INTERACTION OF TRANSVERSE WAVES WITH A TURBULENT PLASMA

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Submitted to JETP editor May 6, 1965

J. Exptl. Theoret. Phys. (U.S.S.R.) **49**, 1332-1344 (October, 1965)

We consider the propagation of transverse waves through a bounded plasma in which strong plasma waves or ion-acoustic waves are excited; the plasma wave spectrum is of the nature of a noise spectrum, with an arbitrary spectral distribution. The intensity of the scattered waves is determined as a function of angle and it is shown that measurements of this parameter can be used as a method of plasma diagnostics.

INTRODUCTION

THE present paper is concerned with a number of effects which arise as a result of the nonlinear interaction of randomly phased transverse waves (radiation) with intense plasma and ion-acoustic waves of a noisy nature (plasma noise) and treats problems that involve the propagation of radiation through a bounded turbulent plasma. We have limited ourselves to the case of a weakly anisotropic nonmagnetized plasma and the point of departure is a system of nonlinear equations for a turbulent plasma that has been given earlier.[1]

Obviously nonlinear processes will be of importance only when the intensities of the radiation and the plasma noise are large compared with the thermodynamic equilibrium values of these parameters. However, it is not necessarily true that the intensities of the transverse (radiation) noise and longitudinal (plasma) noise are always large simultaneously. In fact, because of the very high transparency of a plasma with respect to transverse waves (especially high-frequency waves) the intensity of the radiation in a plasma will as a rule be small. The radiation level is high only if there is some external "specified" source of transverse waves. On the other hand, the level of longitudinal plasma noise can be very high. Hence, we shall not be interested in an initial value problem, but rather in the boundary-value problem that treats propagation of a specified transverse wave generated by some external source through a turbulent bounded plasma. In this case it is natural to treat the problem under the assumption that the transverse radiation is relatively weak, in which case it can be assumed that the transverse waves do not have an important effect on the evolution of the plasma noise; the plasma noise spectrum can then be regarded as some specified function of the wave vector **k**, the time and coordinates.[1)] In this connection, in the equation which gives the intensity of the radiation we can neglect all quadratic terms.

Furthermore, simple estimates show that of all the nonlinear processes in which transverse waves participate the only important ones are the decay ("fusion") interactions between one longitudinal wave (plasma l or ion-acoustic s) and two transverse waves t, that is to say, (t, t, l) and (t, t, s) decay interactions.[2)] All of the other interactions are found to be unimportant and can be neglected. The equation obtained for radiation intensity is then found to be linear; to facilitate the development it is expedient to investigate the interaction of the radiation with plasma waves or with ion-acoustic waves separately. A case in which both kinds of waves are excited simultaneously in the plasma can be studied using the results given below.

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1)A more detailed analysis of this question given earlier[2] shows that this assumption is valid in practice for all presently available radiation sources.

2)It should be noted that the classification of nonlinear interactions adopted in[1] and by certain other authors is somewhat arbitrary in the case of finite absorption; strictly speaking the decay interaction is nothing more than resonance scattering corresponding to the vanishing of the denominator which appears in the general expression for the scattering probability. This classification is appropriate in the case of low absorption since it allows us to simplify the system of original equations and to introduce a convenient method for describing various nonlinear interactions.

1. INTERACTION OF RADIATION WITH PLASMA WAVES [3)]

Thus, let us consider the propagation of radiation through a plasma in which intense plasma waves are excited, these being of a noise-like nature with a spectral energy density $W_l(t, \mathbf{r}, \mathbf{k})$ which is an arbitrary function of the time t, the coordinates $\mathbf{r}$, and the wave vector $\mathbf{k}$. Using the expression given in [1] for the probability of (t, t, l) decay, we can write the equations for the spectral intensity of the radiation $I(t, \mathbf{r}, \omega, \Omega)$ in the form[4)]

$$\hat{D}I(\omega, \Omega) = -\gamma_l(\Omega)I(\omega, \Omega) + \int d\Omega_1 \left[\frac{\omega-\omega_e}{\omega+2\omega_e}\gamma_l^{(+)}(\Omega, \Omega_1)I(\omega+\omega_e, \Omega_1) + \frac{\omega+\omega_e}{\omega-2\omega_e}\gamma_l^{(-)}(\Omega, \Omega_1)I(\omega-\omega_e, \Omega_1)\right], \quad (1.1)$$

$$\hat{D} = \frac{\partial}{\partial t} + v(\omega)\mathbf{n}\frac{\partial}{\partial \mathbf{r}}, \quad v(\omega) = c^2\frac{k(\omega)}{\omega},$$

$$k(\omega) = \frac{[\omega^2 - \omega_e^2]^{1/2}}{c},$$

$$\gamma_l(\Omega) = \int d\Omega_1[\gamma_l^{(+)}(\Omega, \Omega_1) + \gamma_l^{(-)}(\Omega, \Omega_1)], \quad (1.2)$$

$$\gamma_l^{(\pm)}(\Omega, \Omega_1) = \frac{\pi}{16}\omega_e\frac{(\omega^2 \pm 2\omega\omega_e)^{1/2}}{\omega}[1+\mathbf{n}\mathbf{n}_1]\frac{\omega_e}{c}\mathbf{k}_{(\pm)}^2\frac{W_l(\mathbf{k}_{(\pm)})}{n_e mc^2},$$

$$\mathbf{k}_{(\pm)} = \pm\mathbf{n}_1 k(\omega \pm \omega_e) \mp \mathbf{n}k(\omega).$$

Here, the symbols $\Omega = \{\theta, \varphi\}$ and $\Omega_1 = \{\theta_1, \varphi_1\}$ denote the ensemble of angular coordinates with unit vectors $\mathbf{n}$ and $\mathbf{n}_1$ which specify the direction of propagation of the radiation, ω is the frequency, $v(\omega)$ is the group velocity, n_e is the electron plasma density, ω_e is the electron plasma frequency, c is the velocity of light, $d\Omega_1 = \sin\theta_1 d\theta_1 d\varphi_1$, and the quantities $W_l(t, \mathbf{r}, \mathbf{k})$ and $I(\omega, \Omega)$ are normalized so that

$$\int W_l(\mathbf{k})d\mathbf{k} = U_l, \quad \int\frac{I(\omega, \Omega)\,d\omega\,d\Omega}{v(\omega)} = U_t, \quad (1.3)$$

where U_l and U_t are the mean energy densities of the plasma waves and the transverse waves respectively.

