EFFECT OF PARTICLE COLLISIONS ON PLASMA FLUCTUATIONS

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Plasma fluctuations are analyzed with account taken of binary particle collisions. The binary collisions are described within the framework of the kinetic equation through the use of a model collision integral which satisfies conservation of particles, momentum and energy (the Bhatnagar-Gross-Krook integral). A single component non-isothermal plasma is investigated. The fluctuation-dissipation theorem is used to find a general expression for the correlation function for the random forces. Taking account of binary collisions between particles results in an additional correlation of the random forces in velocity space. General expressions are obtained for the spectral distribution of the particle density fluctuations and the temperature fluctuations. The dependence of the fluctuation spectrum on particle density, temperature, and binary collision frequency is investigated. The relation between fluctuations in a collisionless plasma and fluctuations in hydrodynamics is also studied, as is the scattering of electromagnetic waves by fluctuations of density and temperature. It is shown that temperature fluctuations have an important effect on long wavelength scattering characterized by small frequency changes.

THE electrodynamic properties of plasma have been studied in detail in the limiting cases represented by low or high values of the binary collision frequency. In the first case the plasma is usually described by means of a kinetic equation in which the binary collision integral is neglected. In the second case the properties of the plasma are described by the hydrodynamic equations.

It is of considerable interest to study the electrodynamic properties of a plasma in the intermediate case represented by arbitrary values of the binary collision frequency and to establish the relation between the two limiting cases indicated above. A basic difficulty in taking account of the effect of binary collisions on the electrodynamic properties of a plasma lies in the complexity of the solution of the kinetic equation when the correct collision integral is used. The problem can be simplified considerably if a model collision integral is used in the kinetic equation. A model collision integral of this kind, which satisfies conservation of particles, momentum and energy, was proposed by Bhatnagar, Gross, and Krook, who investigated the effect of binary collisions on the dispersion of longitudinal oscillations in a plasma.[1]

In the present paper, using the kinetic equation with a model collision integral we study the effect of binary collisions on plasma fluctuations.

Plasma fluctuations have been considered earlier for the collisionless case,[2] and in the hydrodynamic case, in which the collision frequency is very large.[3] The introduction of a model collision integral makes it possible to study plasma fluctuations for arbitrary values of the effective binary collision frequency.

The plasma motion will be described by the kinetic equation for the electron distribution function $f(\mathbf{v}, \mathbf{r}, t)$:

$$\frac{\partial f}{\partial t}+\mathbf{v}\frac{\partial f}{\partial \mathbf{r}}+\frac{e}{m}\mathbf{E}\frac{\partial f}{\partial \mathbf{v}}=\left[\frac{\partial f}{\partial t}\right], \tag{1}$$

in which the collision integral is written in the form proposed by Bhatnagar, Gross and Krook

$$\left[\frac{\partial f}{\partial t}\right]=\nu\frac{n}{n_0}\{n\Phi-f\},$$
$$\Phi=\left(\frac{m}{2\pi T}\right)^{3/2}\exp\left[-\frac{m(\mathbf{v}-\mathbf{u})^2}{2T}\right]; \tag{2}$$

where $n(\mathbf{r}, t)$, $\mathbf{u}(\mathbf{r}, t)$ and $T(\mathbf{r}, t)$ are the density, macroscopic velocity, and temperature of the plasma electrons, which are given respectively by

$$n=\int f d\mathbf{v}, \quad \mathbf{u}=\frac{1}{n}\int \mathbf{v} f d\mathbf{v}, \quad T=\frac{m}{3n}\int (\mathbf{v}-\mathbf{u})^2 f d\mathbf{v}, \tag{3}$$

n_0 and T_0 are the equilibrium values of the electron density and temperature, ν is the effective frequency of binary collisions, which will be assumed to be independent of the relative velocities of the colliding particles.

In the Bhatnagar-Gross-Krook model it is assumed that particles emanating from a given point r at time t have a Maxwellian velocity distribution centered about the macroscopic velocity **u** and temperature T. The loss of particles at a given point in phase space as a consequence of binary collisions is treated in (2) in the same way as in the usual Boltzmann integral. It is easy to verify that the collision integral in (2) conserves particles, energy and momentum and that it also satisfies an H theorem.

