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BREMSSTRAHLUNG FROM THE ELECTRONS OF A MEDIUM AND HARD VAVILOV-CERENKOV RADIATION

V. V. BATYGIN

M. I. Kalinin Leningrad Polytechnic Institute

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We consider the additional bremsstrahlung from a charged particle, due to interaction of the particle with the long wave electromagnetic field in matter and with the electron ensemble of the outer shells. The latter effect is taken into account via the transverse and longitudinal permeabilities and permittivities of the medium. General expressions are derived for the probability of emission of a hard photon accompanied by emission or absorption of a soft photon. Two-quantum Cerenkov radiation is considered in detail for the case when one of the emitted photons is hard and the other is soft. This radiation is a new mechanism of hard-photon production. It is much weaker than bremsstrahlung but should nevertheless be observable. The correction to bremsstrahlung of electrons on electrons, obtained by taking into account electron coupling in matter, is calculated for a certain model of the electron plasma. The correction may be as large as 50%.

1. INTRODUCTION

In the usual Cerenkov effect, no hard quanta can be emitted, since the refractive index of the medium is $n \leq 1$ for such quanta. However, hard quanta with energy approaching that of a radiating ultrarelativistic particle can occur in the two-quantum Cerenkov effect, accompanied by soft quanta whose refractive index is $n > 1$.[1] Such hard radiation is essentially similar to bremsstrahlung. The difference is that the excess momentum is taken up not by an individual nucleus or electron, as in the ordinary bremsstrahlung, and not by an aggregate of nuclei, as in the case of bremsstrahlung at ultrahigh energies,[2-4] but by the ensemble of electrons of the medium and by the long-wave field in the substance. The energy and momentum of the charged particle passing through the substance is transferred in this case to two photons—a hard vacuum photon, and a soft one that interacts essentially with the charges of the medium. The emission probability of these photons can be expressed in terms of the retarded Green's function of the long-wave radiation in the medium[5] (Sec. 2). The latter is connected with the permittivities and permeabilities of the medium.[6]

If the medium is transparent, the soft transverse photon leaves the medium together with a hard one. We are then faced with a clear cut manifestation of a new, Cerenkov mechanism of production of hard photons when fast charged particles pass through a medium. This effect is observable under certain conditions (Sec. 3). If the medium is an absorbing one for the soft quantum, then the energy is transferred from the particle to the hard photon and to the medium as a whole.

Notice should be taken of the important role played by spatial dispersion of the dielectric constant when hard photons are emitted in conjunction with longitudinal soft quanta. If the momentum transferred to the medium is small and its order of magnitude is that of the characteristic momenta of the electrons of the medium, then the dynamic properties of the electron ensemble play an essential role. The excitations of the medium that receives such a momentum will have a collective character. On the other hand, if the momentum transferred to this medium is large compared with the aforementioned characteristic momenta, then the excitation of the medium will have a single-particle character, constituting simply the excitation of an individual electron. In the first of these cases we deal with two-quantum Cerenkov emission (hard quantum plus soft longitudinal quantum), in the second with bremsstrahlung on the valence electrons of the medium. Both effects are described by a single theory with a dielectric constant that takes spatial dispersion into account.

The hard radiation produced when a charged particle passes through a medium containing a large number of soft photons was considered in several papers.[7, 8, 1] We are considering here the produced two-quantum emission, both spontaneous and induced, which therefore always accompanies the ordinary bremsstrahlung—unlike the radiation considered in [7, 8, 1].

2. PROBABILITY OF EMISSION OF A HARD QUANTUM IN A MEDIUM

We shall separate $A^h(x)$ and $A^s(x)$ of the electromagnetic field, the hard and soft parts

$$A_\mu(x) = A_\mu^s(x) + A_\mu^h(x), \quad (1)$$

where $x = (\mathbf{x}, it)$ and $\mu = 1, 2, 3, 4$. The hard part is the sum over the photon states with momentum $\mathbf{k}$ larger in order of magnitude than the reciprocal of the lattice constant k_0. For such values of $\mathbf{k}$, the photon states that enter in $A^h(x)$ can be regarded as vacuum states with energy $k = |\mathbf{k}|$ ($h = c = 1$). The soft part includes long-wave states of the electromagnetic field with wave vectors $|\mathbf{q}| \lesssim k_0$. These states are strongly influenced by interactions with the medium whose energies are $\omega_\mathbf{q} \neq |\mathbf{q}|$ in the transparency regions, where the photon concept is meaningful; on the other hand, in the absorption regions, the photon concept becomes generally meaningless.

We shall regard the long-wave field together with the medium as a single system with Hamiltonian H_0, so that

$$A_\mu^s(x) = e^{iH_0t} A_\mu^s(\mathbf{x})\, e^{-iH_0t}.$$

The long-wave electromagnetic wave satisfies Maxwell's macroscopic equations and the quantum commutation relations.[9] We shall assume the transverse and longitudinal dielectric constants $\varepsilon_t(\mathbf{q},\omega)$ and $\varepsilon_l(\mathbf{q},\omega)$ which enter in these equations, or the equivalent constants $\varepsilon(\mathbf{q},\omega)$ and $\mu(\mathbf{q},\varepsilon)$, to be known phenomenological functions of $\mathbf{q}$ and of the frequency ω. The field of the atomic nuclei and of the internal electron shells is assumed, as usual, to be non-quantized and is not included in $A_\mu^s(x)$. By medium we mean here the aggregate of outer-shell electrons situated in the averaged field of the nuclei, and of the inner-electron shells. We shall not consider here effects connected with the influence of fields of individual atomic remnants.

