OPEN ELECTRON TRAJECTORIES AND RESISTIVITY OSCILLATIONS IN ZINC

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The resistivity oscillation amplitude (with a period of 6.2×10^{-5}/Oe) as a function of magnetic field direction and measuring current is measured in pure zinc single crystals within magnetic fields up to 24 kOe at 1.3°K. The results indicate that the region of resistivity oscillations coincides with the region of magnetic field directions for which open cross sections of the Fermi surface arise in the basal plane. The oscillation amplitude is determined by electrons that belong simultaneously to open and closed zinc Fermi surfaces. The experimental results suggest that magnetic breakdown of the Fermi surface occurs in zinc.

EVIDENCE for the Shubnikov—de Haas effect (Sh-dH), i.e., resistivity oscillations in a magnetic field, was first reported in [1]. A small peak was observed at H ≈ 8 kOe on an otherwise smooth curve representing the dependence of resistivity on a magnetic field parallel to the [0001] direction.

Renton,[2] working with purer zinc samples, obtained a very clear picture of oscillatory resistivity peaks. The peak separation as a function of the reciprocal magnetic field was linear, thus indicating with certainty that Sh-dH exists in zinc. The oscillatory period was 6.0×10^{-5}/Oe. The occurrence of Sh-dH was subsequently confirmed by several investigators.[3-5]

It was concluded from a comparison of the periods in the de Haas—van Alphen and Shubnikov—de Haas effects[6,7] that the resistivity oscillations are associated with the so-called needle-shaped part of the zinc Fermi surface, which contains 5×10^{-6} electrons per atom. (This part comprises about one-millionth of the volume of the entire Fermi surface of zinc.)

Stark[8] was the first to point out that the amplitude of these oscillations is many orders of magnitude greater than any theoretical prediction. He suggested that the observed oscillations do not represent a Sh-dH effect but may be associated with magnetic breakdown of the zinc Fermi surface. Magnetic breakdown should occur between the needle-shaped surface in the third zone and the "monster" in the second zone.

The concept of magnetic breakdown, which was introduced by Cohen and Falicov,[9] is the following. While moving in a magnetic field on one part of the Fermi surface an electron has a non-vanishing probability of reaching another part of the Fermi surface separated from the first part by an energy gap. The corresponding trajectories can differ in principle from those existing without magnetic breakdown. For example, open trajectories can be formed from elements of closed trajectories and vice versa. Magnetic breakdown can greatly change the galvanomagnetic properties of metals, and this can influence our conclusions regarding the topology of the Fermi surface in the absence of a magnetic field.

According to the almost-free-electron model[10] and OPW calculations[11,12] the zinc Fermi surface should not have open cross sections parallel to the (0001) plane. However, experimental work has indicated that zinc has open trajectories in fields higher than 3 kOe.[13] This disagreement can be accounted for by magnetic breakdown on the Fermi surface between a needle and the monster, which are separated by a small energy gap.

Open cross sections of the Fermi surface and large-amplitude resistivity oscillations can thus have the same cause, magnetic breakdown. We can therefore expect to discover phenomena that will show directly the relationship between the aforementioned two properties of conduction electrons in zinc. The present work is an attempt to discover the interrelationship between open cross sections of the Fermi surface and resistivity oscillations in zinc.

1. SAMPLES AND MEASURING PROCEDURE

Since the foregoing effects are observed in a magnetic field having its direction close to a sixfold axis, in our galvanomagnetic measurements we investigated zinc single crystals where the

Angle*	Samples							
	Zn-1	Zn-2	Zn-3	Zn-4	Zn-5	Zn-6	Zn-7	Zn-8
φ, deg	2	15	25	30	10	20	30	30
θ_1, deg	90	90	90	90	82	84	68	62

*φ is the angle between the $(1\bar{2}10)$ plane and a plane that passes through the sample axis and [0001]. θ_1 is the angle between the sample axis and [0001]. Both angles were determined within $\pm 1°$.

measuring current flowed either parallel or nearly parallel to the basal plane.

The zinc single crystals were grown by the Obreimov-Shubnikov method or were cut from a large single crystal by spark erosion. The orientation of the sample axes with respect to the crystallographic axes was determined optically within $\pm 1°$. During the measurements the orientations of the samples could be determined precisely from the anisotropy of their resistance in the magnetic field. All samples were of about the same purity, which was characterized by the resistivity ratio between room temperature and 4.2°K: $\rho(300°K)/\rho(4.2°K) \approx 18{,}000$. The orientations of the sample axes are given in the accompanying table.