Thus, the determination of the radiation intensity $I(t, \mathbf{r}, \omega, \Omega)$ is reduced to the solution of the integral equation (1.1) in which the coefficients are functions of the frequency ω, the angles θ, and φ, the coordinate $\mathbf{r}$ and the time t; in turn these depend on the spectral distribution of the plasma noise. We will not try to obtain a solution to this complicated equation in general form; instead we consider two simplified limiting cases in which a solution can be found in analytic form and analyzed in detail. The first case, which generally obtains under laboratory conditions, treats a highly collimated beam of transverse waves with a narrow frequency spectrum $\Delta\omega < \omega_e$. The second case, which is appropriate for conditions typical of interaction of radiation with matter in hot stars, treats a broad spectrum in which the radiation intensity $I(\omega)$ varies very weakly over the range $[\omega, \omega + \omega_e]$. The first case, which has direct applications to plasma diagnostics, will be analyzed in greater detail; the second will be analyzed in terms of its general features.

A. Interaction of a narrow-band signal with a turbulent plasma. Assume that on a bounded plasma there is incident along the z axis a highly collimated and spatially bounded beam of transverse waves (signal) whose intensity $I(t, \mathbf{r}, \omega, \Omega)$ is nonzero only within the narrow frequency range $[\bar{\omega} - \Delta\omega/2, \bar{\omega} + \Delta\omega/2]$ (where $\Delta\omega < \omega_e$) and the solid angle $\Delta\Omega$ (i.e., for $0 \le \theta < \Delta\theta$). The problem consists of determining the intensity of the transverse waves at an arbitrary time at any point in space (both inside and outside the plasma).

An elementary analysis of (1.1) shows that transmission of such a signal through a turbulent plasma results in attenuation of the signal by virtue of scattering and generation of scattered or satellite waves at combination frequencies given by

$$\omega \approx \omega + m\omega_e \quad (m = \pm 1, \pm 2, \pm 3, \ldots).$$

In general the intensity of the satellites falls off rapidly with increasing m. In particular, for a highly collimated beam in which the divergence angle $\Delta\theta$ and the transverse dimensions d are small (or the dimensions of the plasma are not very large) this reduction is found to be so fast that in all cases of practical interest the intensity of the scattered waves is found to be negligibly small as compared with the primary signal as is the intensity of higher order satellites as compared with the intensity of the first two ($m = \pm 1$). Hence we shall only consider the solution for the primary signal and the first two satellites.[5)]

[3)]Similar problems have been studied by the author and Tsytovich and the interaction between high-frequency radiation and relativistic plasma waves has been described qualitatively.[3,4]

[4)]We note that when $\omega < 2\omega_e$ the coefficient $\gamma_l^{(-)}$ must be taken equal to zero.

[5)]It should be noted that the number of red satellites ($m < 0$) is always bounded because the frequency ω cannot be smaller than ω_e. In particular, if the frequency of the primary signal $\omega < 2\omega_e$ then there are obviously only two violet satellites ($m > 0$).

It follows from the definition (1.2) that the coefficients γ_l, $\gamma_l^{(+)}$, and $\gamma_l^{(-)}$ are very sensitive to the spectral distribution of the plasma noise $W_l(\mathbf{k})$; in particular these coefficients vanish when $W_l(\mathbf{k}) = 0$. In general this dependence will be very complicated, but in any case the intensity of the longitudinal noise must fall off sharply as a region of strong absorption is approached. We denote by k_m the wave number above which the damping of the longitudinal waves is so large that the function $W_l(k)$ can be set equal to zero when $k > k_m$. Furthermore, let $\theta^{(+)}$ represent the maximum angle for which the wave "fusion" can occur. This angle is obviously determined from the condition

$$2\bar{\omega}^2 + 2\bar{\omega}\omega_e - \omega_e^2 - 2(\bar{\omega}^2 + 2\bar{\omega}\omega_e)^{1/2}(\bar{\omega}^2 - \omega_e^2)^{1/2} \times \cos\theta_l^{(-)} = c^2 k_m^2. \quad (1.4)$$

We assume now that the incident signal is highly collimated so that its angular divergence $\Delta\theta \ll \theta_l^{(+)}$. In this case it can be shown that (1.1) reduces to a simple system of linear differential equations:

$$\hat{D}_0 I_0(\omega, \Omega) = -\gamma_{l,0}(\Omega) I_0(\omega, \Omega) \quad \text{for} \quad m = 0, \quad (1.5)$$

$$\hat{D}_m I_m(\omega, \Omega) = -\gamma_{l,m}(\Omega) I_m(\omega, \Omega) + \int d\Omega_1 \left[\frac{\omega + (m+1)\omega_e}{\omega + (m-2)\omega_e}\right] \times \gamma_{l,m}^{(-)}(\Omega, \Omega_1) I_{m-1}(\omega, \Omega_1) \quad \text{for } m > 0, \quad (1.6)$$

$$\hat{D}_m I_m(\omega, \Omega) = -\gamma_{l,m}(\Omega) I_m(\omega, \Omega) + \int d\Omega_1 \left[\frac{\omega + (m-1)\omega_e}{\omega + (m+2)\omega_e}\right] \times \gamma_{l,m}^{(+)}(\Omega, \Omega_1) I_{m+1}(\omega, \Omega_1) \quad \text{for } m < 0,$$

$$D_m = \frac{\partial}{\partial t} + v_m \mathbf{n}\frac{\partial}{\partial \mathbf{r}}, \qquad v_m = v(\omega + m\omega_e), \quad (1.7)$$

$$\gamma_{l,m} = \int d\Omega_1 [\gamma_{l,m}^{(+)} + \gamma_{l,m}^{(-)}], \qquad \gamma_{l,m}^{(\pm)} = \gamma_l^{(\pm)}|_{\omega \to \omega + m\omega_e},$$

where the function

$$I_m(\omega, \Omega) = I(\omega + m\omega_e, \Omega),$$

$$\bar{\omega} - \Delta\omega/2 \leqslant \omega \leqslant \omega + \Delta\omega/2 \quad (1.8)$$

determines the intensity of the m-th satellite.