The kinetic equation (1) is supplemented by the equation for the self-consistent field **E**:

$$\operatorname{div}\mathbf{E} = 4\pi e(n - n_0). \tag{4}$$

The ion motion in the plasma is neglected since the ions appear only as a means of balancing the equilibrium electric charge. The plasma motion is described completely by the system of equations (1) and (4).

The electromagnetic fluctuations in the plasma with collisions taken into account are investigated by means of the fluctuation-dissipation theorem;[4] on the right side of the kinetic equation we introduce a random force $y(\mathbf{v}, \mathbf{r}, t)$:

$$\frac{\partial f}{\partial t} + \mathbf{v}\frac{\partial f}{\partial \mathbf{r}} + \frac{e}{m}\mathbf{E}\frac{\partial f}{\partial \mathbf{v}} = \left[\frac{\partial f}{\partial t}\right] + y. \tag{5}$$

Having taken as the generalized thermodynamic velocity $\dot{x}$ the function

$$\dot{x} = \left[\frac{\partial f}{\partial t}\right] + y$$

we find the corresponding generalized thermodynamic force X from the relation

$$X = -\frac{\partial \dot{S}}{\partial \dot{x}},$$

where $\dot{S}$ is the time derivative of the entropy of the system

$$\dot{S} = -\int\left\{\left[\frac{\partial f}{\partial t}\right] + y\right\}\ln f\, d\mathbf{v}\, d\mathbf{r}. \tag{6}$$

We limit ourselves to fluctuations close to thermodynamic equilibrium and assume that the deviations of quantities from their equilibrium values are small. In this case the thermodynamic velocity $\dot{x}$ is a linear function of the thermodynamic force X:

$$\dot{x} = -\hat{\gamma}X + y, \quad \hat{\gamma}X = \int\gamma(\mathbf{v}, \mathbf{v}')X(\mathbf{v}')\,d\mathbf{v}'.$$

Introducing the kinetic coefficient γ in this relation directly determines the correlation function for the random forces

$$\langle y(\mathbf{v}, \mathbf{r}, t)\, y(\mathbf{v}', \mathbf{r}', t')\rangle = 2\gamma(\mathbf{v}, \mathbf{v}')\,\delta(\mathbf{r} - \mathbf{r}')\,\delta(t - t'). \tag{7}$$

Assuming that the fluctuation deviation δf from the equilibrium value of a distribution function f_0 is small and using (6) and (2) we find

$$\gamma(\mathbf{v}, \mathbf{v}') = \nu\Big\{f_0(v)\,\delta(\mathbf{v} - \mathbf{v}') - \frac{1}{n_0}\left[1 + \frac{m}{T_0}\mathbf{v}\mathbf{v}' + \frac{3}{2}\left(1 - \frac{mv^2}{3T_0}\right)\times\left(1 - \frac{mv'^2}{3T_0}\right)\right]f_0(v)f_0(v')\Big\}. \tag{8}$$

The spectral distribution of the correlation function for the random forces is then given by the expression

$$\langle y(\mathbf{v})\,y(\mathbf{v}')\rangle_{k\omega} = 2\gamma(\mathbf{v}, \mathbf{v}'). \tag{9}$$

Using the kinetic equation (5) and the equation for the self-consistent field (4) the fluctuation deviation of the distribution function and the consequent fluctuations in density and temperature in the plasma can be expressed in terms of the random forces; we can then carry out an averaging over the random forces by means of (9). Thus, from (4) and (5) the space-time component of the Fourier fluctuations in the density δn and temperature δT are found from

$$\begin{aligned} \alpha_{11}\delta n_{k\omega}/n_0 + \alpha_{12}\delta T_{k\omega}/T_0 &= Y^1_{k\omega}, \\ \alpha_{21}\delta n_{k\omega}/n_0 + \alpha_{22}\delta T_{k\omega}/T_0 &= Y^2_{k\omega}, \end{aligned} \tag{10}$$