The interaction between a fast fermion $\psi(x)$ passing through the medium and the field $A_\mu(x)$ is described by a scattering matrix

$$S = T \exp\Big[- \int d^4x N[\bar{\psi}(x)\gamma_\mu\psi(x)A_\mu(x)]\Big],$$

where γ_μ are Dirac matrices and e is the fermion charge. We are interested in the second-order matrix element, describing the transition of the "medium + long-wave field" system from an initial state i to a final state f, with emission of a hard photon having a momentum $\mathbf{k}$ and polarization e_μ^λ ($\lambda = 1, 2$), and scattering of the particle from the state with momentum $\mathbf{p}_0$ and spin r_0 to the state $\mathbf{p}$, r:

$$\begin{aligned} S_{f\mathbf{p}\mathbf{k},\, i\mathbf{p}_00} = {} & 2^{-1}e^2 \int d^4x_1 \int d^4x_2 T\,(\mathbf{p}\,|\,N\bar{\psi}(x_1)\gamma_\mu\psi(x_1) \\ & \times N\bar{\psi}(x_2)\gamma_\nu\psi(x_2)\,|\,\mathbf{p}_0)\, T\,(f\mathbf{k}\,|\,A_\mu^h(x_1)\,A_\nu^h(x_2) \\ & + A_\mu^s(x_1)\,A_\nu^s(x_2) + A_\mu^s(x_1)A_\nu^h(x_2) + A_\mu^h(x_1)\,A_\nu^s(x_2)\,|\,i0). \end{aligned} \quad (2)$$

The first term in (2) describes the emission of two hard quanta by a free particle, and is forbidden by the conservation laws. The second term describes the Cerenkov effect with emission of two soft photons, in which we are not interested. The contribution of the hard photon to the Cerenkov radiation is due to the last two terms.

Equation (2) contains matrix elements of the long-wave field in a homogeneous medium, which can be written in the form (see [10])

$$(A_\mu^s(x))_{fi} = (A_\mu^s(0))_{fi}e^{-i(\mathbf{q}\mathbf{x}-\omega t)}, \quad (3)$$

where $\mathbf{q} = \mathbf{P}_f - \mathbf{P}_i$ and $\omega = E_f - E_i$ are the momentum and energy transferred from the particle to the "medium + long-wave field" system; $\mathbf{P}_i$, $\mathbf{P}_f$ and E_i, E_f are the initial and final momentum and energy of this system. The momentum and energy are carried away by the soft photons in the transparency region, and by the medium as a whole in the absorption region.

Using (3), we obtain the matrix element

$$\begin{aligned} S_{f\mathbf{p}_0\mathbf{k},\, i\mathbf{p}_00} = {} & \left(\frac{2\pi}{k}\right)^{1/2} (2\pi)^4 e^2 \{\bar{u}_{\mathbf{p}r}\hat{e}_\lambda S^c(p_0 - q)\gamma_\mu u_{\mathbf{p}_0r_0}(A_\mu^s(0))_{fi} \\ & + (A_\nu^s(0))_{fi}\, \bar{u}_{\mathbf{p}r}\gamma_\nu S^c(p_0 - k)\hat{e}_\lambda u_{\mathbf{p}_0r_0}\}\, \delta^4(p_0 - p - k - q) \\ \equiv {} & M_{f\mathbf{p}\mathbf{k},\, i\mathbf{p}_00}\delta^4(p_0 - p - k - q) \end{aligned} \quad (4)$$

and the probability of emission of a single photon per unit path

$$dw_{\mathbf{k}\mathbf{q}} = \frac{1}{2v}\sum_{\substack{if\\ rr_0\lambda}} |M_{f\mathbf{p}\mathbf{k},\, i\mathbf{p}_00}|^2 \rho_i \frac{d\mathbf{k}\,d\mathbf{q}}{(2\pi)^6}\,\delta^4(p_0 - p - k - q), \quad (5)$$

averaged over the initial velocities of the medium with density matrix $\rho(H_0)$, whose diagonal matrix element is $\rho_i = \exp[(F - E_i)/T]$, where T is the temperature of the medium; the particle is polarized and its velocity is v.

Formula (5) includes, besides $|M|^2$, a bilinear combination of matrix elements of the long-wave field, which can be expressed (see [5,6]) in terms of the retarded Green's function D^R of the long-wave field:

$$D_{\mu\nu}^R(\mathbf{q},\omega)=-i\int d\mathbf{x}\int_0^\infty dt\exp[-i\mathbf{q}\mathbf{x}+i(\omega+i\delta)t]$$
$$\times\operatorname{Sp}\{\rho(H_0)[A_\mu(x)A_\nu(0)-A_\nu(0)A_\mu(x)]\},$$

where $\delta\to+0$. Going over in this expression to expansion in terms of the eigenfunctions and taking the imaginary part, we obtain

$$\sum_{if}\rho_i(A_\nu^s(0))_{fi}^*(A_\mu^s(0))_{fi}\delta^4(p_0-p-k-q)$$
$$=\frac{N_\omega+1}{\pi(2\pi)^3}g_\nu(-\operatorname{Im}D_{\mu\nu}^R(\mathbf{q},\omega)),$$
$$\mathbf{q}=\mathbf{p}_0-\mathbf{p}-\mathbf{k},\qquad\omega=E_0-E-k,\tag{6}$$

where $g_\nu=+1$ when $\nu=1, 2, 3$ and $g_\nu=-1$ when $\nu=4$, $N_\omega=(e^{\omega/T}-1)^{-1}$.

As is well known,[6] by using a transverse gauge for which div $\mathbf{A}=0$ we get

$$D_{\mu\nu}^R(\mathbf{q},\omega)=(1-\delta_{\mu4})(1-\delta_{\nu4})(\delta_{\mu\nu}-q_\mu q_\nu/\mathbf{q}^2)D_t(\mathbf{q},\omega)$$
$$-\delta_{\mu4}\delta_{\nu4}D_l(\mathbf{q},\omega).\tag{7}$$

The transverse and longitudinal parts D_t and D_l of the Green's function are expressed in terms of the constants ε_t, ε_l or ε,μ, viz.,

$$D_t(\mathbf{q},\omega)=\frac{4\pi}{\omega^2\varepsilon_t(\mathbf{q},\omega)-\mathbf{q}^2}=\frac{4\pi\mu(\mathbf{q},\omega)}{\omega^2\varepsilon(\mathbf{q},\omega)\mu(\mathbf{q},\omega)-\mathbf{q}^2}$$
$$D_l(\mathbf{q},\omega)=\frac{4\pi}{\mathbf{q}^2\varepsilon_l(\mathbf{q},\omega)}=\frac{4\pi}{\mathbf{q}^2\varepsilon(\mathbf{q},\omega)}.\tag{8}$$