The samples were mounted conventionally for galvanomagnetic measurements at helium temperatures. Each sample was positioned approximately perpendicular to the field between the poles of an electromagnet. The orientation of the sample axis with respect to the magnetic field was controlled by a micrometer screw that could tip the cryostat from a vertical position. The maximum deviation of the sample axis from a position perpendicular to the magnetic field was $\sim 10°$. The behavior of resistivity in a magnetic field is not greatly changed at such small angles of inclination compared with the case $\mathbf{J} \perp \mathbf{H}$. Such inclinations and the rotation of the magnet in the horizontal plane enabled the use of the same sample to investigate two-dimensional regions of crystallographic directions close to [0001]. The signal from the potential leads of the sample was amplified by a F-18 instrument before being fed to the y-coordinate of the automatic register. The x-coordinate of the same register recorded a signal proportional to the magnetic field or to the rotation angle of the electromagnet. The magnetic field was introduced automatically at the rate of 2 kOe per minute; the magnet was rotated 8° per minute. Measurements were obtained at 1.3°K.

Resistivity oscillations are observed against a background of monotonic increase, with the exception of the case when $\mathbf{H} \parallel [0001]$. This background has hindered quantitative measurements of the resistivity oscillation amplitude. For the purpose of compensating the monotonic part of the resistance in a magnetic field the zinc sample was accompanied by a polycrystalline tin sample. The emf in the potential leads of the tin sample was in series with the measuring circuit of the zinc sample. The compensating emf was selected by varying the current through the tin. By obtaining the best compensating conditions during the process of measurement the oscillating resistance supplement $\Delta\rho_{osc}(H)$ was determined.

2. EXPERIMENTAL RESULTS

A. Dimensions of the stereographic projection of magnetic field directions associated with open Fermi surfaces of zinc. The stereographic projection of (special) magnetic field directions for which open cross sections of the Fermi surface appear in zinc was investigated in [13]. It was here shown that open cross sections of the Fermi surface appear when 1) the magnetic field is parallel to the basal plane, 2) the magnetic field is parallel to the $\{1\bar{1}00\}$ planes and the angle θ measured from the [0001] direction lies in the interval $6° < \theta < 42°$ (the so-called $\langle 1\bar{1}00\rangle$ "whiskers" in the stereographic projection of special magnetic field directions), and 3) the magnetic field is parallel to any direction for the angles $0° < \theta < 6°$ (the "two-dimensional region" of $\langle 0001\rangle$ directions).

It was later found that the two-dimensional region has a radius not greater than 2°, and for $\theta > 2°$ open cross sections remain only in the $\{1\bar{1}00\}$ and $\{1\bar{2}10\}$ planes; the length of the $\langle 1\bar{2}10\rangle$ whiskers does not exceed 6°.[4] However, Reed and Brennert[3] did not observe $\langle 1\bar{2}10\rangle$ whiskers. For our investigations it was important to know the dimensions of the $\langle 1\bar{2}10\rangle$ whiskers and of the $\langle 0001\rangle$ two-dimensional region. We therefore carefully measured the resistivity anisotropy of sample Zn-1. The axis of this sample was inclined at different angles from the position for which $\mathbf{J} \perp \mathbf{H}$; the declinations lay in the plane passing through the sample axis and [0001]. The most important

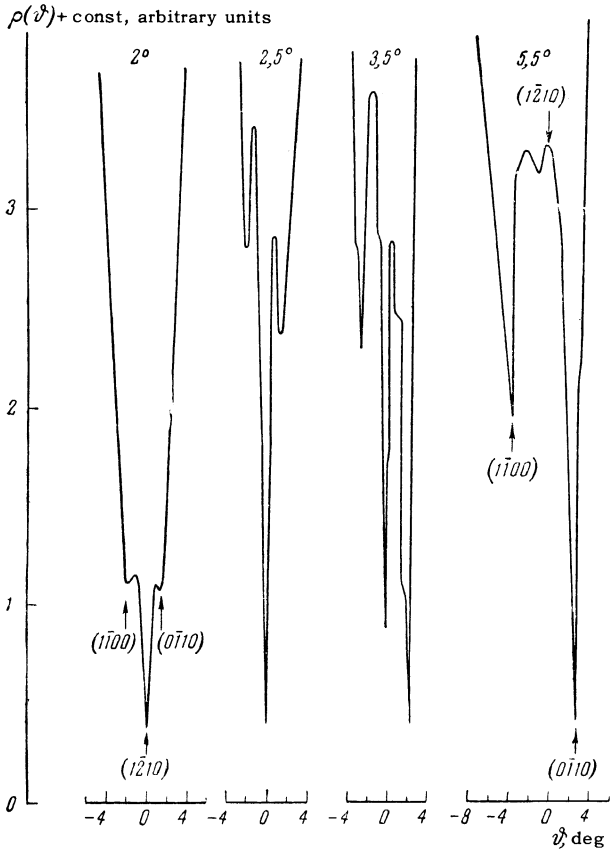

FIG. 1. Resistivity of sample Zn-1 vs. rotation angle of magnetic field. H = 10 kOe, T = 4.2°K. The curves are marked with the value of the angle between the (0001) plane and the magnetic field rotation axis. The curves are shifted by an arbitrary constant amount along the ordinate axis.