where the right sides of the relations are determined by the random forces

$$Y_{k\omega}{}^1 = \frac{i}{n_0}\int\frac{y_{k\omega}(\mathbf{v})\,d\mathbf{v}}{\omega - \mathbf{k}\mathbf{v} + i\nu}, \qquad Y_{k\omega}{}^2 = \frac{im}{n_0T_0}\int\frac{v^2y_{k\omega}(\mathbf{v})\,d\mathbf{v}}{\omega - \mathbf{k}\mathbf{v} + i\nu}, \tag{11}$$

while the coefficients α_{ij} are given by

$$\alpha_{11} = 1 - \frac{i\nu}{\omega + i\nu}J - 2\frac{\Omega^2 + i\nu\omega}{(\omega + i\nu)^2}z^2(J - 1),$$

$$\alpha_{12} = \frac{1}{2}\frac{i\nu}{\omega + i\nu}[J - 2z^2(J - 1)],$$

$$\alpha_{21} = 3 - 2\frac{i\nu}{\omega + i\nu}[(z^2 + 1)J - z^2] - 2\frac{\Omega^2 + i\nu\omega}{(\omega + i\nu)^2}[2(z^2 + 1)J - 2z^2 - 3]z^2,$$

$$\alpha_{22} = 3 - \frac{i\nu}{\omega + i\nu}[(2z^4 + z^2 + 1)J - 2z^4 - 2z^2],$$

$$\Omega^2 = \frac{4\pi n_0e^2}{m}, \quad z = x + iy = (\omega + i\nu)/\sqrt{2}kv_T,$$

$$v_T = \sqrt{T_0/m}. \tag{12}$$

The function J(z) which appears in (12) is given by

$$J(z)=\frac{z}{\sqrt{\pi}}\int_{-\infty}^{+\infty}\frac{e^{-t^2}}{z-t}\,dt. \qquad (13)$$

We now solve the system in (10) for δn and δT, making use of (11) and (9); in this way we obtain the following general formulas for the spectral distributions of the fluctuations in density and temperature in a plasma with binary collisions between particles taken into account:

$$\langle\delta n^2\rangle_{k\omega}=\frac{|\alpha_{22}|^2\langle Y_1^*Y_1\rangle_{k\omega}-2\,\mathrm{Re}\,\alpha_{12}\alpha_{22}^*\langle Y_1^*Y_2\rangle_{k\omega}+|\alpha_{12}|^2\langle Y_2^*Y_2\rangle_{k\omega}}{|\alpha_{11}\alpha_{22}-\alpha_{12}\alpha_{21}|^2},$$

$$\langle\delta T^2\rangle_{k\omega}=\frac{T_0^2}{n_0^2}\times\frac{|\alpha_{11}|^2\langle Y_2^*Y_2\rangle_{k\omega}-2\,\mathrm{Re}\,\alpha_{11}\alpha_{21}^*\langle Y_1^*Y_2\rangle_{k\omega}+|\alpha_{21}|^2\langle Y_1^*Y_1\rangle_{k\omega}}{|\alpha_{11}\alpha_{22}-\alpha_{12}\alpha_{21}|^2},$$

$$\langle\delta n\delta T\rangle_{k\omega}=\frac{T_0}{n_0}\,\frac{\alpha_{22}\alpha_{11}^*\langle Y_2^*Y_1\rangle_{k\omega}-\alpha_{12}\alpha_{11}^*\langle Y_2^*Y_2\rangle_{k\omega}-\alpha_{22}\alpha_{21}^*\langle Y_1^*Y_1\rangle_{k\omega}+\alpha_{12}\alpha_{21}^*\langle Y_1^*Y_2\rangle_{k\omega}}{|\alpha_{11}\alpha_{22}-\alpha_{12}\alpha_{21}|^2}. \qquad (14)$$