In accordance with (6) and (7), the probability (5) of emission of a hard vacuum photon with momentum $\mathbf{k}$ and transfer to the medium of a momentum $\mathbf{q}$ and an energy ω, summed over the final polarization states of the radiating charged particle and the photons and averaged over the initial states of the medium and over the initial polarization state, takes the form

$$dw_{\mathbf{kq}}=dw_{\mathbf{kq}}^t+dw_{\mathbf{kq}}^l.\tag{9}$$

Here $dw_{\mathbf{kq}}^t$ and $dw_{\mathbf{kq}}^l$ are the probabilities of spontaneous and stimulated emission of a hard photon by the particle, due to the interaction with the medium via the transverse and longitudinal long-wave fields. In the case of a transparent medium, these will be the probabilities of emission of a hard photon accompanied by a transversely or longitudinally polarized soft photon.

We present the final expressions:

$$dw_{\mathbf{kq}}^t=\frac{e^4(N_\omega+1)\,d\mathbf{k}\,d\mathbf{q}}{2(2\pi)^5kEE_0}\mathscr{L}_t[-\operatorname{Im}D_t(\mathbf{q},\omega)],\tag{10a}$$

$$dw_{\mathbf{kq}}^l=\frac{e^4(N_\omega+1)\,d\mathbf{k}\,d\mathbf{q}}{(2\pi)^5kEE_0}\mathscr{L}_l[-\operatorname{Im}D_l(\mathbf{q},\omega)];\tag{10b}$$

$$\mathscr{L}_t=\frac{1}{\varkappa_1^2\varkappa_2^2}\Big\{4(\varkappa_1+\varkappa_2)^2-4\varkappa_1\varkappa_2(\varkappa_1+\varkappa_2)-\varkappa_1\varkappa_2(\varkappa_1^2+\varkappa_2^2)$$
$$+2(\eta^2-\nu^2)[\varkappa_1\varkappa_2(\varkappa_1+\varkappa_2)-(\varkappa_1^2+\varkappa_2^2)]$$
$$-2(\eta^2-\nu^2)^2\varkappa_1\varkappa_2+\frac{\eta^2-\nu^2}{\eta^2}\Big[-\frac{4}{\alpha^2}[\varkappa_1(1-\xi)+\varkappa_2]^2$$
$$+2\frac{\nu}{\alpha}(1-\xi)\varkappa_1\varkappa_2^2+4\frac{\nu}{\alpha}(\varkappa_1+\varkappa_2)^2-4\frac{\nu}{\alpha}\xi\varkappa_1(\varkappa_1+\varkappa_2)$$
$$-2\frac{\nu}{\alpha}\varkappa_1^2\varkappa_2-2\nu^2\varkappa_1\varkappa_2^2+\frac{1}{2}\varkappa_1\varkappa_2(\varkappa_1^2+\varkappa_2^2)$$
$$-\frac{2}{\alpha^2}[1+(1-\xi)^2](\eta^2-\nu^2)\varkappa_1\varkappa_2$$
$$+\frac{2}{\alpha}(2-\xi)\nu(\eta^2-\nu^2)\varkappa_1\varkappa_2+(\eta^2-\nu^2)(\varkappa_1+\varkappa_2)$$
$$\times(\varkappa_1+\varkappa_2-\varkappa_1\varkappa_2)+(\eta^2-\nu^2)^2\varkappa_1\varkappa_2\Big]\Big\};\tag{11}$$

$$\mathscr{L}_l=\frac{1}{\varkappa_1^2\varkappa_2^2}\Big\{-\frac{8}{\alpha^2}[\varkappa_1(1-\xi)+\varkappa_2]^2+8\frac{\nu}{\alpha}(\varkappa_1+\varkappa_2)^2$$
$$+4\frac{\nu}{\alpha}\varkappa_1\varkappa_2(\varkappa_2-\varkappa_1)-8\frac{\nu\xi}{\alpha}\varkappa_1(\varkappa_1+\varkappa_2)$$
$$-4\frac{\nu\xi}{\alpha}\varkappa_1\varkappa_2(\varkappa_1+\varkappa_2)+\varkappa_1\varkappa_2(\varkappa_1^2+\varkappa_2^2)-4\nu^2\varkappa_1\varkappa_2^2$$
$$+(\eta^2-\nu^2)\Big[-4\frac{1+(1-\xi)^2}{\alpha^2}\varkappa_1\varkappa_2+8\frac{\nu}{\alpha}\varkappa_1\varkappa_2$$
$$+2(\varkappa_1+\varkappa_2)^2-2\varkappa_1\varkappa_2(\varkappa_1+\varkappa_2)\Big]+2(\eta^2-\nu^2)^2\varkappa_1\varkappa_2\Big\};\tag{12}$$

$$\alpha=m/E_0,\qquad\eta=\mathbf{q}/m,\qquad\nu=\omega/m,\qquad\xi=k/E_0;$$

$$\varkappa_1=\frac{(p_0-q)^2+m^2}{m^2}=\frac{2(E_0\omega-\mathbf{p}_0\mathbf{q})+\mathbf{q}^2-\omega^2}{m^2},$$
$$\varkappa_2=\frac{(p_0-k)^2+m^2}{m^2}=\frac{2(E_0k-\mathbf{p}_0\mathbf{k})}{m^2}.\tag{13}$$

A contribution to (10)-(13) is made by the absorption regions, in which

$$\varepsilon_{t,l}=\varepsilon'_{t,l}(\mathbf{q},\omega)+i\varepsilon''_{t,l}(\mathbf{q},\omega),$$

as well as by the transparency regions, in which $\varepsilon''\to+0$ and the imaginary parts of the Green's functions are δ-like. The probability $dw_{\mathbf{kq}}$ in (9) represents an addition to the ordinary bremsstrahlung $dw_{\mathbf{kq}}^{\mathrm{br}}$,[10] due to the interaction of the particle with the field of the atomic nuclei and the internal shells.