regions of the angular dependences of resistivity $\rho_H(\vartheta)$ for different angles of inclination are shown in Fig. 1, where ϑ is the rotation angle of the magnetic field, with $\vartheta = 0$ arbitrary in this and subsequent figures. The function $\rho_H(\vartheta)$ is seen to have narrow minima for the directions H ∥ $\{1\bar{1}00\}$ and H ∥ $(1\bar{2}10)$. For metals having an even number of conduction electrons per atom these minima arise for magnetic field directions leading to open cross sections of the Fermi surface. [14] The measurements for different angles of inclination showed that the resistivity minimum for H ∥ $\{1\bar{2}10\}$ exists only for $\theta \lesssim 5°$; the minima for **H** ∥ $\{1\bar{1}00\}$ can be observed clearly down to $\theta \approx 2°$. It was therefore concluded that the length of a $\langle 1\bar{2}10 \rangle$ whisker is 5° ± 0.5°, and that the radius of the two-dimensional region does not exceed 2°. [14]

We also note that the angular separation of the given minima from the minimum for **H** ∥ (0001) enables us in some instances to determine very accurately the orientation of the magnetic field axis of rotation with respect to the crystal axes. When the rotation axis of the magnetic field is parallel to the sample axis the orientation of the latter can be determined.

B. Resistivity oscillation amplitude vs. the angle θ when the rotation axis of the magnetic field is parallel to (0001). This case was realized in samples Zn-1, Zn-2, Zn-3, and Zn-4 for **J** ⊥ **H**. On the other hand, the same situation could be realized by a suitable inclination of the Zn-5 or Zn-6 axis in the magnetic field.

For the given case the angular dependence $\rho(\theta)$ has two deep narrow minima—when **H** ∥ (0001) and **H** ∥ [0001]. (The angles θ and ϑ coincide here.) Figure 2 shows ρ(H) measured for **H** ∥ [0001] and some magnetic field directions close to this. The oscillation amplitude decreases sharply as θ increases. For $\theta > 3°$ oscillations were revealed only by using compensation and enhancing the sensitivity of the measuring circuit; for $\theta > 5°$ resistivity oscillations were not observed within the limits of accuracy.

For the purpose of observing quantitatively the anisotropy of the oscillation amplitude, the oscillating supplement $\Delta\rho_{osc}$(H) was separated from ρ(H). In $\Delta\rho_{osc}$(H) it was most convenient to observe the changes of peak amplitudes in magnetic fields of about 3.2, 4.1, and 5.5 kOe. The amplitudes of these peaks are represented by A_5, A_4, and A_3 in

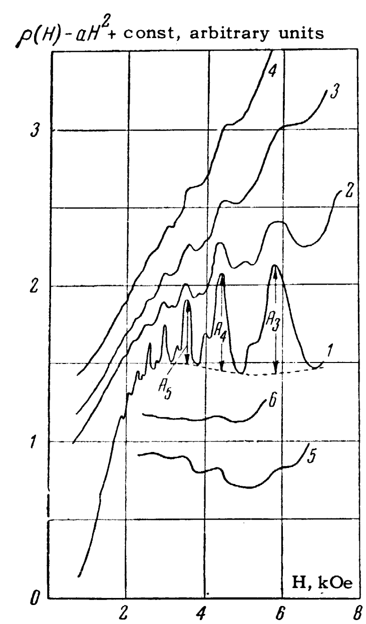

FIG. 2. Resistivity vs. magnetic field at 1.3°K. Curve 1 – sample Zn-5, **H** ∥ [0001]; curves 2, 3, 4, 5, 6 – sample Zn-1 with the magnetic field forming the angle 0, 0.5, 1, 2, or 3° with the [0001] direction. The sensitivity was identical for all curves. The measuring current was 0.25 A for curves 2–4, and 1 A for curves 5 and 6. The factor a was zero for curves 1–4.

Fig. 2; here the subscripts n = 3, 4, 5 etc. of A are the serial indices of the oscillation maxima as a function of H^{-1} (n = 0 for $H \rightarrow \infty$). The values of A_3, A_4, and A_5 were taken to be unity when **H** ∥ [0001], in which case they have the ratios 2.5:1.5:1. The dependence of A_3, A_4, and A_5 on the angles ϑ, θ and the direction of the current **J** was also investigated.

In our measurements the maximum value of θ did not exceed 28°. In the range $0° \le \theta \le 28°$ the period of the oscillations depends only slightly on θ. Therefore the amplitudes A_3, A_4, and A_5 were not corrected for the shifting of oscillation peaks in a magnetic field as θ increases. Also, A_3, A_4, and A_5 behave alike, except in a single case that will be considered here. Therefore the results shown in the figures pertain mainly to A_4.