The correlation functions $\langle Y_i^*Y_j\rangle_{k\omega}$ appearing in (14), like the coefficients α_{ij}, are expressed in terms of the function J(z):

$$\langle Y_1^*Y_1\rangle_{k\omega}=\sqrt{2}\frac{n_0}{kv_T}\Big\{\mathrm{Re}\,i\frac{J}{z}-y\Big[\Big|\frac{J}{z}\Big|^2+2|J-1|^2+\frac{1}{6|z|^2}|(2z^2-1)J-2z^2|^2\Big]\Big\},$$

$$\langle Y_1^*Y_2\rangle_{k\omega}=\sqrt{2}\frac{n_0}{kv_T}\Big\{\mathrm{Re}\frac{2i}{z}[(z^2+1)J-z^2]-y\Big[\frac{2}{|z|^2}J^*[(z^2+1)J-z^2]+2(J-1)^*[2(z^2+1)J-2z^2-3]+\frac{1}{3|z|^2}[(2z^2-1)J-2z^2]^*[(2z^4+z^2+1)J-2z^4-2z^2]\Big]\Big\},$$

$$\langle Y_2^*Y_2\rangle_{k\omega}=\sqrt{2}\frac{n_0}{kv_T}\Big\{\mathrm{Re}\frac{2i}{z}[2(z^4+2z^2+2)J-4z^4-5z^2]-2y\Big[\frac{2}{|z|^2}|(z^2+1)J-z^2|^2+|2(z^2+1)J-2z^2-3|^2+\frac{1}{3|z|^2}|(2z^4+z^2+1)J-2z^4-2z^2|^2\Big]\Big\}. \qquad (15)$$

The requirement that the expression in the denominator of (14) vanish represents the dispersion equation for longitudinal oscillations in the plasma:

$$\Big(3-2i\frac{y}{z}J\Big)\Big[1-i\frac{y}{z}J-(p+2ixy)(J-1)\Big]+\frac{i}{2}\frac{y}{z}(1+p-2x^2)[2z^2(J-1)-J]=0,$$

$$p=\Omega^2/k^2v_T^2.$$

The relation in (14) allows us to investigate the effect of particle collisions on the nature of the plasma fluctuations over the whole range of values of effective collision frequencies starting from the collisionless case and going up to the hydrodynamic limit.1) Neglecting collisions the spectral distributions for the fluctuations in density and temperature in a plasma are given by

$$\langle\delta n^2\rangle_{k\omega}=\sqrt{2\pi}\frac{n_0}{kv_T}\frac{e^{-z^2}}{|1-p(J-1)|^2},$$

$$\langle\delta T^2\rangle_{k\omega}=\sqrt{2\pi}\frac{T_0^2}{n_0kv_T}e^{-z^2}\times\Big\{4(z^4+2z^2+2)|1-p(J-1)|^2-4\,\mathrm{Re}(z^2+1)\{3-p[2(z^2+1)J-2z^2-3]\}[1-p(J-1)]^*+|3-p[2(z^2+1)J-2z^2-3]|^2\Big\}\times\Big\{|1-p(J-1)|^2\Big\}^{-1} \qquad (16)$$

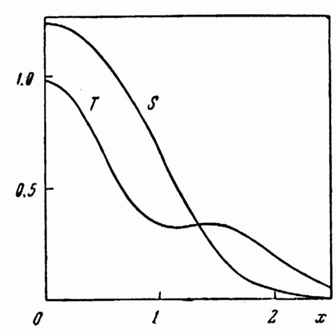

FIG. 1. Spectral distributions of the correlations of density and temperature in a collisionless plasma for $p = 0.2$ ($p = 1/a^2k^2$, $S(x) = \langle\delta n^2\rangle_{kx} \times (2n_0)^{-1}$, $T(x) = n_0\langle\delta T^2\rangle_{kx}/2T_0^2$, $x = \omega/\sqrt{2}kv_T$).