The condition for applicability of (1.5) for I_0 the intensity of the main signal is then

$$\gamma_{l,0}(0)[d + s\Delta\theta] \ll [\theta_l^{(+)}]^3/[\Delta\theta]^2. \quad (1.9)$$

However (1.6), which applies to the satellites, is subject to the more stringent condition

$$\gamma_{l,0}(0)[d + s\Delta\theta] \ll \theta_l^{(+)}. \quad (1.10)$$

In (1.9) and (1.10) the symbol d denotes the characteristic transverse dimensions of the beam (say the diameter) on entrance into the plasma, while the symbol s denotes distance along the beam for which the solutions of (1.5) and (1.6) still hold, assuming that (1.9) and (1.10) are satisfied respectively.[6)] It then follows that when $\Delta\theta \ll \theta_l^{(+)}$ the applicability conditions of (1.5) for $I_0(t, \mathbf{r}, \omega, \Omega)$ are much less stringent than of (1.6) for the satellites. In particular, for a strictly parallel beam the change in intensity of the primary signal in general (i.e., for any values of the path s) is independent of the amplitude of the scattered wave.

The solution of (1.5) can be found by elementary methods and is of the form

$$I_0(t, s, \rho, \omega, \Omega) = \Psi\left(t - \frac{s_0(\rho)}{c} - \frac{s - s_0(\rho)}{v_0}, \rho, \omega, \Omega\right) \times \exp\left[-\int_{s_0(\rho)}^{s} ds' \gamma_{l,0}\left(t - \frac{s - s'}{v_0}, s', \rho, \omega, \Omega\right)\right], \quad (1.11)$$

where the new space variables s and ρ are related to the old variables $\{\mathbf{r}\}$ by the expressions:

$$s = (\mathbf{n}\mathbf{r}), \quad \rho = \mathbf{r} - \mathbf{n}s;$$

$\Psi(t - s_0(\rho)/c, \rho, \omega, \Omega)$ is a specified function of the time, coordinates, frequency and angle which represents the intensity of the incident signal at the boundary of the plasma. The quantity $s_0(\rho)$ is a function which describes, in terms of the variables s and ρ, the equation of the "trailing" surface $s = s_0(\rho)$ at which we assign the boundary conditions for the incident signal and the scattered waves.

In order to estimate the characteristic distance $z_l = v_0/\gamma_{l,0}$ in which the intensity of the incident signal is reduced by a factor of e we compute the damping factor $\gamma_{l,0}(\Omega)$ for a high-frequency signal $\bar{\omega} \gg \omega_e$ under the assumption that the spectral energy density of the plasma noise is independent of wave vector, being given by

$$W_l(k) = \begin{cases} 3U_l/4\pi k_m^3 & k < k_m \\ 0 & k > k_m \end{cases}. \quad (1.12)$$

Substituting (1.12) in (1.7), neglecting quantities of order ω_e as compared with $\bar{\omega}$ and integrating over angle we find

6)We note also that (1.9) is a criterion for the application of (1.5) only if we are interested in the distribution of intensity of the main signal within a solid angle which does not exceed the initial solid angle (i.e., $\theta < \Delta\theta$). If this condition is not satisfied the criterion in (1.9) must be replaced by (1.10).

$$\gamma_{l,0}(\Omega)=\frac{3\pi}{2}\omega_e\left(\frac{\bar{\omega}}{\omega_e}\right)^2\left(\frac{\omega_e}{ck_m}\right)^3\frac{U_l}{n_e m_e c^2}$$
$$\times\left[1-\frac{4}{3}\sin^2\frac{\theta_l}{2}+\sin^4\frac{\theta_l}{2}\right]\sin^4\frac{\theta_l}{2}, \quad (1.13)$$

$$\sin\frac{\theta_l}{2}=\begin{cases} ck_m/2\bar{\omega} & 2\bar{\omega}\geqslant ck_m \\ 1 & 2\bar{\omega}\leqslant ck_m \end{cases}. \quad (1.14)$$

As the signal frequency $\bar{\omega}$ is increased the damping factor $\gamma_{l,0}$ first increases as the square of the frequency and then, when $\bar{\omega} = ck_m/2$, reaches a maximum value given by

$$\gamma_{l,max}=\frac{\pi}{4}\omega_e\left(\frac{\omega_e}{ck_m}\right)\frac{U_l}{n_e m_e c^2}, \quad (1.15)$$

finally, when $\bar{\omega} > ck_m/2$ the damping factor becomes inversely proportional to $\bar{\omega}^2$. However, the damping factor associated with the binary collisions γ_c is

$$\gamma_c\approx\omega_e\left(\frac{\omega_e}{\bar{\omega}}\right)^2\frac{L}{16\pi N_D}, \quad (1.16)$$

where $N_D = n_e(v_{Te}/\omega_e)^3$ is the number of particles in a Debye sphere, m_e is the mass, $v_{Te}^2 = T_e/m_e$ is the square of the electron thermal velocity in the plasma and L is the Coulomb logarithm.

Since $N_D \gg 1$, it follows from (1.13) and (1.16) that if the noise level is high the damping due to scattering on plasma oscillations can frequently be several orders of magnitude greater than the collisional damping. For example, when

$$\bar{\omega}=ck_m/2\approx(c/v_T)\omega_e, \qquad U_l\approx n_e T_e$$

the ratio

$$\gamma_{l,0}/\gamma_c\approx N_D(v_{Te}/c)\gg 1.$$

We now turn to the analysis of the solutions of (1.6) for the scattered waves, in particular for the first two satellites ($m = \pm 1$). An analysis of this kind will interest us primarily from the point of view of studying the possibility of using these effects for plasma diagnostics.