Figure 3 shows the dependence of A_4 on θ for several zinc samples. It is seen that the anisotropic character of the oscillation amplitude does not depend on the current direction in the basal plane. In the present case the current direction is perpendicular to the rotation plane of the magnetic field.

C. Oscillation amplitude vs. the angle ϑ when the magnetic field rotation axis is not parallel to (0001). For this case we used samples whose axes were not parallel to (0001), as well as others in which the axes were parallel to (0001) but not parallel to the rotation axis of the magnetic field. As already mentioned, the latter situation is brought about by tipping the samples in the field.

The dependence of the oscillation amplitude on ϑ is characteriaed by the maxima of the $A_n(\vartheta)$ curves in Fig. 4. The maximum amplitudes are observed when the magnetic field is parallel to $\{1\bar{1}00\}$ and $\{1\bar{2}10\}$ planes. With deviation from these directions the oscillation amplitude falls off sharply. With an increasing angle between (0001) and the magnetic field rotation axis, the character of the anisotropy of $A_n(\vartheta)$ does not vary beginning at 5°, but the maxima of A_n decrease slowly. Thus when the angle between (0001) and the magnetic field rotation axis increases from 0° to 20°, the maxima of A_n are reduced about one-half for **H** ∥ $\{1\bar{1}00\}$.

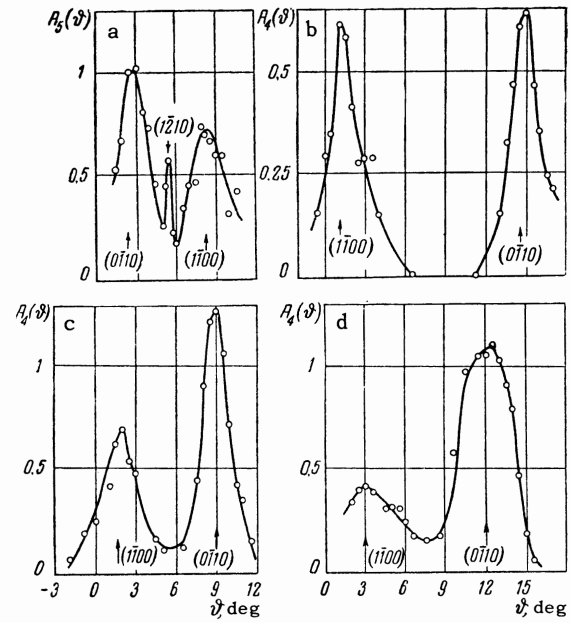

FIG. 4. Oscillation amplitude vs. rotation angle of magnetic field at 1.3°K. a – Sample Zn-1, θ_2 = 3.5° [the angle between the magnetic field rotation axis and the plane (0001)]; b – sample Zn-1, θ_2 = 10°; c – sample Zn-5, θ_2 = 5°; d – sample Zn-6, θ_2 = 6°.

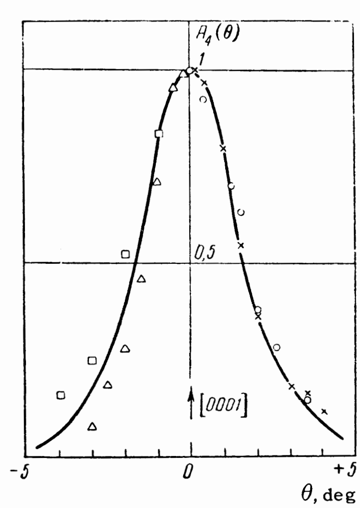

FIG. 3. Oscillation amplitude at H ≈ 4 kOe vs. the angle between the magnetic field and [0001] at 1.3°K for different samples: ○ – Zn-2, **H** ∥ (φ = 15°); × – Zn-5, **H** ∥ (φ = 20°); △ – Zn-1, **H** ∥ $(10\bar{1}0)$; □ – Zn-4, **H** ∥ $(1\bar{2}10)$.

Another characteristic of the $A_n(\vartheta)$ curves is the asymmetry which increases with the azimuthal angle φ defining the orientation of the sample axis.

Finally, we note that when the magnetic field approaches a $\{1\bar{2}10\}$ plane (for $2° < \theta < 5°$) the dependence of A_n on the magnetic field changes in character. For any magnetic field direction in general the oscillation amplitude increases monotonically with the field (Fig. 2). Exceptions occur for **H** ∥ $\{1\bar{2}10\}$ and for directions deviating from the plane by an angle ϑ not exceeding ±1.5°. Within the indicated limits of ϑ and θ, both the amplitude and location of each maximum of the $\Delta\rho_{osc}(H)$ curves depends on the angle between $\{1\bar{2}10\}$ and the magnetic field (Fig. 5). It can be seen that both the shape and relative height of the maximum of $A_n(\vartheta)$ for **H** ∥ $\{1\bar{2}10\}$ (Fig. 4a) will depend strongly on the index of the oscillation peak.