In the shortwave case ($p = (ak)^{-2} \ll 1$, a is the Debye radius) these distributions are characterized by wide maxima at zero frequency. In Fig. 1 we show the spectral distribution of the fluctuations in a collisionless plasma for the case $p = 0.2$. In the longwave region ($p \gg 1$) the

*Density fluctuations in a fluid have been studied in[5] on the basis of a kinetic equation with a model collision integral. Hydrodynamic fluctuations have also been studied in[6].

spectral distributions (16) are characterized by delta function peaks at the frequency corresponding to the longitudinal plasma oscillations $\omega_p = (\Omega^2 + 3k^2v_T^2)^{1/2}$; the spectral distribution of the temperature fluctuations has the same sharp peak at zero frequency, which is related to the entropy waves in the plasma.

Assuming that the frequency of binary collisions is small and expanding (14) in powers of ν we can find the correction to the distribution (16) due to binary collisions. The expressions for the spectral distributions (14) are also simplified considerably in the hydrodynamic limit (large values of the effective collision frequency). Expanding (14) in inverse power of ν we find

$$\langle\delta n^2\rangle_{k\omega} = \frac{4n_0}{\nu} \times \frac{(6\omega^2 + 5k^2v_T^2)\,k^2v_T^2}{|3\omega(\omega^2 - \Omega^2 - {}^5/_3 k^2v_T^2) + i\nu^{-1}(9\omega^2 - 5\Omega^2 - 5k^2v_T^2)\,k^2v_T^2|^2},$$

$$\langle\delta T^2\rangle_{k\omega} = \frac{4T_0^2}{n_0\nu} \times \frac{[5(\omega^2 - \Omega^2 - k^2v_T^2)^2 + {}^8/_3\,\omega^2k^2v_T^2]\,k^2v_T^2}{|3\omega(\omega^2 - \Omega^2 - {}^5/_3 k^2v_T^2) + i\nu^{-1}(9\omega^2 - 5\Omega^2 - 5k^2v_T^2)\,k^2v_T^2|^2},$$

$$\langle\delta n\delta T\rangle_{k\omega} = \frac{4T_0}{\nu} \times \frac{[5(\omega^2 - \Omega^2 - k^2v_T^2) + 4\omega^2]\,k^4v_T^4}{|3\omega(\omega^2 - \Omega^2 - {}^5/_3 k^2v_T^2) + i\nu^{-1}(9\omega^2 - 5\Omega^2 - 5k^2v_T^2)\,k^2v_T^2|^2}. \tag{17}$$

The relation in (17) can also be obtained directly from the hydrodynamic system of equations for the fluctuations given by Landau and Lifshitz.[3] We note that the hydrodynamic system of equations [3] can itself be obtained from a kinetic equation (5) by the moment method if the hydrodynamic random forces are defined by the relations

$$s_{ij} = \frac{m}{\nu}\int v_iv_jy\,d\mathbf{v}, \quad g_i = \frac{m}{2\nu}\int v^2v_iy\,d\mathbf{v}. \tag{18}$$

According to [3] the correlation function for the hydrodynamic random forces is expressed in terms of the coefficients of viscosity and thermal conductivity. Finding the correlation functions for s_{ij} and g_i by means of (9) and (8) and making a comparison with the hydrodynamic formulas it is easy to determine the coefficients of viscosity and thermal conductivity for the Bhatnagar-Gross-Krook model:

$$\eta = n_0T_0/\nu, \quad \zeta = 0, \quad \varkappa = 5n_0T_0/2m\nu. \tag{19}$$

Now, in (17) we allow the effective collision frequency to approach infinity ($\nu \to \infty$), obtaining the formula for the spectral distributions for the ideal hydrodynamic case:

$$\langle\delta n^2\rangle_{k\omega} = 2\pi n_0\frac{k^2v_T^2}{\Omega^2 + k^2v_s^2}\left\{\frac{k^2v_T^2}{\Omega^2 + k^2v_T^2}\left(\frac{c_p}{c_v} - 1\right)\delta(\omega) + \frac{1}{2}[\delta(\omega - \omega_s) + \delta(\omega + \omega_s)]\right\},$$