It follows from (1.6) and (1.7) that the angular distribution of the intensity of the scattered waves is related to the spectral energy density of the longitudinal waves $W_l(\mathbf{k})$. In the general case, however, this relation is an integral one so that the value of the intensity $I(\omega, \Omega)$ corresponding to some fixed value of the angles θ and φ is determined by the value of the plasma noise over a finite region of $\Delta\mathbf{k}$ (which depends on $\Delta\theta$) rather than at discrete values; thus, we can only get information on the value of the spectral noise density as averaged over this range $\Delta\mathbf{k}$. In other words, the relation between the angular distribution of the scattered waves and the noise spectrum is generally not unique. However, this ambiguity is reduced for the first two satellites (and only these two) as the divergence angle $\Delta\theta$ is reduced; when $\Delta\theta = 0$, i.e., in a strictly parallel beam, this ambiguity disappears altogether. In this case a knowledge of the angular distribution of the scattered radiation allows us to determine completely the spectral distribution of the plasma noise for values of the wave vector $\mathbf{k} = \mathbf{k}_{(\pm)}(\omega, \Omega)$.

In this connection we assume below that the incident signal is a parallel beam such that

$$\Psi\left(t-\frac{s_0(\rho)}{c},\rho,\omega,\Omega\right)=\bar{\Psi}\left(t-\frac{s_0(\rho)}{c},\rho,\omega\right)\frac{\delta(\cos\theta-1)}{\pi}. \quad (1.17)$$

Furthermore, since the dimensions of a plasma are generally much smaller than the characteristic distance $z_l = v_0/\gamma_{l,0}$ under laboratory conditions we also assume that the exponential factors of the type $\exp(\int ds'\gamma_{l,m})$ are equal to unity.

Taking account of (1.17) we have

$$I_{\pm1}(t,s,\rho,\omega,\Omega)=\frac{1}{v_{\pm1}}\int_{s_0(\rho)}^{s}\Phi_{\pm1}\left(t-\frac{s-s'}{v_{\pm1}},s',\rho,\omega,\Omega\right)ds', \quad (1.18)$$

$$\frac{1}{v_{\pm1}}\Phi_{\pm}(t,s,\rho,\omega,\Omega)=\frac{\pi}{16}\frac{\omega\pm\omega_e}{\omega}\frac{k(\omega\pm\omega_e)}{k(\omega)}\frac{\omega_e^2}{c^2}$$
$$\times\tilde{k}_{\pm}^2\frac{W_l(t,s,\rho,\tilde{\mathbf{k}}_{(\pm)})}{n_e m_e c^2}$$
$$\times\bar{\Psi}\left(t-\frac{s_0(\rho)}{c}-\frac{s-s_0(\rho)}{v_0},\rho,\omega\right)[1+\cos^2\theta],$$

$$\tilde{\mathbf{k}}_{(+)}=\mathbf{n}k(\omega+\omega_e)-\mathbf{n}_0k(\omega),$$
$$\tilde{\mathbf{k}}_{(-)}=-\mathbf{n}_0k(\omega-\omega_e)+\mathbf{n}k(\omega), \quad (1.19)$$

where $\mathbf{n}$, as before, is a unit vector in the direction of propagation of the scattered wave while $\mathbf{n}_0$ is a unit vector in the direction of propagation of the incident wave; θ is the angle between the vectors $\mathbf{n}$ and $\mathbf{n}_0$.

When $k > k_m$ the function $W_l(\mathbf{k}) = 0$ and it follows from (1.18) and (1.19) that the scattered radiation corresponding to the red ($m = -1$, $\omega \approx \omega + \omega_e$) and the violet ($m = 1$, $\omega \approx \omega + \omega_e$) satellites is concentrated inside cones with opening angles which are smaller than $\theta_l^{(-)}$ and $\theta_l^{(+)}$ respectively. The values of the limiting angles depend on k_m and the frequency of the primary signal, being determined from the relations

$$2\bar{\omega}^2\pm2\bar{\omega}\omega_e-\omega_e^2-2(\bar{\omega}^2\pm2\bar{\omega}\omega_e)^{1/2}(\bar{\omega}^2-\omega_e^2)^{1/2}$$
$$\times\cos\theta_l^{(\pm)}=c^2k_m^2. \quad (1.20)$$

In the case of the high-frequency signals

$\bar{\omega} \gg \omega_e$ the angles $\theta_l^{(+)}$ and $\theta_l^{(-)}$ are approximately equal, with $\theta_l^{(+)} \approx \theta_l^{(-)} \approx \theta_l$, where θ_l is given by (1.14).

In view of the definition of the vectors $\tilde{\mathbf{k}}_{(\pm)}$ it is evident that the scattered waves at frequency $\omega \approx \bar{\omega} - \omega_e$ (red satellite) disappear if the noise consists only of longitudinal waves propagating in the opposite direction to the direction of propagation of the main signal; only the violet satellite (at frequency $\omega \approx \bar{\omega} + \omega_e$) is generated in this case. When the longitudinal waves propagate in the same direction as the incident beam, the red satellite is stronger than the violet satellite.

If the intensity of the incident signal and the plasma noise spectrum do not change appreciably during the transit time of the transverse wave from one plasma boundary to the other the system can be regarded as stationary; it is then easy to find an expression for the radiation flux $S_{(\pm)}(\omega, \theta, \varphi)$ scattered in the direction $\mathbf{n}$ passing through a surface Σ which bounds the plasma with respect to the surrounding space.[7] This expression is of the form

$$S_{(\pm)}(\omega, \theta, \varphi) = V\frac{\bar{S}_0}{\sigma}\frac{\pi}{16}\frac{\omega \pm \omega_e}{\omega}\frac{k(\omega \pm \omega_e)}{k(\omega)}\frac{\omega_e^2}{c^2} \times \tilde{k}_{(\pm)}^2\frac{\overline{W}_l(\tilde{\mathbf{k}}_{(\pm)})}{n_e m_e c^2}[1 + \cos^2\theta], \quad (1.21)$$

where $\overline{W}_l$ is the spectral density of the longitudinal noise energy averaged over the plasma volume V occupied by the incident radiation, $\overline{S}_0(\omega)$ is the total radiation flux integrated over angles θ and φ, and $\sigma \approx \pi d^2/4$ is the cross section of the incident beam; hence

$$\bar{S}_0 = \int_\sigma \bar{\Psi}\mathbf{n}_0\, d\boldsymbol{\sigma},$$

$$S_{(\pm)} = \int_\Sigma I_{\pm 1}\mathbf{n}\, d\boldsymbol{\sigma}, \int_V W_l \bar{\Psi} d\mathbf{r} = \frac{V}{\sigma}\bar{S}_0\bar{W}_l. \quad (1.22)$$