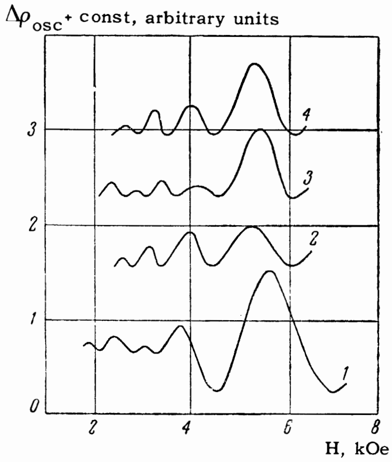

FIG. 5. Oscillating supplement to resistivity vs. magnetic field for sample Zn-1 at T = 1.3°K and θ_2 = 3.5° (see Fig. 4a). For curve 1, **H** ∥ (1$\bar{2}$10); for curves 2, 3, and 4 the magnetic field was tipped 0.4°, 0.8°, and 1.4°, respectively from (1$\bar{2}$10).

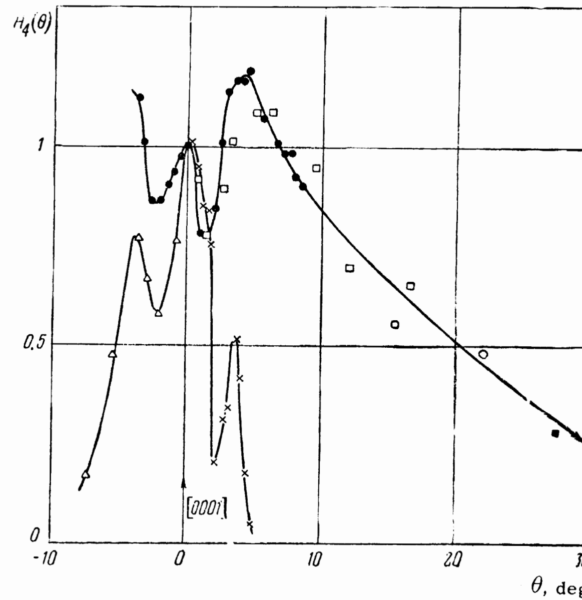

FIG. 6. Resistivity oscillation amplitude for H ≈ 4 kOe vs. the angle between the magnetic field and [0001] at 1.3°K for different samples: × – Zn-1, **H** ∥ (1$\bar{2}$10); ● – Zn-4, **H** ∥ (1$\bar{1}$00); □ – Zn-6, **H** ∥ (1$\bar{1}$00); ○ – Zn-7, **H** ∥ (1$\bar{1}$00); ■ – Zn-8, **H** ∥ (1$\bar{1}$00); △ – Zn-2, magnetic field parallel to the plane defined by φ = 15°. A_4 = 0 at θ ≈ 40 (extrapolated).

D. Oscillation amplitude vs. the angle θ in planes passing through [0001], H, and J. This case was realized for a small number of zinc samples by varying the orientation of the sample axis with respect to the magnetic field, i.e., by tipping the axis from the position for which J ⊥ H. This was done in planes passing through the sample axis and [0001]. These planes are defined uniquely by φ. The dependence of the oscillation amplitude on θ for some planes is shown in Fig. 6. $A_n(\theta)$ has two maxima in each of the {1$\bar{1}$00} and {1$\bar{2}$10} planes (for θ = 0° and θ ≈ 4°) and a minimum at θ ≈ 2°. For θ > 4°, in the {1$\bar{1}$00} plane $A_n(\theta)$ decreases monotonically as θ increases. At θ ≈ 40° resistivity oscillations evidently disappear in this plane. In the case of {1$\bar{2}$10} resistivity oscillations could not be observed even with θ ≈ 5°.

For planes at intermediate values of φ (0° < φ < 30°), $A_n(\theta)$ decreases monotonically as θ increases. For planes with φ close to 0° the $A_n(\theta)$ curves practically coincide with the case described in Paragraph B (with the exception of the case for φ = 0°).

E. Conclusions. Our results can be summarized as follows:

1. Resistivity oscillations in zinc occur within a narrow range of magnetic field directions close to the {1$\bar{1}$00} and {1$\bar{2}$10} planes. The maximum oscillation amplitudes are reached for **H** ∥ {1$\bar{1}$00} and **H** ∥ {1$\bar{2}$10}.