$$\langle\delta T^2\rangle_{k\omega} = 2\pi\frac{T_0^2}{mn_0c_v}\,\frac{k^2v_s^2}{\Omega^2 + k^2v_s^2}\left\{\frac{\Omega^2 + k^2v_T^2}{k^2v_s^2}\delta(\omega) + \frac{1}{2}\left(1 - \frac{c_v}{c_p}\right)[\delta(\omega - \omega_s) + \delta(\omega + \omega_s)]\right\},$$

$$\langle\delta n\delta T\rangle_{k\omega} = 2\pi\frac{T_0}{mc_v}\,\frac{k^2v_T^2}{\Omega^2 + k^2v_s^2}\left\{-\delta(\omega) + \frac{1}{2}[\delta(\omega - \omega_s) + \delta(\omega + \omega_s)]\right\},$$

$$c_p = 5/2m, \quad c_v = 3/2m, \quad v_s^2 = v_T^2c_p/c_v. \tag{20}$$

According to (20) the spectral distributions for the fluctuations are characterized by peaks at frequencies corresponding to acoustic oscillations $\omega_s = (\Omega^2 + k^2v_s^2)^{1/2}$ and at zero frequency, corresponding to entropy waves in the plasma.

It is interesting to note that the spectral distributions of simultaneous fluctuations of density and temperature in the plasma are the same for the case of high collision frequencies and for the collisionless case;

$$\langle\delta n^2\rangle_k = n_0\frac{k^2v_T^2}{k^2v_T^2 + \Omega^2}, \quad \langle\delta T^2\rangle_k = \frac{T_0^2}{mn_0c_v},$$
$$\langle\delta T\delta n\rangle_k = 0. \tag{21}$$

It follows from the last relation in (21) that the fluctuations in density and temperature are statistically independent, in accordance with well-known results.[4]

In Fig. 4 we show the spectral distributions for the fluctuations in density and temperature for intermediate values of the effective collision frequency for various values of the parameter p = 0; 1; 4; 10. According to these curves, taking account of collisions in the shortwave region leads to the appearance of the acoustic maximum in the spectrum of density fluctuations; the magnitude of this peak increases with increasing collision frequency. The width of the peak is determined by the viscosity and thermal conductivity of the plasma. In the long-wave case, taking account of collisions leads to a displacement of the Langmuir peak in the density fluctuation spectrum in the direction of low frequencies. This shift is due to the change in the nature of the dispersion of the high-frequency waves in the plasma caused by particle collisions.

In contrast with the isothermal case, which is usually treated, it is found that temperature fluctuations in a plasma are very important, especially in the low-frequency region. It follows from (20) that when $y \gg 1$ the ratio of $T(\omega = 0)$ to $S(\omega = 0)$ is $(1 + p)^2$; in the longwave region $(p > 1)$ this ratio is considerably greater than unity. The effect of the relative depression of the level of low-frequency density fluctuations is due to the compensation of charge fluctuations at frequencies far from the plasma frequency. The width of the peak in the low-frequency fluctuations is determined by the coefficient of thermal conductivity of the plasma.

The existence of fluctuations in density and temperature implies the possibility of scattering of electro-magnetic waves propagating in the plasma. If we assume that the frequency of the incident wave ω_0 is much greater than the collision frequency ν and take account of the explicit dependence of the transverse dielectric constant of the plasma on density and temperature, it is an easy matter to derive the following expression for the current which induces the scattered radiation:

$$\mathbf{J}_{k'\omega'} = \frac{i\Omega^2}{4\pi\omega_0}\left[\left(1 + \frac{k_0^2 v_T^2}{\omega_0^2}\right)\frac{\delta n_{q\Delta\omega}}{n_0} + \frac{k_0^2 v_T^2}{\omega_0^2}\,\frac{\delta T_{q\Delta\omega}}{T_0}\right]\mathbf{E}_{k_0\omega_0}. \tag{22}$$

Here, $\mathbf{E}_{k_0\omega_0}$ is the amplitude of the incident wave, $\Delta\omega$ and q are the change in frequency and wave vector due to scattering. Using (22) we can express the coefficient for scattering of electromagnetic waves in the plasma directly in terms of the spectral distributions of the fluctuations in density and temperature:

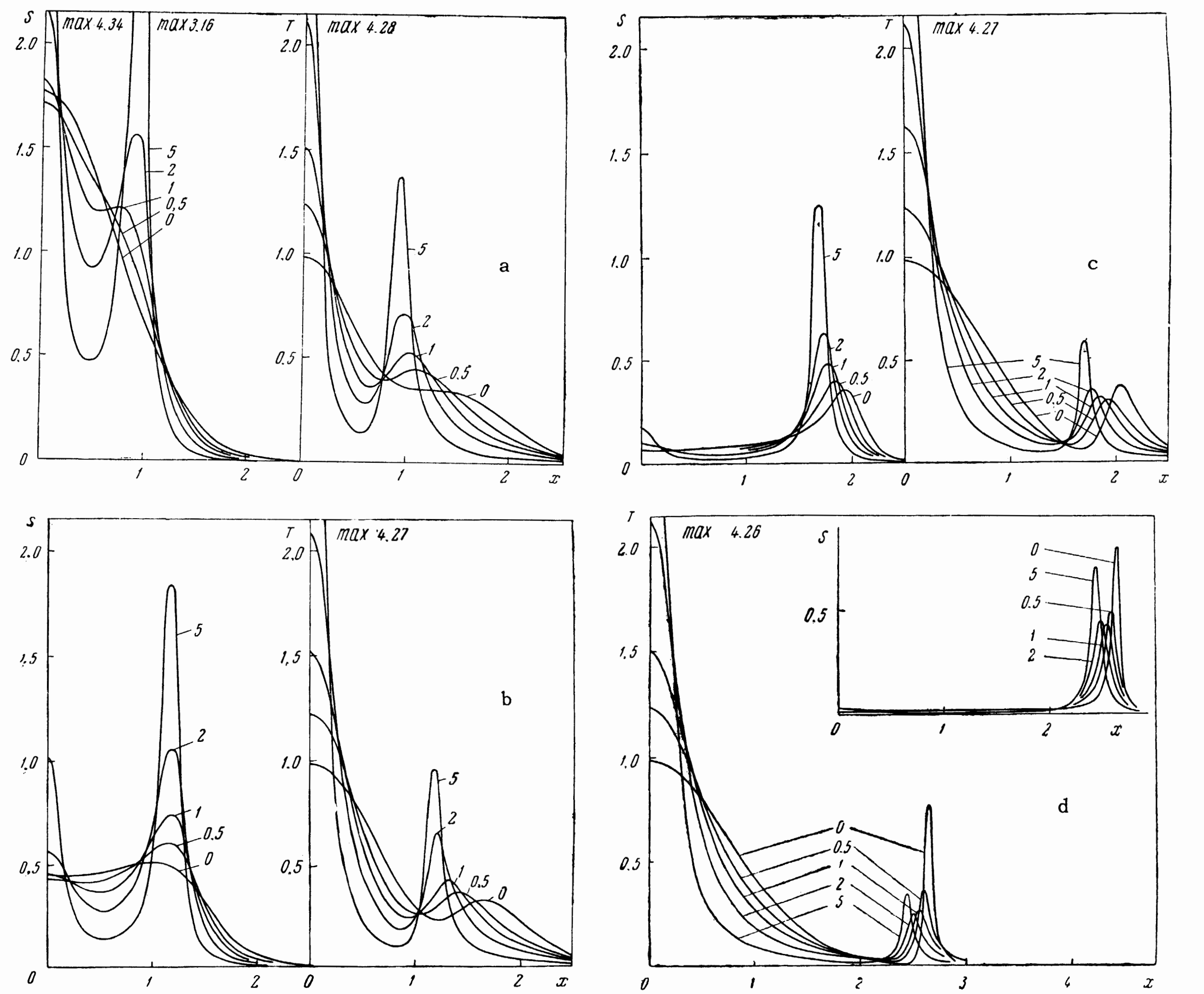

FIG. 2. Spectral distributions S(x, y) and T(x, y) as a function of collision frequency. The parameter $y = \nu/\sqrt{2}kv_T$ assumes the following values 0; 0.5; 1; 2; 5 for the cases a) p = 0; b) p = 1; c) p = 4; d) p = 10.