Thus, the transmission of a beam of transverse waves through a turbulent plasma results in its scattering by the plasma noise and a consequent reduction in beam intensity. Under these conditions, using (1.11) and a specified intensity at the plasma boundary we can determine the intensity at all later times at any point in space. On the other hand, for a well-collimated beam (1.18) and (1.19) relate the angular distribution of the scattered radiation, to the spectral energy density of the longitudinal waves in the plasma. In other words, from the known function $I_{\pm 1}(\omega, \Omega)$, using (1.18) and (1.19), we can determine the value of the function $W_l(\mathbf{k})$ for $\mathbf{k} = \mathbf{k}_{(\pm)}(\omega, \theta, \varphi)$ averaged over the path of the scattered beam. In turn, the function $I_{\pm 1}(\omega, \Omega)$ can be found by direct measurement of the angular distribution of the intensity of the scattered waves at frequencies $\omega \approx \bar{\omega} \pm \omega_e$.

As an example we consider in somewhat greater detail the particular case in which the spectral energy density of the plasma waves is independent of wave number, being given by (1.12). In this case, it follows from (1.21) that the scattered radiation is axially symmetric and concentrated within a cone with opening angle of the order of the limiting angle $\theta_l^{(\pm)}$, determined in accordance with (1.20); this angle depends only on the frequency of the signal $\bar{\omega}$ and the value of k_m. Under these conditions the intensity of the scattered radiation is a minimum in the direction of propagation of the primary signal ($\theta = 0$) and increases rapidly with increasing θ, reaching a peak at $\theta \lesssim \theta_l^{(\pm)}$; it then falls off rapidly, almost to zero, when $\theta > \theta_l^{(\pm)}$. Thus, for high-frequency signals $\omega \gg ck_m/2$ the quantity $\theta_l^{(\pm)} \approx \theta_l \ll 1$ and all the radiation is directed in the forward direction; at relatively low frequencies $\bar{\omega} \lesssim ck_m/2$ the maximum radiation intensity is found $\theta \approx \pi$.

For practical purposes it is of interest to estimate the ratio R_l which is the ratio of the flux of scattered radiation integrated over angles θ and φ

$$\bar{S}(\omega) = \int S_{(+)}(\omega, \Omega)\, d\Omega \approx \int S_{(-)}(\omega, \Omega)\, d\Omega$$

to the incident flux $\bar{S}_0$. Substituting (1.12) in (1.21) and integrating over angles θ and φ we find

$$R_l = \bar{S} / \bar{S}_0 = \gamma_{l,0}\Delta z / c, \quad (1.23)$$

where $\Delta z \approx V/\sigma$ is the plasma length along the incident beam while the damping factor $\gamma_{l,0}$ is determined from (1.13). It follows from the expression for $\gamma_{l,0}$ that there is some optimum frequency $\omega_{opt} = ck_m/2$ at which the ratio of the scattered radiation to the incident radiation is a maximum; this frequency is given by

$$R_{l,\,max} = \frac{\pi}{4}\frac{\omega_e}{ck_m}\frac{U_l}{n_e m_e c^2}\frac{\omega_e \Delta z}{c}. \quad (1.24)$$

This result indicates that the maximum effect in any particular case is obtained by choosing the frequency appropriate to the parameters of the plasma being studied. Thus, to investigate the noise in a high-temperature low-density plasma it is desirable to use radiation in the millimeter region; for a cold high-density plasma, however, it might be more desirable to use infrared and optical

[7]Partial reflection of the transverse wave occurs at the plasma boundary but will be neglected here.

n_e		$3\cdot10^{11}$ cm^{-3}	$3\cdot10^{13}$ cm^{-3}	$3\cdot10^{15}$ cm^{-3}	$3\cdot10^{16}$ cm^{-3}
$\bar{\omega} = 10^{12}$ sec^{-1}	$\frac{n_e m_e c^2}{U_e}\cdot R_l$	$3\cdot10^{-2}$	$3\cdot10^{-3}$	$\bar{\omega} < \omega_e$	$\bar{\omega} < \omega_e$
	$\frac{n_e m_e c^2}{U_s}\cdot R_s$	$5\cdot10^{-3}$	$5\cdot10^{-2}$	$\bar{\omega} < \omega_e$	$\bar{\omega} < \omega_e$
$\bar{\omega} = 8\cdot10^{14}$ sec^{-1}	$\frac{n_e m_e c^2}{U_e}\cdot R_l$	$4\cdot10^{-7}$	$4\cdot10^{-4}$	$4\cdot10^{-1}$	12
	$\frac{n_e m_e c^2}{U_s}\cdot R_s$	$5\cdot10^{-8}$	$5\cdot10^{-5}$	$5\cdot10^{-2}$	1
$R_T = \frac{8\pi}{3}\left(\frac{e^2}{m_e c^2}\right)^2 n_e \Delta z$		$1{,}5\cdot10^{-12}$	$1{,}5\cdot10^{-10}$	$1{,}5\cdot10^{-8}$	$1{,}5\cdot10^{-7}$

radiation (lasers) for diagnostic purposes.