2. When the magnetic field direction departs from [0001] in a {1$\bar{1}$00} plane the oscillation amplitude depends strongly on the direction of the current J: a) When **J** ∥ {1$\bar{1}$00} the oscillation amplitude decreases slowly with growing θ (beginning with θ ≈ 4°), so that for θ ≈ 20° the oscillation amplitude is about one-half of its value for θ = 0°; b) when **J** ⊥ {1$\bar{1}$00}, the oscillation amplitude decreases sharply as θ increases. As θ increases from 0° to 3–4° the amplitude drops by a factor of about 10; as θ increases further, resistivity oscillations cannot be observed. Thus, for a fixed angle θ > 3° the oscillation amplitude in a {1$\bar{1}$00} plane reaches its maximum for **J** ∥ {1$\bar{1}$00} and its minimum for **J** ⊥ {1$\bar{1}$00}. This applies also to {1$\bar{2}$10} for 3° < θ < 5° and to directions departing not more than 3–4° from {1$\bar{1}$00}.

For directions of **J** lying between the two indicated directions the oscillation amplitude decreases smoothly with increase of the angle between the current and the plane passing through [0001] and the selected magnetic field direction. This results in asymmetric maxima on the $A_n(\vartheta)$ curves (Fig. 4).

3. When **J** ∥ (0001), then when **H** deviates from [0001] in any crystallographic plane passing through [0001], the oscillation amplitude drops to less than 1/10 of its value (at θ ≈ 4°). This is accounted for essentially by the considerations in Paragraphs Nos. 1 and 2.

3. DISCUSSION OF RESULTS

We shall here consider the resistivity oscillations having amplitudes not smaller than (1/10)th of their magnitude for **H** ‖ [0001]. Since the amplitudes were measured with accuracy not exceeding 5% of the value for **H** ‖ [0001], we have included practically all field directions for which oscillations were observed. These will be called "giant" oscillations.

The experimental results show that the region in which anisotropy exists and the amplitude of giant resistivity oscillations in zinc are determined by an open Fermi surface. Indeed, the magnetic field directions for which giant oscillations were observed coincide with the magnetic field directions for which open Fermi surface sections appear in the basal plane (Fig. 7), and the oscillation amplitude for a fixed direction of **H** depends essentially on the direction of the current **J**. This dependence characterizes only electrons having open trajectories on the Fermi surface.

This relationship between resistivity oscillations and open Fermi surfaces could be a trivial result of proportionality between the amplitudes of oscillating resistivity-tensor components and the components of its classical part. This is shown by considering the zinc electrical conductivity $\sigma_{\alpha\beta}$ in a magnetic field. Since the giant oscillations correspond in their period to the needle-shaped surface, in the Sh-dH effect it is reasonable to retain the quantum conductivity supplement $\Delta\sigma_{\alpha\beta}^{\text{needle}}$, which is associated only with this portion of the Fermi surface:

$$\sigma_{\alpha\beta} = \sigma_{\alpha\beta}{}^0 + \Delta\sigma_{\alpha\beta}^{\text{needle}} \quad (\alpha; \beta = x, y, z); \tag{1}$$

here $\sigma_{\alpha\beta}^0$ is the sum of classical parts of the conductivity tensors from both the closed and open pieces of the Fermi surface.

To evaluate $\Delta\sigma^{\text{needle}}$ we shall use Eq. (6.23) of [15] for one surface in the case of small quantum numbers:

$$\Delta\sigma \sim \sigma_0\left(\frac{\hbar\omega}{E_F}\right)\omega\tau, \tag{2}$$

where σ_0 is the classical conductivity (proportional to the number N of electrons, E_F is the Fermi energy, $h\omega$ is the separation of Landau levels, ω is the frequency of revolution in a closed orbit, and τ is the relaxation time. (We shall henceforth assume that τ is identical for all pieces of the zinc Fermi surface.)

For a needle surface in fields $H \approx 5$ kOe we have $h\omega/E_F \sim 1/3$ and $\omega\tau \sim 10^3$. Then

$$\Delta\sigma^{\text{needle}} \approx 10^3\, \sigma_0{}^{\text{needle}}, \tag{3}$$

i.e., for one piece of the surface we actually have $\Delta\sigma \gg \sigma_0$. However, for zinc the total conductivity σ_0 is proportional to the total number N_0 of electrons and should considerably exceed σ_0^{needle}, since

$$\sigma_0^{\text{needle}}/\sigma_0 \sim N^{\text{needle}}/N_0 \approx 10^{-6}.$$

We can therefore assume for the conductivity tensor components

$$\Delta\sigma_{\alpha\beta}^{\text{needle}} \ll \sigma_{\alpha\beta}{}^0. \tag{4}$$

In view of the foregoing we write the oscillation additions to the resistivity tensor components as follows: [16]

$$\Delta\rho_{\alpha\beta} = (\mathrm{Det}\,|\rho_{\alpha\beta}{}^0|\, \varepsilon_{\alpha kl}\varepsilon_{\beta pq}\sigma_{kp}{}^0 - \rho_{\alpha\beta}{}^0\rho_{eq}{}^0)\,\Delta\sigma_{eq}, \tag{5}$$

where ϵ_{ikl} is the third order antisymmetric unit tensor and $\rho_{\alpha\beta}^0$ is the classical part of the resistivity tensor.