$$\frac{d\Sigma}{do\,d\omega'}=\frac{1}{4\pi}\left(\frac{e^2}{mc^2}\right)^2\left(\frac{\omega'^2\varepsilon(\omega')}{\omega_0^2\varepsilon(\omega_0)}\right)^{1/2}(1+\cos^2\vartheta)$$
$$\times\left\{\left(1+\frac{k_0^2v_T^2}{\omega_0^2}\right)^2\langle\delta n^2\rangle_{q\Delta\omega}\right.$$
$$+2\,\mathrm{Re}\frac{n_0}{T_0}\frac{k_0^2v_T^2}{\omega_0^2}\left(1+\frac{k_0^2v_T^2}{\omega_0^2}\right)\langle\delta n\delta T\rangle_{q\Delta\omega}$$
$$\left.+\left(\frac{n_0}{T_0}\frac{k_0^2v_T^2}{\omega_0^2}\right)^2\langle\delta T^2\rangle_{q\Delta\omega}\right\}, \tag{23}$$

where ϑ is the scattering angle.

In order to investigate the effect of temperature fluctuations on scattering of waves in a plasma we consider shortwave ($p \lesssim 1$) and long-wave fluctuations separately. If $p \lesssim 1$ the scattering occurs the same way as in a neutral gas[4] and we can assume $\Delta\omega \ll \omega_0$. In this case (23) and (21) can be used to determine the scattering cross-section integrated over frequency:

$$\frac{d\Sigma}{do}=\frac{n_0}{2}\left(\frac{e^2}{mc^2}\right)^2(1+\cos^2\vartheta)\left[\left(1+\frac{k_0^2v_T^2}{\omega_0^2}\right)^2\frac{1}{1+p}\right.$$
$$\left.+\frac{2}{3}\left(\frac{k_0^2v_T^2}{\omega_0^2}\right)^2\right],$$
$$p=\frac{\Omega^2}{q^2v_T^2}. \tag{24}$$

The second term in (24) is due to the temperature fluctuations, whose contribution is small because $k_0^2v_T^2/\omega_0^2 \ll 1$ in the incident wave. However, in the short-wave region the fluctuations in density and temperature are of the same order, as follows from Fig. 1, and the weak dependence of the dispersion of the electromagnetic waves on temperature is due to the insignificant effect of temperature on scattering.

When $p \gg 1$ it is found that the plasma effects are important for scattering. If the plasma is cold and dense the fluctuations can be hydrodynamic in nature. In the limit of ideal hydrodynamics, on the basis of (23) and (20) we obtain the following expression for the scattering of electromagnetic waves on isobaric fluctuations of the entropy:

$$\frac{d\Sigma}{do\,d\omega'}=n_0\left(\frac{e^2}{mc^2}\right)^2(1+\cos^2\vartheta)\frac{(1-pk_0^2v_T^2/\omega_0^2)^2}{(1+p)(3+5p)}\delta(\omega'-\omega_0). \tag{25}$$

In the longwave case ($q^2v_T^2$, $k_0^2v_T^2 \ll \Omega^2$) the expression $pk_0^2v_T^2/\omega_0^2$, which corresponds to the contribution of the temperature in the cross-section, reaches values of the order of unity. Thus, when $p \gg 1$ the relative rise in the level of low-frequency temperature fluctuations can have an important effect on the scattering of transverse waves characterized by small frequency changes. In particular, it follows from (25) that the angular distribution of the scattered radiation is changed:

$$\left.\frac{d\Sigma}{do}\right|_{\omega'\approx\omega_0}\sim(1+\cos^2\vartheta)(1+\varepsilon(\omega_0)-2\cos\vartheta)^2. \tag{26}$$

The results given in (24)–(26) are independent of the collision model. At finite collision frequencies the scattering cross-section for an arbitrary change of frequency will be described by (23), with the spectral distributions of the fluctuations given by the curves in Fig. 2.

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