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SPIN-MAGNETOPHONON RESONANCE AND THE MAGNETORESISTANCE OSCILLATIONS PRODUCED BY IT IN SEMICONDUCTORS

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A theory of a new internal resonance in semiconductors, called spin-magnetophonon resonance by the authors, and the theory of various aspects of this phenomenon in galvanomagnetic effects, are developed on the basis of the theory previously proposed by the authors for interactions between electrons and optical phonons leading to transitions involving spin flip.[13] It is shown that in the case in which the spin sub-bands of the Landau magnetic bands are split by a quantizing magnetic field, spin-magnetophonon resonance leads to new oscillations (compared to what the magnetophonon resonance theory yields) of the transverse ρ_{xx} and longitudinal ρ_{zz} magnetoresistance. The character of the extremum on the oscillation curve of the longitudinal magnetoresistance at the point corresponding to the resonance value of the magnetic field depends on the presence of some additional electron-spin-flip scattering mechanism, which interferes with the scattering by optical phonons (electron scattering with spin flip by acoustic phonons has been considered by us); it also depends on the relation between this mechanism and other scattering mechanisms. The equilibrium electron distribution is assumed to be of the Boltzmann type. The results of the theory are in agreement with the experimental data.

In recent years, a theory of magnetophonon resonance (MPR) in semiconductors has been developed,[1-8] and a significant number of researches have been carried out (see, for example, [9-12]) on its experimental detection and study.

MPR is an internal resonance which can be compared with the corresponding "external" resonance—cyclotron resonance. Actually, both these resonances are the consequence of the features of the density of states of electrons in a magnetic field and are connected with electron transfer from the bottom of one Landau magnetic band to the bottom of the next higher band. The energy required for this purpose is drawn from different sources: this is the energy of the external electromagnetic field in the case of cyclotron resonance, and the energy of optical phonons in the case of MPR.

In this paper we construct a theory for the oscillations of the galvanomagnetic kinetic coefficients, brought about by a new form of internal resonance, called by us spin-magnetophonon resonance (SMPR). SMPR is associated with electron transitions between spin subbands of the same or of different Landau bands—transitions with spin flip,[1)] —which are accompanied by absorption of an optical phonon with energy $\hbar\omega_0$ (ω_0 is the limiting frequency of long wave optical phonons). In this case, the following relation is satisfied:

$$\hbar\omega_0 = K\hbar\omega_c \pm |g|\mu_0 H$$

(K is an integer, $K \geq 0$ for the plus sign and $K \geq 1$ for the minus sign; $\omega_c = eH/mc$ is the cyclotron frequency, $-e$ the charge on the electron, m the effective mass of the electron, c the velocity of light, g the effective spectroscopic splitting factor, $\mu_0 = e\hbar/2m_0c$ the Bohr magneton, m_0 the mass of the free electron, and H the external magnetic field).

[1)]Here and below we defined transitions with spin flip as transitions between states which arise as a consequence of the removal of the Kramers degeneracy by the magnetic field. This is done for convenience and brevity of terminology. It must be remembered that strictly speaking these states (and this only in the case of a diagonal matrix of the total characteristic magnetic moment of the electron) are eigenstates of the operator of the total characteristic magnetic moment of the electron itself, which appears as a consequence of a strong spin-orbit interaction.

SMPR can also be compared with external resonance—paramagnetic resonance with conduction electrons in quantizing magnetic fields (in the infrared absorption region). As in the case of a comparison of MPR with cyclotron resonance, the similarity of the internal and external resonance lies in the transitions of the electron between the same initial and final states, while the difference lies in the sources from which the energy necessary for transitions between these states is drawn.

The theory of SMPR is constructed on the basis of the theory developed previously by the authors[13] for the interaction of the characteristic magnetic moment of the electron with the optical vibrations of the lattice. As numerical estimates in [13] have shown, the interaction with optical phonons leading to spin flip in semiconductors with strong spin-orbit coupling, for example in InSb, is characterized by a constant d with the dimensions of energy, comparable in magnitude with the constant of the deformation potential E_0.