For $\Delta\rho_{xx}$ we have

$$\Delta\rho_{xx} = \mathrm{Det}\,|\rho_{\alpha\beta}{}^0|\,(\sigma_{yz}{}^0\Delta\sigma_{zy} - \sigma_{yy}{}^0\Delta\sigma_{zz} + \sigma_{zy}{}^0\Delta\sigma_{yz} - \sigma_{zz}{}^0\Delta\sigma_{yy}) + (\rho_{xx}{}^0)^2(\Delta\sigma_{xx} + \Delta\sigma_{xy} + \Delta\sigma_{xz}). \tag{6}$$

When open and closed sections of the Fermi surface exist simultaneously in high magnetic fields [17] ($\gamma_0 = 1/\omega\tau \ll 1$), then

$$\mathrm{Det}(\rho_{\alpha\beta}{}^0) \sim \gamma_0{}^{-2}, \quad (\rho_{xx}{}^0)^2 \sim \gamma_0{}^{-4}.$$

Therefore the first term in (6) can be neglected,

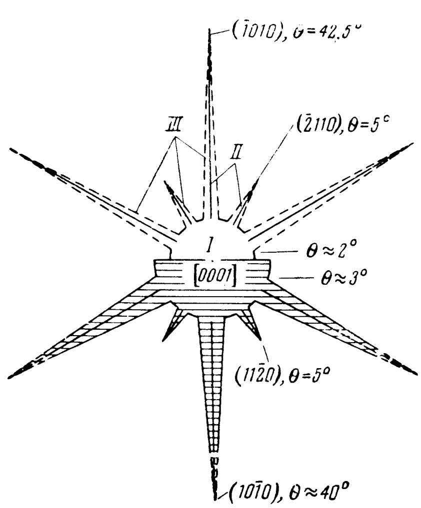

FIG. 7. The upper half of this figure is a projection, on the basal plane, of the magnetic field directions associated with open cross sections of the zinc Fermi surface: I – two-dimensional region, II – whiskers, III – region of greatly elongated cross sections. The lower (shaded) half is a projection, on the basal plane, of the region of magnetic field directions for which "giant" resistivity oscillations were observed.

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$$\Delta\rho_{xx} \approx (\rho_{xx}{}^0)^2(\Delta\sigma^{xx} + \Delta\sigma^{xy} + \Delta\sigma^{xz}). \tag{7}$$

Since ρ_{xx}^0 depends mainly on an open surface we can expect the latter to influence the Sh-dH oscillation amplitude. However, two circumstances prevent us from assuming that the usual Sh-dH effect occurs in zinc:

1. We estimate the relative value of $\Delta\rho_{xx}$ from (7):

$$\Delta\rho_{xx}/\rho_{xx}{}^0 \approx \Delta\sigma_{xx}^{\text{needle}}/\sigma_{xx}{}^0 \ll 1.$$

From the experimental results obtained by Stark [4] and in the present work we have

$$\Delta\rho_{xx} / \rho_{xx}{}^0 \approx 1/3.$$

2. When open and closed cross sections of the Fermi surface exist simultaneously the dependence of resistivity on the angle α between the current and the mean direction of open cross sections is given by [17]

$$\rho_{xx}{}^0 = BH^2\cos^2\alpha + C. \tag{8}$$

Equation (7) shows that the dependence of the resistivity oscillation amplitude on α results only from the dependence of ρ_{xx}^0 on α. Therefore the maximum and minimum oscillation amplitudes should be reached at the maximum and minimum values of the resistivity (for a given magnetic field direction), i.e., $\alpha = 0°$ and $90°$, respectively. We observed exactly the opposite effect experimentally; the maximum of $\Delta\rho_{xx}^{osc}$ is reached at the minimum resistivity (when $\alpha = 90°$), and the minimum value is reached for maximum resistivity (when $\alpha = 0°$).

We shall assume that giant resistivity oscillations in zinc are associated with electrons belonging to open and closed Fermi surface regions simultaneously. In other words, the oscillation amplitude and its dependence on the H and J directions are associated with magnetic breakdown on the zinc Fermi surface. Magnetic breakdown should occur only between a closed needle and an open Fermi surface region, if the open surface exists without the magnetic field, or between needles and the monster, if Harrison's model is correct. In the first case magnetic breakdown should considerably increase the density of open trajectories in the basal plane. In the second case open trajectories should appear; this amounts to a fundamental change in the topology of the zinc Fermi surface in a magnetic field.

At the present time we have no way of determining the topology of the Fermi surface in the absence of a magnetic field. However, both cases are equivalent for a qualitative evaluation of the experimental findings. For the sake of simplicity we shall consider the second case: magnetic breakdown occurs across a small energy gap between needles and the monster.