There is a certain connection between the quantities η and ν in formulas (10)-(13); this connection follows from the energy and momentum conservation laws. Namely, the energy transferred to the medium is

$$\nu=\frac{\omega}{m}=\frac{E_0-E-k}{m}=\frac{1-\xi}{\alpha}$$

$$-\frac{1}{m}[(\mathbf{p}_0-\mathbf{k}-\mathbf{q})^2+m^2]^{1/2}=\eta_{\|}-\frac{\xi}{2(1-\xi)}\frac{\alpha^2+\vartheta^2}{\alpha}$$

$$-\frac{\xi}{1-\xi}\vartheta\eta_\perp\cos\varphi-\frac{\alpha\eta_\perp^2}{2(1-\xi)}, \tag{14}$$

where $\eta_{\|}=\eta\cos\theta$, $\eta_\perp=\eta\sin\theta$ are the components of $\boldsymbol{\eta}$ parallel and perpendicular to $\mathbf{p}_0$, θ is the angle between $\mathbf{p}_0$ and $\boldsymbol{\eta}$, ϑ is the angle between $\mathbf{p}_0$ and $\mathbf{k}$, and φ is the angle between the planes $(\mathbf{p}_0, \mathbf{k})$ and $(\mathbf{p}_0, \boldsymbol{\eta})$. In the derivation of (14) we have assumed that the particle is ultrarelativistic ($\alpha \ll 1$, $\vartheta \ll 1$) and that $\eta \lesssim 1$.

The derivation presented here, together with formulas (9)–(14), is applicable also when the hard radiation is the result of extracting a momentum $\mathbf{q}$ and an energy ω from the medium. The only difference lies in the substitution

$$q\to -q,\quad \mathbf{q}\to-\mathbf{q},\quad \omega\to-\omega$$
$$(\boldsymbol{\eta}\to-\boldsymbol{\eta},\ \nu\to-\nu,\ \theta\to\pi-\theta).$$

The factor $N_\omega+1$ then becomes $-N_\omega$;

$$\varepsilon'(-\mathbf{q},-\omega)=\varepsilon'(\mathbf{q},\omega),\ \varepsilon''(-\mathbf{q},-\omega)=-\varepsilon''(\mathbf{q},\omega).$$

In the transparency region such a process can be regarded as scattering by the electromagnetic excitations present in the medium and transformation of these excitations into transverse hard photons.[7,8,1] Our formulas (10) and (11) overlap the formulas of Tsytovich,[11] which describe two-quantum Cerenkov radiation in the case when the medium is transparent, one of the quanta is a hard vacuum quantum, and the other is soft and transverse. Our formulas are valid also in the case of a medium that absorbs soft quanta.

Expressions (10b) and (12) generalize the corresponding formulas of the paper by Gailitis and Tsytovich[8] to include the case of an absorbing medium and an energy transfer $\nu \neq 0$.

3. HARD SPONTANEOUS CERENKOV RADIATION (TRANSVERSE SOFT PHOTON)

The probability of hard Cerenkov radiation is proportional to $N_\omega+1$, while the probability of hard radiation following absorption of soft quanta is proportional to N_ω. When $T \ll \omega$ the occupation number is $N_\omega \ll 1$ and the principal role in the equilibrium medium is played by spontaneous radiation, which we shall now consider. Induced Cerenkov radiation, stimulated by a flux of soft transverse photons passing through the medium, was considered in [1].

In the case of a transparent medium, the imaginary part of D^R in (10a) is δ-like:

$$-\operatorname{Im}D_t(\mathbf{q},\omega)=-4\pi\operatorname{Im}\frac{1}{\omega^2\varepsilon_t-\mathbf{q}^2+i\delta}$$
$$=\frac{2\pi^2}{nvm^2}\delta(\nu-\nu_0), \tag{15}$$

where $n(\nu)=\sqrt{\varepsilon_t(\nu)}$ is the refractive index of the medium, $\nu_0(\eta)$ the solution of the dispersion equation $\eta=\nu n(\nu)$, and $V=d\nu/d\eta$ is the group velocity. The quantity (15) is proportional to $\delta(E_0-E-k-\omega)$ and expresses the energy conservation law. We assume that the particle is ultrarelativistic. $\eta \lesssim 10^{-5}$–10^{-4} in the case of an optical soft photon ν, and formula (14) relating ν and η becomes simpler, for the two terms with $\eta_\perp$ can be discarded. For such values of η we can disregard spatial dispersion, too.

Expressing $\cos\theta=\eta_{\|}/\eta$ in terms of ν, η, ξ, and ϑ with the aid of (14) we rewrite (15) in the form

$$-\operatorname{Im}D_t(\mathbf{q},\omega)=\frac{2\pi^2V}{m^2\eta^2}\delta(\cos\theta-\cos\theta_0), \tag{16}$$

where

$$\cos\theta_0=\frac{1}{n(\nu)}+\frac{\xi}{1-\xi}\frac{\alpha^2+\vartheta^2}{2\alpha\nu n(\nu)}. \tag{17}$$

It is seen from (16) that in the case of double Cerenkov radiation the soft photon is emitted inside the Cerenkov cone, the apex angle of which is

$$\theta_C=\arccos\left(\frac{1}{n}+\frac{\alpha^2+\nu\alpha}{2n}\right).$$

Since $\cos\theta_0 \le 1$, the necessary condition for the emission of a hard photon accompanied by a soft transverse photon is $n(\nu) > 1$, as in the case of single-quantum Cerenkov radiation. Formulas (15) and (16) are applicable also to a relatively soft absorbing medium, when $\varepsilon'' \ll \varepsilon'^{3/2}$, in which case the width $-\operatorname{Im}D_t$ relative to $\cos\theta$ is small compared with unity, which is the interval of appreciable variation of $\mathcal{L}_t$ as a function of $\cos\theta$.

