mathrm{Re}\,\Pi(q, \omega) = n_e \frac{q_e^2}{\omega^2},$$

$$\mathrm{Im}\,\Pi(q_e, \omega) = n_e(2\pi\beta)^{1/2}(2q_e)^{-1}(1 - e^{-\beta\omega}) \times \exp\left[-\frac{\beta}{2}\left(\frac{\omega}{q_e} - \frac{q_e}{2}\right)^2\right].$$

Bearing this in mind, we write the integral in (12) in the form

$$\int_{\omega/p}^{q_1} \frac{dq_e}{q_e^3} S(q_e) \frac{\operatorname{Im}\Pi(q_e,\omega)}{(\omega^2-\omega_p^2)^2+\omega^4 q_e^{-4}[4\pi \operatorname{Im}\Pi(q_e,\omega)]^2}, \quad (12a)$$

where $\omega_p = (4\pi ne^2/m)^{1/2}$ is the plasma frequency.

At small q_e we can neglect the dependence of S on q_e, so that the contribution of (12) to $W_3(\omega)$ is proportional to

$$n_e S_0 \frac{\omega^4}{(\omega^2-\omega_p^2)^2} \int_{\omega/p}^{q_1} \frac{dq_e}{q_e^4} \exp\left(-\frac{\beta}{2}\frac{\omega^2}{q_e^2}\right), \quad \omega \neq \omega_p. \quad (13)$$

For large momentum transfers, the integral with respect to q_e takes the form

$$\int_{q_1}^{2p} \frac{dq_e}{q_e^4} S(q_e) \exp\left[-\frac{\beta}{2}\left(\frac{\omega}{q_e}-\frac{q_e}{2}\right)^2\right]. \quad (14)$$

The integrand decreases rapidly on both sides of the region $q_e \sim \sqrt{2\omega}$, because the argument of the exponential contains the large quantity $-\beta\omega(\omega/\kappa^2)$ at the lower limit and $-\beta p^2$ at the upper limit. Therefore only a small region on the integration contour, in the vicinity of the point $q_e = \sqrt{2\omega}$, is important, and (14) is equal to

$$S(\sqrt{2\omega}) / 4\omega^2\sqrt{\beta}, \quad (15)$$

which does not depend on q_1. The sharp maximum in the integrand of (14) corresponds to the fact that pair collisions of the electrons will be the most probable, and the presence of weak interaction between the slow electrons, as expected, will not exert a noticeable influence on these collisions.

It follows from (15) that the main part of $W_3(\omega)$ depends on ω like ω^{-2}, so that we can assume that S depends little on ω when $q_e \ll p$. When $q_e \sim p$, that is, in large-angle electron-electron scattering, the structure of $W_1(\omega)$ and of the corresponding part of $W_3(\omega)$ is such that the contribution of these terms to $W(\omega)$ is proportional to the integral

$$\int \frac{dq_e}{q_e^4}[1+S(q_e)] \exp\left[-\frac{\beta}{2}\left(\frac{q_e}{2}\right)^2\right]. \quad (16)$$

But this expression does not depend on ω at all when ω is small, and in this sense no assumption whatever need be made concerning the behavior of $S(q_e)$ for large q_e. We see from (14) that (16) is a small quantity so long as perturbation theory itself is valid. The presence of the term (16) is due to the fact that all the conduction electrons acquire the recoil momentum, and this makes large-angle electron-electron scattering possible.

The integral in (13) is proportional to $\exp[-\beta\omega^2/q_1]$. Therefore (13) makes a contribution equally negligible, compared with (15), as the integration regions in (14) far from $q_e \approx \sqrt{2\omega}$. Thus, when $\omega \neq \omega_p$ we should expect the following dependence of W on ω:

$$W(\omega) \sim A/\omega^2. \quad (17)$$

However, when $\omega \approx \omega_p$ the plasma term (13) can become large. In this case we can no longer neglect Im Π in the denominator of the integrand in (12a). In addition, we must take into account the fact that the frequency of the plasma oscillations depends on the wave vector, since

$$\operatorname{Re}\Pi(q_e,\omega) = n\left(\frac{q_e}{\omega}\right)^2\left(1+3\frac{q_e^2}{\beta\omega^2}\right).$$