Blount has given the probability for an electron transition between two trajectories separated by a gap E_g that is independent of H: [18]

$$W = \exp(-\pi m^* c E_g{}^2 / 8\hbar e H E_F). \tag{9}$$

Stark [8] has commented that in a magnetic field quantization of electron orbits and the appearance of Landau levels should result in an oscillating dependence of the gap width E_g on H. This would result in oscillations of the probability of electron transitions and therefore in an oscillating magnetic-field dependence of resistivity. The oscillatory period would in the first approximation be determined by the needle-shaped surface.

We shall now evaluate qualitatively the amplitude of resistivity oscillations and its dependence on the directions of the magnetic field and the current. For this purpose we shall consider the idealized case in which for a minimum gap width we have $0 < W < 1$, and for a maximum gap we have $W \approx 0$. To determine the accompanying variation of resistivity we use Eq. (8) of [14]:

$$\rho = \rho_0 \frac{D\cos^2\alpha + \lambda_1\gamma_0{}^2}{[(\Delta V/V + CD)^2 + C'D]\gamma_0{}^2 + \lambda\gamma_0{}^4}, \tag{10}$$

where $\Delta V = V_1 - V_2$ is the discompensation of electron and hole volumes of the Fermi surface, resulting from open or greatly extended sections, with $V = (V_1 + V_2)/2$; C, C′, λ, and λ_1 are of the same order of magnitude; D is proportional to the thickness of the open trajectory layer.

When $W \approx 0$ the basal plane contains no open trajectories: $D = 0$ and $\Delta V = 0$. Then

$$\rho = \rho_0 \frac{\lambda_1}{\lambda} \frac{1}{\gamma_0{}^2} = B_1 H^2, \tag{11}$$

where B_1 = const. For $0 < W < 1$ open trajectories exist in the basal plane only in certain directions determined by the projections shown in Fig. 7. Since $D \neq 0$ and $V_1 \neq V_2$, the resistivity for these field directions can be represented by

$$\rho \approx B_2 H^2 \cos^2\alpha + C_2. \tag{12}$$

For other directions we have, as previously, $\rho = B_1H^2$. This shows that resistivity oscillations will exist only for magnetic field directions leading to open Fermi surface sections; in this case the oscillation amplitude will be

$$A_n \approx (B_1 - B_2\cos^2\alpha)H_n{}^2. \tag{13}$$

The maximum and minimum amplitudes will be reached with $\alpha = 90°$ and $\alpha = 0°$, respectively.

Similar properties should be exhibited by regions of H directions for which highly elongated closed electron trajectories exist. (These regions exist near the whiskers of the projection in Fig. 7.) The oscillation amplitude should here depend strongly on the angle ϑ_1 between open and elongated sections. This is associated with the fact that in the narrow region $\vartheta \leq \gamma_0$ the highly elongated closed trajectories do not permit compensation of V_1 and V_2. In the angular region $\vartheta \approx \gamma_0$ the expression for the resistivity becomes (11) instead of (12), so that the oscillation amplitude (for $\alpha = 90°$) must have a narrow peak at $\vartheta = 0$. The half-width of this maximum will decrease as 1/H when the magnetic field is increased.

We also have the special case H ∥ [0001], when there are no open trajectories even for $W \neq 0$, with $V_1 \neq V_2$. Then ρ_{xx} = const and we have the oscillation amplitude $A_n \approx B_1 H_n^2$.

The ideal case here considered thus accounts qualitatively for the observed anisotropy, the amplitude, and the dependence on current direction that are exhibited by resistivity oscillations in zinc. In any real case, of course, $0 < W < 1$ in a magnetic field and both open and closed electron trajectories always exist. This results in a complex magnetic field dependence of the coefficients B_1 and B_2, but does not alter fundamentally the foregoing qualitative explanation of giant resistivity oscillations in zinc. We note furthermore that the complicated form of $A_n(\theta)$ for $0° < \theta < 4°$ (Fig. 6) is evidently associated with the fact that the density of open trajectories and the width of the energy gap depend on θ.

We shall finally consider briefly the nonmonotonic growth of the resistivity oscillation amplitude that is observed near the $\langle 1\bar{2}10 \rangle$ whisker. Figure 2 shows that the resistivity oscillation minima are split. This can be accounted for by spin splitting of Landau levels. Calculations by Bennett and Falicov[19] have yielded the value 89 of the g-factor for electrons of the needle; the dependence on θ is here given as $g(\theta) \sim \cos\theta$. Our measurements at small angles θ in directions approximately parallel to $\{1\bar{1}00\}$ showed that the characteristic shape of the oscillation curves is independent of both θ and φ. However, near $\{1\bar{2}10\}$ the oscillation curves are greatly modified; one maximum disappears while others grow. This behavior of the oscillation maxima must be associated with a strong dependence of the g-factor of needle electrons on φ. This in turn must be associated with anisotropy of the needle surface in the basal plane. However, further experimental work is needed here.

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179