The energy $k=E_0\xi$ of the hard photon can be obtained from (17). It depends uniquely on ν, ϑ, and θ_0:

$$\xi=\frac{2\alpha(n\cos\theta_0-1)}{\alpha^2+\vartheta^2+2\alpha(n\cos\theta_0-1)} \tag{18}$$

and increases with decreasing ϑ and θ. The largest value of ξ is attained for fixed ν when $\vartheta=\theta=0$:

$$\xi_{max}(\nu)=[1+\alpha^2/f(\nu)]^{-1},$$
$$f(\nu)=2\alpha\nu(n-1). \tag{19}$$

The upper limit of the hard spectrum corresponds to the maximum of the function $f(\nu)$. This maximum is usually attained in the ultraviolet part of the spectrum near the upper limit of the region $n > 1$ (at $\nu \sim 10^{-5}$–10^{-4}). It is seen from (19) that when $\alpha \lesssim 10^{-5}$–10^{-4} ($E_0 \gtrsim (10^4$–$10^5)m$), the hard quantum can carry away a larger part of the charged-particle energy. The minimum of k at which the hard photon is vacuum and our calculation is applicable is attained at $k_{min} \sim 10^8\ cm^{-1}$ ($\xi_{min} \sim 10^{-2}\alpha$).

To find the spectrum and the angular distribution of the hard photons it is necessary to integrate (10a) with respect to

$$dq = 2\pi m^3\eta^2 d\eta d\cos\theta = 2\pi m^3 V^{-1} n^2 \nu^2 d\nu d\cos\theta.$$

Integration with respect to $d\cos\theta$ is immediately carried out with the aid of (16). The remaining integration with respect to ν cannot be carried out in general form. It is carried out over the range of values of ν defined by the condition

$$f(\nu) = 2\alpha\nu[n(\nu) - 1] \geqslant \frac{\xi}{1-\xi}(\alpha^2 + \vartheta^2), \qquad (20)$$

which follows from the requirement $\cos\theta_0 \le 1$. We denote this region by $\Delta(\xi, \theta)$. Its dimensions increase with increasing energy E_0 (decreasing α) and with decreasing ξ, and also ϑ. When $\xi \sim \xi_{min}$ and $\vartheta \lesssim \alpha$, and for arbitrary $E_0 \gg m$, the region $\Delta(\xi, \vartheta)$ is defined in practice by the condition $n(\nu) > 1$, With increasing ξ and ϑ the region $\Delta(\xi, \vartheta)$ decreases and tends to zero.

Retaining in (11) only the terms of lowest order in α^2, ϑ^2, and $\nu\alpha \ll 1$, we obtain for the spectrum and the angular distribution of the hard quanta, after integrating with respect to $\cos\theta$ and ν, the expression

$$dw^t_{\xi,\vartheta} = \frac{me^4}{4\pi^2}\left\{\frac{\xi}{1-\xi}\left[2 + \frac{\xi^2}{1-\xi} - 4\alpha^2\frac{\vartheta^2}{(\alpha^2+\vartheta^2)^2}\right]\right.$$
$$\times \int\limits_{\Delta(\xi,\vartheta)} d\nu + \frac{1+(1-\xi)^2}{2(1-\xi)}\left[4\frac{\alpha^2}{(\alpha^2+\vartheta^2)^2}\frac{1-\xi}{\xi}\right.$$
$$\times \int\limits_{\Delta(\xi,\vartheta)} \frac{(n^2-1)^2}{n^2}\nu^2 d\nu - 4\frac{\alpha}{\alpha^2+\vartheta^2}\int\limits_{\Delta(\xi,\vartheta)}\frac{n^2-1}{n^2}\nu\, d\nu$$
$$\left. - \int\limits_{\Delta(\xi,\vartheta)}\left(1-\frac{1}{n^2}\right)d\nu\right\} d\xi\, d\Omega, \qquad (21)$$

where $d\Omega = 2\pi\vartheta\, d\vartheta$ is the element of solid angle for the directions **k**. The angular distribution (21) has the same width $\Delta\vartheta \sim \alpha$ as in the case of bremsstrahlung. The principal role in (21) with $E_{10} \gg 10^3\ m\sqrt{\xi}$ is played by the second term. The emission probability will in this case be

$$dw_{\xi\vartheta} = \frac{me^4}{2\pi^2}\frac{1+(1-\xi)^2}{\xi}\frac{\alpha^2}{(\alpha^2+\vartheta^2)^2}$$
$$\times\left[\int\limits_{\Delta(\xi,\vartheta)}\frac{(n^2-1)^2}{n^2}\nu^2 d\nu\right] d\xi\, d\Omega. \qquad (22)$$

When $\xi \sim \xi_{min}$, this formula is valid for arbitrary $E_0 \gg m$. For larger $\xi \sim \xi_{max}$ it is necessary, generally speaking, to take into account all the terms in (21). When $E_0 \gtrsim 10^4 m$ formula (22) is suitable for arbitrary ξ.

We obtain the spectrum of the hard Cerenkov radiation by integrating (21) with respect to ϑ from zero to

$$\vartheta = \vartheta_{max} = \left[f(\nu)\frac{1-\xi}{\xi} - \alpha^2\right]^{1/2} \qquad (23)$$

under the sign of integration with respect to ν. The remaining integration with respect to ν is over the region $\Delta(\xi) \equiv \max \Delta(\xi, \vartheta) = \Delta(\xi, 0)$. We shall not write out here the relatively cumbersome expression for the spectrum which is obtained upon integration. We present the result only for $E_0 \gg 10^3 m\sqrt{\xi}$:

$$\frac{dw_\xi^t}{d\xi} = \frac{me^4}{2\pi}\frac{1+(1-\xi)^2}{\xi}\int\limits_{\Delta(\xi)}\frac{(n^2-1)^2}{n^2}\nu^2 d\nu. \qquad (24)$$

The probability (24) vanishes at the upper limit $\xi = \xi_{max}$, since $\Delta(\xi_m) = 0$. The dependence on the mass ($1/m^2$, since $\nu \sim 1/m$) is the same as in the case of bremsstrahlung. Consequently, a noticeable role can be played only by the radiation of the electron; the heavy particles radiate a negligible amount of hard photons by the Cerenkov mechanism.