For purposes of illustration, we show in the table values of the ratio R_l for a plasma layer of thickness Δz = 10 cm in which $\omega_e/k_m = 3\cdot10^8$ cm/sec for several different values of the density n_e and frequency $\bar{\omega}$. For comparison we also give values of the ratio

$$R_T = \frac{8\pi}{3}\left(\frac{e^2}{m_e c^2}\right)^2 n_e \Delta z$$

corresponding to pure Thomson scattering. These values indicate that if W_l the plasma noise level is high the intensity of the scattered radiation lies within the sensitivity of presently available methods of measurement.

B. Interaction between the plasma and radiation with a wide frequency spectrum. We now consider the interaction between a turbulent plasma and radiation characterized by a wide frequency spectrum, which extends from $\omega \gtrsim \omega_e$ to some maximum frequency ω_m. This case is of interest, in particular, in astrophysics and would characterize the interaction between radiation and the turbulent atmosphere of a hot star.

In the general case (1.1), which gives the radiation intensity $I(\omega, \Omega)$, is extremely complicated. Hence we shall analyze this equation for a particular case which is fairly general but in which a considerable mathematical simplification is possible. Specifically, we shall consider the high-frequency region $\omega \gg \omega_e$ and assume that the radiation intensity $I(\omega, \Omega)$ varies weakly over the interval $[\omega, \omega + \omega_e]$ so that

$$\omega_e \partial \ln I / \partial\omega \ll 1$$

and quantities of order $\omega_e/\omega \ll 1$ can be neglected compared with unity. Under these conditions (1.1) becomes

$$\hat{D}I(\omega, \Omega) = -I(\omega, \Omega)\int d\Omega_1 \varkappa^{(+)}(\Omega, \Omega_1) + \int d\Omega_1 \varkappa^{(+)}(\Omega, \Omega_1) I(\omega, \Omega_1) + \omega_e \int d\Omega_1 \varkappa^{(-)}(\Omega, \Omega_1)\,\omega \times \frac{\partial}{\partial\omega}[I(\omega, \Omega_1)/\omega] + \omega_e^2 \int d\Omega_1 \varkappa^{(+)}(\Omega, \Omega_1)\,\omega^2 \times \frac{\partial}{\partial\omega}\left[\omega^{-2}\frac{\partial I(\omega, \Omega_1)}{\partial\omega}\right], \quad (1.25)$$

where the quantities $\varkappa^{(+)}(\Omega, \Omega_1)$ and $\varkappa^{(-)}(\Omega, \Omega_1)$ depend on the plasma noise spectrum, being determined by the relations

$$\varkappa^{(\pm)}(\omega, \Omega, \Omega_1) = \frac{\pi}{16}\omega_e[1 + (\mathbf{n}\mathbf{n}_1)^2]\frac{\omega_e}{c}k_0^2\frac{W_l(\mathbf{k}_0) \pm W_l(-\mathbf{k}_0)}{n_e m_e c^2},$$

$$\mathbf{k}_0 = \frac{\omega}{c}(\mathbf{n}_1 - \mathbf{n}). \quad (1.26)$$

The first two terms in (1.25) are the basic terms and correspond to pure elastic scattering of radiation on longitudinal waves; the third term, which is proportional to the first derivative with respect to frequency ω, is nonvanishing only for anisotropic noise $W_l(\mathbf{k})$ and leads to an asymmetry in the scattering associated with the asymmetric spectrum of plasma noise. Finally the last term, which generally saturates at a low order of magnitude, takes account of the broadening of the radiation spectrum due to the frequency change in the wave fusion and decay interactions; this term derives from the change in the total radiation energy associated with the change in the plasma noise energy.

If the weak variation in the radiation spectrum is not important, small terms proportional to the frequency derivative can be neglected and the intensity equation becomes a radiation transport equation:[5]

$$\hat{D}I(\Omega) = -I(\Omega)\int d\Omega_1 \varkappa^{(+)}(\Omega, \Omega_1) + \int d\Omega_1 \varkappa^{(+)}(\Omega, \Omega_1) I(\Omega_1). \quad (1.27)$$

In the particular case of isotropic turbulence

$$\varkappa^{(+)}(\Omega,\Omega_1)=\frac{\pi}{2}\omega_e\frac{\omega_e\omega^2}{c^5 n_e m_e}W_l\left(2\frac{\omega}{c}\sin\frac{\theta}{2}\right)\times[1+\cos^2\theta]\sin^2\frac{\theta}{2} \tag{1.28}$$

and the scattering coefficient

$$\bar{\varkappa}=\frac{1}{c}\int d\Omega_1\varkappa^{(+)}=\frac{\pi}{4}\left(\frac{\omega_e}{\omega}\right)^2\times\int_0^{k_m}dk_1\left[2-\left(\frac{k_1 c}{\omega}\right)^2+\frac{1}{4}\left(\frac{k_1 c}{\omega}\right)^4\right]\frac{W_l(k_1)k_1^3}{n_e m_e c^2} \tag{1.29}$$

is independent of the angles θ and φ.

To estimate the effect of scattering of radiation on a turbulent plasma which could occur in the atmosphere of a hot star we assume that the spectral density of the noise energy $W_l(\mathbf{k})$ is independent of wave number and given by (1.12). We also assume that in hot stars the mean thermal velocity $v_{Te} \approx c$, $k_m \approx \omega_e/c$, the frequency $\omega \gg \omega_e$, and consequently that the quantity $2\omega/ck_m$ is generally greater than unity, so that

$$\varkappa=\frac{3\pi}{32}\left(\frac{\omega_e}{\omega}\right)^2\frac{\omega_e}{c}\frac{U_l}{n_e m_e c^2}. \tag{1.30}$$

On the other hand, the Thomson scattering coefficient is

$$\varkappa_T=\frac{8\pi}{3}\left(\frac{e^2}{m_e c^2}\right)^2 n_e=\frac{1}{6\pi}\frac{\omega_e^4}{n_e c^4}. \tag{1.31}$$

It then follows that at relatively low frequencies the optical scattering of plasma waves can be several orders of magnitude greater than the Thomson scattering. Assuming, as an example, that $k_m = \omega_e/c$, $U_l \approx n_e m_e c^2$ and $n_e = 10^{20}$ cm^{-3} we find that the ratio κ/κ_T will be greater than unity in the frequency region from $\omega \approx 5\times10^{14}$ sec^{-1} up to $\omega \approx 5\times10^{18}$ sec^{-1} and that at $\omega \approx 5\times10^{14}$ sec^{-1} the ratio $\kappa/\kappa_T \approx 10^8$. For a thermonuclear plasma with density $n \approx 10^{15}$ cm^{-3} and temperature $T_e \approx 100$ keV this frequency range extends from $\omega \approx 10^{12}$ sec^{-1} to $\omega \approx 2\times10^{17}$ sec^{-1} and at $\omega = 10^{12}$ sec^{-1} the ratio $\kappa/\kappa_T \approx 5\times10^{10}$.