The integral in (12) takes now the form

$$\int_{\omega/p}^{q_1} \frac{dq_e}{q_e^4} S(q_e) \exp\left[-\frac{\beta}{2}\left(\frac{\omega}{q_e}\right)^2\right]\left\{\frac{(\omega^2-\omega_+^2)^2(\omega^2-\omega_-^2)^2}{\omega^4} + C\omega^4 q_e^{-6} \exp\left[-\beta\left(\frac{\omega}{q_e}\right)^2\right]\right\}^{-1};$$

$$C = (2\pi)^3 n_e^2 \beta (1-e^{-\beta\omega})^2,$$

$$\omega_\pm^2 = {}^1\!/_2\omega_p^2[1 \pm (1+12q_e^2/\beta\omega_p^2)^{1/2}]. \quad (18)$$

The integrand has a sharp maximum when $\omega^2 - \omega_\pm^2 = 0$ and $\omega^2 - \omega_p^2 \ll \omega_p^2$. This occurs when

$$q_e^2 = q_0^2 = {}^1\!/_3\beta(\omega^2-\omega_p^2). \quad (19)$$

Since $q_{min} = \omega/p$, q_0 will lie on the integration contour if

$$\omega^2 - \omega_p^2 > 3\omega^2/\beta p^2. \quad (20)$$

The integrand at the maximum takes the form

$$S_0\beta(\omega^2-\omega_p^2)[3C\omega^4]^{-1} \exp\left[\frac{3}{2}\frac{\omega^2}{\omega^2-\omega_p^2}\right]. \quad (21)$$

If $\omega^2 - \omega_p^2 \ll \omega_p^2$, so that q_0 is sufficiently close to the lower limit of integration, then the main contribution to the integral is made by the section of the integration contour close to q_0. With increasing q_e, the integrand in (18) decreases rapidly, and Eq. (18) does not depend on q_1 both when q_1 lies in the region $\omega/p \ll q_1^2 \ll \omega$ and when $\omega \sim \omega_p$. We note that the inequalities that determine q_1 when $\omega \neq \omega_p$ and when $\omega \sim \omega_p$ are quite weak and are satisfied for all possible values of n_e in semiconductors.

The estimate (18) shows that when $\omega \sim \omega_p$ the maximum possible contribution to W from plasma effects is proportional to

$$S_0\omega_p^2(\omega^2-\omega_p^2)^{-1}. \quad (22)$$

At the same time, when $\omega \sim \omega_p$ the pair collisions make a contribution $\sim S_0$ to W, so that collisions with small momentum transfer turn out to have an effect

$$[\omega_p^2 / (\omega^2 - \omega_p^2)]_{max} \sim \beta p^2 \tag{23}$$

times larger than the pair collisions.

6. CONCLUSION

Thus, the spectrum of the electrons reflected with small energy loss is described by formula (17). In addition to this monotonic part of the spectrum, resonance can occur at a frequency close to plasma frequency. A resonance of this kind can be expected in semiconductors with large carrier density. In the opposite case, this peak may coalesce in the experiment with the peak of elastic reflection, leading to a broadening of the latter upon reflection from the conducting crystals, compared with the case of reflection from dielectrics, when the width of the peak is determined only by the energy lost during phonon emission. If ω_p is small and close to the phonon frequencies, additional damping arises, connected with the electron-phonon interaction. This corresponds formally to the fact that in this case we cannot neglect in the δ function of (3) the change in the lattice energy compared with the change in the energy of the electronic subsystem. The role of multiple collisions between the fast electrons and the electronic subsystem is relatively small in semiconductors. A simple examination leads to the rather obvious conclusion that when $\omega\tau \gg 1$ (τ is the mean free time of the fast electron in the target) multiple peaks appear with frequencies $2\omega_p$, $3\omega_p$, etc., and some broadening of the resonance lines arises, due to the successive collisions with small and large momentum transfer.

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