Let us compare (24) with the probability of bremsstrahlung from an ultrarelativistic particle at $E_0 \gg m/e^2 Z^{1/3}$:[10]

$$\frac{dw_\xi^t}{dw_\xi^{br}} \sim \frac{m^3\bar{\Delta}^3(\bar{n}-1)^2}{10Z^2e^2N_\alpha}, \qquad (25)$$

where N_α is the number of nuclei per unit volume, Z the atomic number of the nuclei, $\bar{n}$ the average value of the refractive index in the upper ultraviolet part of the region $\Delta(\xi)$, the significant widths of which we shall denote by $\bar{\Delta} \sim \Delta(\xi_m)$. Formulas (22), (24), and (25), are suitable also in the case when the absorption in the region $\bar{\Delta}$ is not too large.

In condensed media, for $\bar{\Delta} \sim 10^{-4}$, $N_\alpha \sim 10^{22}$–$10^{23}\ cm^{-3}$, $(\bar{n}-1)^2 \sim 10$ and ξ not too close to ξ_m, we get $dw_\xi^t/dw_\xi^{br} \sim (10^{-2}$–$10^{-1})/Z^2$. This fraction increases with increasing $\bar{\Delta}$ and $\bar{n}$. In light elements with $Z^2 \lesssim 10$, under favorable cases, it can reach 10%, but a more probable value is ~0.1%–

1%. In gases there is very little Cerenkov radiation since $(\bar{n}-1)^2/N_\alpha \sim N_\alpha$ and decreases with decreasing N_α.

The angular distribution of the bremsstrahlung and hard Cerenkov radiation is essentially the same. Therefore the ratio of the probability of radiation in a zero angle $dw^{t}_{\xi,\vartheta=0}/dw^{br}_{\xi,\vartheta=0}$ is of the same order of magnitude, $(10^{-2}-10^{-1})/Z^2$, as the ratio (25) of the probabilities integrated over the angle. However, if both the hard quantum and the momentum transfer (soft quantum) are directed forward, then the probability of single-quantum bremsstrahlung vanishes in the first Born approximation, unlike the probability of hard Cerenkov radiation. The latter can play in this case the principal role, along with the double bremsstrahlung.[12]

The loss to hard Cerenkov radiation has at $E_0 \gtrsim 10^4 m$ an order of magnitude

$$-dE_0/dx = E_0 \int_{\xi_{min}}^{\xi_{max}} \xi\, dw_\xi \sim w_t \xi_m E_0 \sim m^2 e^4 \bar{\Delta}^3 \alpha^{-1} (\bar{n}-1)^2$$

$$\sim 10^{-5} m \frac{E_0}{m} \sim 10 \frac{E_0}{m} \text{ eV/cm}.$$

The ratio of this loss to the bremsstrahlung loss is small, of the same order as the probability ratio dw^t_ξ/dw^{br}_ξ. However, hard Cerenkov loss like bremsstrahlung loss, increases in proportion to E_0. Consequently, at sufficiently high electron energies the two losses become comparable, and then the hard Cerenkov loss exceeds the single-quantum Cerenkov radiation loss

$$(-dE_0/dx)_{\text{onequ}} \sim m^2 e^2 \bar{\Delta}^2$$

and the ionization loss, which depends little on the energy,

$$(-dE_0/dx)_{\text{ion}} \sim 10 N_a Z e^4 m^{-1} \ln (10^8/\alpha^3 Z^2).$$

This occurs when $E_0 \sim 10^4 m$ and $E_0 \sim 10^6 mZ$, respectively.

We note that hard Cerenkov radiation can be separated from bremsstrahlung by recording the coincidences of the hard and soft photons. The latter should lie in the transparency region in order to be registered. The principal role will be played by the ultraviolet edge of this region, $\bar{\Delta} \sim 10^{-5}$ (in place of $\bar{\Delta} \sim 10^{-4}$, as in the consideration of the total intensity above). Here, according to (25), the number of hard Cerenkov quanta and the accompanying soft quanta decreases by approximately three orders and amounts to

$$N_{\text{twoqu}} \sim m e^4 \bar{\Delta}^3 (\bar{n}-1)^2 \sim 10^{-8} \text{ qu/cm}.$$

This number must be compared with the number of hard bremsstrahlung quanta, which amounts to

$$N_{\text{br}} \sim 10 e^6 Z^2 m^{-2} N_a \sim (10^4 - 10^5) Z^2 N_{\text{twoqu}} \text{ qu/cm},$$

and with the number of soft quanta from the single-quantum Cerenkov effect

$$N_{\text{onequ}} \sim m e^2 \bar{\Delta} \sim 10^3 \text{ qu/cm}.$$

The background of hard bremsstrahlung quanta has an acceptable value. On the other hand, the number of unnecessary soft quanta is larger than the number of the quanta of interest to us by approximately 11 orders of magnitude. However, all the unnecessary quanta propagate along the corresponding Cerenkov cones, whose apex angle amounts to several times 10° in the ultraviolet region which is essential for the two-quantum Cerenkov effect.

There are also soft bremsstrahlung quanta, the number of which has the same order of magnitude as the number given above for the hard bremsstrahlung quanta. These soft quanta propagate essentially at small angles $\vartheta \sim \alpha$. On the other hand, the soft quanta which accompany the hard Cerenkov quanta are distributed more or less uniformly inside the Cerenkov cones. Consequently, there exists a wide angle interval in which the unfavorable soft background is practically nonexistent. Direct observation of the coincidences considered here would yield a direct confirmation of the correctness of the quantum theory of the Cerenkov effect (we are dealing essentially with the manifestation of the recoil effect, which plays only a very small role in the single-quantum Cerenkov effect.