INTERACTION OF RADIATION WITH ION-ACOUSTIC WAVES

We now study the interaction between radiation and a turbulent plasma in which intense ion-acoustic waves are excited. As we shall see below, the interaction of transverse waves with ion-acoustic waves is generally weaker than the interaction with plasma waves; in many cases, however, the former interaction is characterized by a completely different spectrum and angular dependence can be quite distinctive. Furthermore, an investigation of this interaction is of interest from the point of view of plasma diagnostics: just as in the case considered above a measurement of the intensity of the scattered radiation provides information on the spectral energy density of the ion-acoustic noise. In contrast with the work of Akhiezer, [6] in the present work we obtain a general expression for the amplitude of the primary signal and the scattered waves for an arbitrary ion-acoustic noise spectrum.

Thus, assume that a bounded plasma, in which there are excited intense ion-acoustic noise waves, is irradiated (along the z-axis) by a beam of transverse waves (signal) with an angular divergence of order $\Delta\theta$ and a specified spectral distribution. The problem consists of determining the spectral intensity of the radiation $I(t, \mathbf{r}, \omega, \Omega)$ in a given direction $\Omega = \{\theta, \varphi\}$ at an arbitrary time at any point in space. The initial equation for radiation intensity can be easily obtained from equations given earlier [1] and differs from (1.1) in that the probability given there for (t, l, t) decay is replaced by the probability for (t, s, t) decay. This equation can be written in the form

$$\hat{D}I(\omega,\Omega)=-\gamma_s(\Omega)I(\omega,\Omega)+\int d\Omega_1[\gamma_s^{(+)}(\Omega,\Omega_1)I(\omega+\omega_s,\Omega_1)+\gamma_s^{(-)}(\Omega,\Omega_1)I(\omega-\omega_s,\Omega_1)], \tag{2.1}$$

$$\gamma_s^{(\pm)}=\frac{\pi}{16}[1+(\mathbf{n}\mathbf{n}_1)^2]\frac{\omega_e^2}{c^2}v(\omega)\frac{r_{De}^{-2}}{1+k_0^2 r_{De}^2}\frac{W_s(\pm\mathbf{k}_0)}{n_e m_e c^2},$$

$$\mathbf{k}_0=(\mathbf{n}_1-\mathbf{n})\frac{\omega}{c}\frac{v(\omega)}{c},$$

$$\gamma_s=\int d\Omega_1[\gamma_s^{(+)}+\gamma_s^{(-)}],$$

$$\omega_s=\omega_i\left[\frac{k_0^2 r_{De}^2}{1+k_0^2 r_{De}^2}+3k_0^2 r_{Di}\right]^{1/2}. \tag{2.2}$$

Here, ω_e and ω_i are the plasma frequencies, $r_{De} = v_{Te}/\omega_e$ and $r_{Di} = v_{Ti}/\omega_i$ are the Debye radii, $v_{Te} = (T_e/m_e)^{1/2}$ and $v_{Ti} = (T_i/m_i)^{1/2}$ are the thermal velocities while T_e, T_i, m_e, and m_i are respectively the temperature and mass of the electrons and ions in the plasma while $W_s(\mathbf{k})$ is the spectral energy density of the ion-acoustic waves where $\int W_s d\mathbf{k} = U_s$, and U_s is the mean energy density of the acoustic waves.

We shall be interested in two problems: 1) determining the intensity of the primary signal in the plasma, that is the intensity of the radiation at the frequency of the incident signal propagating at an angle θ which does not exceed the initial divergence angle of the incident signal $\Delta\theta$, and 2) determining the intensity of the radiation scattered with a change of frequency at an angle θ which is greater than the initial divergence angle $\Delta\theta$. We limit our analysis to the case of well collimated beams in which $\Delta\theta$ the characteristic divergence angle of the beam transverse waves incident on the plasma is much smaller than the limiting angle θ_s for which radiation can occur in decay (for absorption or fusion) of the scattered t-plasmon. This angle is evidently determined from the relation

$$\sin\frac{\theta_s}{2} = \begin{cases} \dfrac{k_m}{2k(\omega)} & \text{for} \quad k_m \leqslant 2k(\omega) \\ 1 & \text{for} \quad k_m \geqslant 2k(\omega) \end{cases}, \tag{2.3}$$

where $k(\omega)$ is the wave number of the transverse wave, while k_m is the maximum wave number of the ion-acoustic waves; beyond this wave number the intensity of the acoustic oscillations is comparable to the intensity of thermal noise, since we can assume that $W_s(k) = 0$ when $k > k_m$.