It is interesting to note that although the interaction characterized by the constant d appears as the result of taking into account the relativistic terms in the Hamiltonian of the electron in the lattice (account of the spin-orbit term), the constant d is not small, just as the constant Δ of spin-orbit splitting of the valence band is not small.[2)]

The analysis presented in [13] shows that the overwhelmingly predominant contribution to the constant d is made by the short-range part (at distances r of the order of the lattice constant a) of the interaction of the electrons with the optical vibrations of the lattice. The long-range part of the interaction ($r \gg a$) with the polarizing vibrations of the lattice makes a negligible contribution to d, of the order of the ratio of the energy of the electron in the conduction band to m_0c^2.

We note that, in the case of the ordinary interaction (without spin flip) of the electrons with the polarizing vibrations of the lattice, the inverse is the case: the main contribution to the electron scattering is made by the long-range and not the short-range interactions.

The large value of the constant d permits us to hope that the interaction with optical phonons which leads to spin flip can appear in experiments, in particular, in the form of additional oscillations of the magnetoresistance (in comparison with those produced by MPR). The additional oscillations of the magnetoresistance observed in [17-19] can apparently be associated with SMPR.

The present research includes an analysis of the phenomenon of SMPR both for the transverse (σ_{xx}) and longitudinal (σ_{zz}) electrical conductivity in a magnetic field of semiconductors of the type n-InSb. It was shown in [3] that in the case of MPR, in the calculation of the oscillating part of ρ_{zz}, it is necessary to take into account the interference of two scattering mechanisms—by the acoustic and optical phonons. In the case of SMPR, in the computation of ρ_{zz}, it is also necessary to take into account interference, but of four rather than two scattering mechanisms—namely, scattering by acoustical and optical phonons with and without spin flip. The character of the extremum (minimum or maximum) on the curve of ρ_{zz} at the point of the resonance value of the magnetic field is determined by the relations between the different interfering scattering mechanisms.

1. FORMULATION OF THE PROBLEM. MATRIX ELEMENT FOR TRANSITION ON AN ELECTRON WITH SPIN FLIP

The application of the method of effective mass to the conduction electrons in an ideal lattice with account of spin-orbit interaction and in the presence of an external magnetic field permits us to consider the electrons as free particles with an effective mass m and an effective g factor. This means that in the case of a semiconductor with a simple band (nondegenerate, and having a minimum at the center of the Brillouin zone) the Hamiltonian of the electron can be written in the form

$$\mathscr{H}_{\text{el}} = \frac{1}{2m}\left(\mathbf{p} + \frac{e}{c}\mathbf{A}\right)^2 - \frac{g}{2}\mu_0\boldsymbol{\sigma}\mathbf{H}, \qquad (1.1)$$

where $\mathbf{p}$ is the momentum operator, $\mathbf{A} = \mathbf{A}(0, xH, 0)$ is the vector potential of the external magnetic field, directed along the z axis, σ is a vector whose components are the Pauli matrices, σ_x, σ_y, and σ_z.

The eigenfunction of the Hamiltonian (1) is the product of the eigenfunctions of the operator σ_z and a coordinate part characterized by the set of the quantum numbers $\kappa \equiv (n, k_y, k_z)$ and equal to

$$F_\kappa(\mathbf{r}) = \frac{1}{L}\exp\{ik_y y + ik_z z\}\Phi_n\left(\frac{x - x_0}{l_H}\right). \qquad (1.2)$$

Here $\mathbf{k}$ is the wave vector of the electron $x_0 = -\hbar c k_y/eH$, $l_H = (c\hbar/eH)^{1/2}$ is the magnetic length, and $\Phi_n(x - x_0)/l_H$ is a homogeneous os-

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[2)] According to the estimate of Kane, [14] $\Delta = 0.9$ eV for InSb. The validity of this estimate is appreciably strengthened by experiments [15] on the determination of the spectroscopic splitting factor g which, in accord with the theory of Roth et al. [16], is expressed in terms of Δ, the effective mass m and the width of the forbidden band E_g.

cillating function determined by the relations

$$\Phi_n\left(\frac{x-x_0}{l_H}\right)=(\pi l_H^2)^{-1/4}(2^n n!)^{-1/2}$$
$$\times\exp\left[-\frac{1}{2}\left(\frac{x-x_0}{l_H}\right)^2\right]H\left(\frac{x-x_0}{l_H}\right). \quad (1.3)$$

Here $H_n((x-x_0)/l_H)$ is a Hermite polynomial of order n.