4. HARD CERENKOV RADIATION (LONGITUDINAL SOFT PHOTON) AND BREMSSTRAHLUNG IN A DEGENERATE ELECTRON PLASMA

In the case of a transparent medium, the hard radiation due to the interaction with the medium via the longitudinal field is proportional to

$$-\operatorname{Im} D_l(\mathbf{q}, \omega) = \frac{4\pi}{\mathbf{q}^2}\left(-\operatorname{Im}\frac{1}{\varepsilon(\mathbf{q}, \omega)+i\delta}\right)$$

$$= \frac{4\pi}{\mathbf{q}^2}\delta[\varepsilon(\mathbf{q}, \omega)]. \qquad (26)$$

According to (26), the radiating particle gives up to the medium a momentum $\mathbf{q}$ and an energy ω corresponding to the zeroes of the dielectric constant, and consequently to elementary excitations of the medium. As noted by Silin,[13] when allow-

allowance is made for the spatial dispersion and the theory of losses in a medium is used it is unnecessary to distinguish between short-range and long-range losses. Similar considerations can be applied to the question of the radiation of hard photons by a particle. Small values of transferred **q** correspond in this case to collective excitations (plasmons), while large **q** correspond to excitation of individual electrons of the medium. The radiation of a plasmon together with a hard quantum is a double Cerenkov effect, while the radiation of hard quanta upon collision with individual electrons of the medium is bremsstrahlung from the valence electrons of the medium. Both processes are thus limiting cases of the same phenomenon, namely the radiation of hard quanta as a result of interaction between a fast particle and a medium.

Let us illustrate these considerations by calculating with the aid of (10b) and (12) the probability dw_{kq}^{l} of emission of a hard photon when an electron interacts with a degenerate electron plasma described by a more or less realistic dielectric constant:

$$\varepsilon(\mathbf{q}, \omega) = 1 - \frac{\omega_p^2}{\omega^2 - (\mathbf{q}^2/2m)^2 - {}^3/_5 v_0^2 \mathbf{q}^2}, \qquad (27)$$

where $\omega_p = (4\pi N_e e^2/m)^{1/2}$ is a plasma frequency, N_e the density of the valence electrons, and $v_0 = (3\pi^2)^{1/2} N_e^{1/3}/m$ the electron velocity at the Fermi level. (27) is an interpolation formula and includes as limiting cases the expressions

$$\varepsilon_l = 1 - \omega_p^2/\omega^2 \quad (\text{for} \quad q \ll mv_0),$$

$$\varepsilon_l = 1 - \omega_p^2/[\omega^2 - (q^2/2m)^2] \quad (\text{for} \quad q \gg mv_0)$$

and corresponds to elementary excitations:[14]

$$\omega_0 = m\nu_0, \quad \nu_0 = [\nu_p^2 + \rho\nu_p\eta^2 + \eta^4/4]^{1/2}. \qquad (28)$$

Here $\nu_p = \omega_p/m \sim 10^{-5}$ (3.59×10^{-5} for Be, 2.12×10^{-5} for Mg, and 4.83×10^{-5} for C), and $\rho = 3v_0^2/5\nu_p \sim 1$ (0.93 for Be, 0.77 for Mg, and 0.87 for C). Substituting (27) in (26), we obtain

$$-\operatorname{Im} D_l(\mathbf{q}, \omega) = \frac{2\pi\nu_p^2}{m^2\nu_0}\delta(\nu - \nu_0). \qquad (29)$$

When $\eta^2 \gg \nu_p$, the elementary excitations (28) take the form $\omega_0 = m\eta^2/2 = q^2/2m$, corresponding to energy transfer to an individual electron of this medium, which remains in this case nonrelativistic, $\eta^2 \ll 1$. As can be readily verified, in a wide range of values $\nu_p \ll \eta^2 \ll 1$, formulas (10b) and (12) overlap the corresponding formulas for bremsstrahlung from free electrons, written out in accordance with the usual rules of vacuum quantum electrodynamics ([10], p. 327) and pertaining to the case when one of the electrons is at rest prior to the collision and acquires a nonrelativistic velocity after the collision. In this case only two of the eight diagrams are significant; these are the non-exchange diagrams which describe the radiation of an incoming fast electron, and not of the recoil electron. When $\eta^2 \gtrsim 1$, formula (27) no longer is valid, but the electron of the medium is free and the theory given in [10] can be used. When $\eta^2 \sim \nu_p$, the electron of the medium cannot be regarded as free, and the theory of [10] is not valid. We note that it is just the region of small η^2 which makes the main contribution to the radiation probability.

A correct expression for the bremsstrahlung from the electrons of the medium can be obtained by calculating the correction dw'_ξ to the bremsstrahlung from the free electrons; this correction takes into account the disparity between the elementary excitations and the single-particle excitations for small momentum transfers $\eta^2 \sim \nu_p$; we define it as

$$dw_\xi' = dw_{1\xi}^{l} - dw_{1\xi}^{el}, \qquad (30)$$

where $dw_{1\xi}^{l}$ and $dw_{1\xi}^{el}$ are the contributions to the spectrum in accordance with (10b) and in accordance with the theory of bremsstrahlung from free electrons respectively. These contributions come from that part of the region of integration with respect to η^2 in which $\eta^2 \ll 1$—this is the only part that contributes to the correction. The bremsstrahlung spectrum, with allowance for the electron coupling, will then be

$$dw_\xi^{el} = dw_\xi^{l} + dw_\xi'. \qquad (31)$$

The determination of the spectrum of the hard quanta reduces to an integration of (10b) with respect to

$$d\mathbf{q} = m^3\eta_\perp d\eta_\perp d\eta_\parallel d\varphi = {}^1/_2 m^3 d\eta^2 d\eta_\parallel d\varphi$$

and with respect to $d\Omega = \pi\, d\vartheta^2$. We consider only the ultrarelativistic case $\alpha \ll 1$. In formula (12) we should leave here only the terms with $1/\alpha^2$. In formula (29) it is necessary to substitute the expression for ν from (14), and to retain in it all the terms (unlike the procedure in Sec. 3), since now η^2 is not so small. We shall take the angle ϑ to be the argument of the δ- function in (29); then

$$\delta(\nu - \nu_0) = \frac{2\alpha(1-\xi)}{\xi}\left[1 - \frac{s}{(s^2 + \vartheta_0^2)^{1/2}}\right]\delta(\vartheta^2 - \vartheta_1^2),$$

$$s = \alpha\eta_\perp \cos\varphi, \quad \vartheta_0^2 = \alpha^2\left[2\frac{1-\xi}{\xi}\,\frac{\eta_\parallel - \nu_0}{\alpha} - 1\right],$$

$$\vartheta_1 = \sqrt{s^2 + \vartheta_0^2} - s.$$

The integration with respect to $\eta_{\|}$ and η^2, which remains after integration with respect to ϑ^2 and with respect to $\cos\varphi$ should be carried out over the region defined by the inequalities

$$\vartheta_0^2 \geqslant 0, \quad \eta^2 - \eta_{\|}^2 \geqslant 0, \tag{32}$$

the first of which follows from the condition $\vartheta_1^2 \geq 0$, and the second from the condition $\eta_{\perp}^2 \geq 0$.