If

$$\gamma_s(d + s\Delta\theta) \ll \theta_s^3 / (\Delta\theta)^2, \quad \theta < \Delta\theta \tag{2.4}$$

(2.1) shows that the scattered wave has essentially no effect on the intensity of the primary signal so that the second term on the right side of (2.1) can be neglected compared with the first. In this case the solution of (2.1) is elementary, being given by

$$I_0(t, s, \rho, \omega, \Omega) = \Psi\left(t - \frac{s_0(\rho)}{c} - \frac{s - s_0(\rho)}{v(\omega)}, \ \rho, \ \omega, \ \Omega\right)$$

$$\times \exp\left[-\int_{s_0(\rho)}^{s} ds'\gamma_s\left(t - \frac{s - s'}{v(\omega)}, \ s' \ \rho, \ \omega, \ \Omega\right)\right]. \tag{2.5}$$

In order to estimate the characteristic distance $z_s = v(\omega)/\gamma_s$, in which there is an appreciable reduction in the intensity of the incident signal we calculate the damping factor γ_s under the assumption that the spectral energy density of the acoustic waves is independent of wave number, being given by $W_s = 3U_s/4\pi k_m^3$ when $k < k_m$ and vanishing when $k > k_m$. After some simple calculations we find

$$\gamma_s(\omega, \Omega) = \frac{3\pi}{8}\frac{\omega_e^2}{c}\frac{1}{k_m^3 r_{De}^2}\frac{U_s}{n_e m_e c^2} f\left(\sin^2\frac{\theta_s}{2}\right), \tag{2.6}$$

where $(\theta_s/2)$ is determined from (2.3) and

$$f(x) = \frac{1}{2\alpha}\left\{\left[1 + \frac{1}{2\alpha} + \frac{1}{8\alpha^2}\right]\ln(1 + 4\alpha x) - 2x\left[1 + \frac{1}{4\alpha}\right] + x^2\right\},$$

$$\alpha = [k(\omega) r_{De}]^2. \tag{2.7}$$

It follows that the damping factor γ_s is essentially independent of signal frequency in a wide frequency range from $\omega \gtrsim \omega_e$ to $\omega \approx c/2r_{De}$ and that it falls off inversely in proportion to ω^2 [(cf. Eq. (1.13)] when $\omega > c/2r_{De}$. This means that at low frequencies the scattering of transverse waves by acoustic waves can, in principle, be more important than scattering by plasma waves.

We now determine the intensity of the waves scattered at angles $\theta > \Delta\theta$. This problem can be of interest from the point of view of diagnostics of ion-acoustic noise in a plasma. If

$$\gamma_s[d + s\Delta\theta] \ll \theta_s, \quad \theta > \Delta\theta \tag{2.8}$$

simple estimates show that the integral equation (2.1) can be replaced by the simpler equations

$$\hat{D}I(\omega, \Omega) = \int d\Omega_1 [\gamma_s^{(+)}(\Omega, \Omega_1) I_0(\omega + \omega_s, \Omega_1) + \gamma_s^{(-)}(\Omega, \Omega_1) I_0(\omega - \omega_s, \Omega_1)], \tag{2.9}$$

where the function $I_0(\omega, \Omega)$ is the intensity of the primary signal in the plasma, being given by (2.5). The solution of (2.9) is elementary and will not be given here. We shall write an expression for the radiation flux $S(\omega, \Omega)$ propagating in the direction $\mathbf{n}$ through a plasma boundary Σ under the assumption that the incident signal is a beam of parallel rays, that the intensity of the noise W_s is constant over V the plasma volume occupied by the incident radiation, that it varies slowly in the time in which the signal propagates from one boundary of the plasma to the other, and finally that the plasma dimensions are much smaller than the characteristic distance $z_s = v(\omega)/\gamma_s$.

Substituting (2.5) in (2.9) taking account of (1.17), and integrating the right and left sides of (2.9) over the volume we find

$$S(\omega, \Omega) = \int_\Sigma I(\omega, \Omega)\mathbf{n}\, d\sigma = \frac{\pi}{16}\Delta z \frac{\omega_e^2}{c^2}\frac{r_{De}^{-2}}{1 + k_0^2 r_{De}^2}$$

$$\times \frac{W_s(\mathbf{k}_0)\bar{S}_0(\omega + \omega_s(\mathbf{k}_0)) + W_s(-\mathbf{k}_0)\bar{S}_0(\omega - \omega_s(\mathbf{k}_0))}{n_e m_e c^2}$$

$$\times [1 + (\mathbf{n}\mathbf{n}_0)^2], \tag{2.10}$$

where $\mathbf{k}_0 = (\mathbf{n}_0 - \mathbf{n})k(\omega)$; $\mathbf{n}_0$ and $\mathbf{n}$ are unit vectors in the direction of the incident beam and the scattered wave respectively,

$$\bar{S}_0 = \int_0 \Psi\mathbf{n}_0\, d\sigma\, d\Omega$$

is the total flux of incident radiation, while Δz is the characteristic length of the plasma along the incident beam, so that

$$\int_V \Psi d\mathbf{r} = \bar{S}_0 \Delta z.$$

In particular, it follows from (2.10) that the angular distribution of the scattered radiation is uniquely related to the spectral distribution of the ion-acoustic noise at the points $\mathbf{k} = \pm \mathbf{k}_0 (\omega, \Omega)$ while the spectrum is shifted with respect to the spectrum of the incident signal by an amount $\pm \omega_s(k_0)$ determined by (2.2); this shift increases with increasing angle of observation θ from zero at $\theta = 0$ to $\sim\theta$ at $\theta \approx \theta_s$.

As an example which allows us to estimate the intensity of the scattered signal, we find the ratio R_s of the total scattered radiation flux integrated over the angles θ and φ, to the incident flux $\overline{S} = \int S d\Omega$ for the case in which the spectral width of the signal $\Delta\omega \ll \overline{\omega}$ while the spectral energy density of the ion-acoustic waves is constant, being given by $W_s = 3U_s / 4\pi k_m^3$ for $k < k_m$ and zero for $k > k_m$. Integrating (2.10) with respect to the angle we find

$$R_s = \gamma_s \Delta z / c, \qquad (2.11)$$

where γ_s is given by (2.6).

In the table we give values of the ratio R_s corresponding to several different values of the plasma density n and mean signal frequency $\overline{\omega}$ for a hydrogen plasma of thickness $\Delta z = 10$ cm with a temperature ratio $T_e/T_i = 10$, a mean electron thermal velocity $u_{Te} = 3 \times 10^8$ cm/sec and $k_m \approx r_{Di}^{-1}$. These numbers show that for a sufficiently high level of ion-acoustic noise the intensity of the scattered waves also lies within the sensitivity of existing methods of measurement.

Experimental observation of the effects indicated above should, in principle, make available new possibilities for monitoring and diagnosing plasma noise.

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Translated by H. Lashinsky
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