The eigenvalues of the operator (1) are

$$\varepsilon_{\varkappa,s}=\varepsilon_\varkappa - sg\mu_0 H/2, \quad (1.4)$$

where $\epsilon_\kappa = \hbar\omega_c(n+1/2)+\hbar^2 k_z^2/2m$, and $s=\pm 1$ is the spin quantum number.

We recall that the effective mass method is applicable when the wave function of the electron changes only slightly over distances of the order of the lattice constant. Furthermore, we have made use of the two-band approximation of Kane, [14] the condition for the applicability of which in our case reduces essentially to the inequality $\hbar\omega_0/E_g \ll 1$, which is satisfied.

As shown in [13], the part of the operator of electron-phonon interaction, corresponding to transitions with spin flip in scattering by optical phonons, has the form

$$\mathscr{H}^{(sp)}_{el\text{-}ph}=\sum_{i,\mathbf{q}} d\left(\frac{\hbar}{2NM\omega_\mathbf{q}}\right)^{1/2}\left\{e^{i\mathbf{qr}}b_\mathbf{q}^{(i)}\begin{pmatrix}0 & [\mathbf{h}_-\mathbf{e}_\mathbf{q}^{(-,i)}]\\ [\mathbf{h}_+\mathbf{e}_\mathbf{q}^{(-,i)}] & 0\end{pmatrix}\right.$$
$$\times\left(\frac{\mathbf{p}}{\hbar}+\frac{e}{\hbar c}\mathbf{A}+\frac{\mathbf{q}}{2}\right)+e^{-i\mathbf{qr}}b_\mathbf{q}^{(i)+}\begin{pmatrix}0 & [\mathbf{h}_-\mathbf{e}_{-\mathbf{q}}^{(-,i)}]\\ [\mathbf{h}_+\mathbf{e}_{-\mathbf{q}}^{(-,i)}] & 0\end{pmatrix}$$
$$\left.\times\left(\frac{\mathbf{p}}{\hbar}+\frac{e}{c\hbar}\mathbf{A}-\frac{\mathbf{q}}{2}\right)\right\}, \quad (1.5)^*$$

where $\mathbf{h}_\pm = \mathbf{l}_x \pm \mathbf{l}_y$, $\mathbf{l}_x$, $\mathbf{l}_y$ are the unit vectors corresponding to the directions along the x and y axes; (i) is the index of the branch of optical vibrations of the lattice; $\mathbf{e}_\mathbf{q}^{(-,i)}=\mathbf{e}_{1,\mathbf{q}}^{(i)}-\mathbf{e}_{2,\mathbf{q}}^{(i)}$ is the difference of the vectors characterizing the displacements of the atoms of both types[3)] during the vibrations of the lattice. For optical phonons, $\mathbf{e}_\mathbf{q}^{(-,i)}$ differs from zero even in the zeroth approximation in the phonon wave vector $\mathbf{q}$; in interactions with acoustical phonons, (1.5) is also valid, but $\mathbf{e}_\mathbf{q}^{(-,i)}$ for acoustical phonons differs from zero only in terms beginning with first order in $\mathbf{q}$, that is, $\mathbf{e}_\mathbf{q}^{(-,i)}\approx a\mathbf{q}$ (see [13]).

In what follows, for purposes of avoiding complicated expressions, we shall consider the extreme quantum case ($|g|\mu_0 H/T \gg 1$, T is the temperature in energy units), in which the electrons are primarily in the lowest spin subband of the zero Landau magnetic band (band with quantum number $n=0$, $s=-1$).