If we confine ourselves to not too soft a part of the spectrum $\xi \gtrsim 0.1$, then

$$\varkappa_1 \approx -(2/\alpha)(\eta_{\|} - \nu_0),$$

$$\varkappa_2 = 2(1-\xi)(\eta_{\|} - \nu_0)/\alpha - 2(\xi/\alpha^2)s(s^2 + \vartheta_0^2)^{1/2},$$

and after calculating the double integral with respect to $\eta_{\|}$ and η^2 we obtain

$$\frac{dw_\xi'}{d\xi} = 2\frac{e^6 N_e}{m^2}\left\{\frac{1+(1-\xi)^2 - {}^2/_3(1-\xi)}{\xi}\left(\ln\frac{1}{\nu_p(1+\rho)} - 2\ln\frac{2}{\alpha}\,\frac{1-\xi}{\xi}\right) + 2\frac{1+(1-\xi)^2}{\xi} - 2.44\frac{1-\xi}{\xi}\right\},$$

$$\xi \geqslant 0.1. \tag{33}$$

This correction must be added, in accordance with (31), to the expression for the probability dw_ξ^{el}, of the bremsstrahlung from the free electrons, obtained by Garibyan.[15, 10] As a result we obtain an expression for the probability of bremsstrahlung from a degenerate electron plasma, caused by the valence electrons of the medium; this expression takes into account the difference between the aggregate of valence electrons from the free-electron gas:

$$\frac{dw_\xi}{d\xi} = 2\frac{e^6 N_e}{m^2}\left\{\frac{1+(1-\xi)^2 - {}^2/_3(1-\xi)}{\xi}\ln\frac{1}{\nu_p(1+\rho)} + \frac{1+(1-\xi)^2}{\xi} - 1.70\xi - 3.91\frac{1-\xi}{\xi}\right\}. \tag{34}$$

The main difference between (34) and the Garibyan formula is the change in the form of the logarithmic term. The coefficient preceding the logarithm, which depends on ξ, is the same in both formulas. Equation (34) contains $\ln[1/(\nu_p + \rho\nu_p)]$ in place of $2\ln[2(1-\xi)/\alpha\xi]$. As $\xi \to 0$, expression (34) does not acquire in the loss spectrum $\xi\, dw_\xi$ the logarithmic divergence characteristic of free unscreened charges. In this sense, the effect of coupling is analogous to the screening effect. The term with ν_p reflects the transfer of energy to the plasmon, while the term $\rho\nu_p = 3v_0^2/5$ takes into account the binding energy of the plasma electron in the initial state. The sign of the correction (33) can differ, but is most frequently negative. In the case of Be ($E_0 = 5.11 \times 10^9$ eV), the ratio dw_ξ'/dw_ξ^{el} amounts to -65% for $\xi = 0.05$, -48% for $\xi = 0.3$, -33% for $\xi = 0.5$, -20% for $\xi = 0.7$, and $+9\%$ for $\xi = 0.9$. We see that this correction is quite appreciable, especially in the soft part of the spectrum. The contribution made to the total bremsstrahlung spectrum (from the nuclei and from the electrons) will not be so large, since the calculated correction pertains only to radiation from the valence electrons. When $E_0 = 5 \times 10^9$ eV and $\xi \sim 0.1$–0.5, the decrease in the bremsstrahlung probability amounts to approximately $[n/(Z^2 + Z - n)] \times 50\%$, where n is the number of valence electrons. For Be, Li, and C this amounts to $\sim 5\%$.

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[1] V. V. Batygin, JETP **48**, 272 (1965), Soviet Phys. JETP **21**, 179 (1965).

[2] M. L. Ter-Mikaelyan, JETP **25**, 296 (1953).

[3] L. D. Landau and I. Ya. Pomeranchuk, DAN SSSR **92**, 535 and 735 (1953).

[4] A. B. Migdal, JETP **32**, 633 (1957), Soviet Phys. JETP **5**, 527 (1957).

[5] A. I. Alekseev, Collection: Primenenie metodov kvantovoĭ teorii polya k zadacham mnogikh tel (Application of Methods of Quantum Field Theory to Many-body Problems), Atomizdat, 1961.

[6] I. E. Dzyaloshinskiĭ and L. P. Pitaevskiĭ, JETP **36**, 1797 (1959), Soviet Phys. JETP **9**, 1282 (1959).

[7] M. I. Ryazanov, JETP **45**, 333 (1965), Soviet Phys. JETP **18**, 232 (1964).

[8] A. Gaĭlitis and V. N. Tsytovich, JETP **46**, 1726 (1964), Soviet Phys. JETP **19**, 1165 (1965).

[9] A. A. Abrikosov, L. P. Gor'kov and I. E. Dzyaloshinskiĭ, Metody kvantovoĭ teorii polya v statisticheskoĭ fizike (Methods of Quantum Field Theory in Statistical Physics), Fizmatgiz, 1962.

[10] A. I. Akhiezer and V. B. Berestetskiĭ, Kvantovaya élektrodinamika (Quantum Electrodynamics), 2d ed., Fizmatgiz, 1959.

[11] V. N. Tsytovich, DAN SSSR **154**, 76 (1964), Soviet Phys. Doklady **9**, 49 (1964).

[12] L. G. Zazunov and P. I. Fomin, JETP **46**, 1392 (1964), Soviet Phys. JETP **19**, 941 (1964).

[13] V. P. Silin, JETP **37**, 273 (1959), Soviet Phys. JETP **10**, 192 (1960).

[14] V. P. Silin and A. A. Rukhadze, Élektromagnitnye svoĭstva plazmy i plazmopodobnykh sred (Electromagnetic Properties of Plasma and Plasmalike Media), Atomizdat, 1961.

[15] G. M. Garibyan, JETP **24**, 617 (1953).

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