We write out the form of the square of the modulus of the matrix element which in the case $g<0$[4)] defines the transition of an electron from the state $n=0$, $s=-1$, k_y, k_z to a state of higher energy which differs in the value of the spin projection $n'\geq 0$, $s'=1$, k_y', k_z':

$$\frac{1}{\hbar^2}\left|\left(n',k_y',k_z',s'=1\left|e^{i\mathbf{qr}}[\mathbf{h}_-\mathbf{e}_0^{(-,i)}]\left(\mathbf{p}+\frac{e}{c}\mathbf{A}+\frac{\hbar\mathbf{q}}{2}\right)\right.\right.\right.$$
$$\left.\left.\times\right|n=0,s=-1,k_y,k_z\right)\Big|^2=\delta_{k_y',k_y+q_y}\delta_{k_z',k_z+q_z}l_H^{-2}$$
$$\times\frac{u^{n'}}{n'!}e^{-u}\left\{\frac{u}{2}(e_z^{(-,i)})^2+[(e_x^{(-,i)})^2+(e_y^{(-,i)})^2]l_H^2\left(k_z+\frac{q_z}{2}\right)^2\right.$$
$$\left.-e_z^{(-,i)}(e_x^{(-,i)}q_x+e_y^{(-,i)}q_y)\left(k_z+\frac{q_z}{2}\right)\right\}, \quad (1.6)$$

where

$$u=\tfrac{1}{2}l_H^2 q_\perp^2, \quad q_\perp^2=q_x^2+q_y^2. \quad (1.6a)$$

The result (1.6) is obtained by using for the integrals of the form

$$I_{n'n}=\int_{-\infty}^{\infty}d\zeta\exp[-\tfrac{1}{2}(\zeta-\zeta_0)^2-\tfrac{1}{2}(\zeta-\zeta_0')+$$
$$+iq_x l_H\zeta]H_{n'}(\zeta-\zeta_0')H_n(\zeta-\zeta_0) \quad (1.7)$$

the expressions (compare with those given in [20])[5)]

$$I_{n'n}=\sqrt{\pi}2^n(\zeta_0-\zeta_0'+iq_x l_H)^{n'-n}\exp[-\tfrac{1}{2}\tilde{u}+$$
$$+\tfrac{1}{2}iq_x l_H(\zeta_0+\zeta_0')]L_n^{n'-n}(\tilde{u}), \quad n'\geqslant n;$$
$$I_{n'n}=\sqrt{\pi}2^{n'}(\zeta_0'-\zeta_0+iq_x l_H)^{n-n'}\exp[-\tfrac{1}{2}\tilde{u}$$
$$+\tfrac{1}{2}iq_x l_H(\zeta_0+\zeta_0')]L_{n'}^{n-n'}(\tilde{u}), \quad n\geqslant n';$$
$$\tilde{u}=\tfrac{1}{2}[\zeta_0-\zeta_0')^2+(q_x l_H)^2]. \quad (1.8)$$

The square of the modulus of the matrix element for the transition of an electron from the state with $s=1$ to the state with $s=-1$ can be obtained from (1.6) by the simultaneous substitutions $\mathbf{q}\to-\mathbf{q}$, $\mathbf{k}\to\mathbf{k}'$, and $\mathbf{k}'\to\mathbf{k}$.

We shall further consider the manifestation of SMPR in galvanomagnetic effects. We shall consider first the simplest case—the transverse electric conductivity.

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*$[\mathbf{he}]\equiv\mathbf{h}\times\mathbf{e}$.

3)We consider a crystal in which the unit cell has two atoms.

4)For InSb the effective g factor is negative, [16] so that the state with a spin directed against the magnetic field is energetically more favorable.

5)We take the opportunity to note that the expressions given in [20] for integrals of the form (1.7), with a separate phase factor in the form of an arctangent, it is necessary to multiply by $[\mathrm{sign}(\zeta_0-\zeta')]^{n-m}$ in Eq. (8) and by $[\mathrm{sign}(\zeta'-\zeta_0)]^{m-n}$ in Eq. (9).

2. OSCILLATIONS OF THE TRANSVERSE ELECTRIC CONDUCTIVITY

As is well known, the transverse electric conductivity σ_{xx} differs from zero only in the presence of scattering and, in the lowest approximation, is proportional to the scattering probability. Inasmuch as the total scattering probability of the electron is an additive quantity, it suffices for the evaluation of the effect of SMPR on σ_{xx} to consider only the contribution made by the spin-flipping interaction of electrons with the optical phonons. It is natural that this contribution, which is proportional to $\exp(-\hbar\omega_0/T)$, is small in comparison with the background noise brought about by other scattering mechanisms (in particular, by acoustical phonons), and can itself appear only close to the resonance values of the magnetic field $K\hbar\omega_c + |g|\mu_0 H = \hbar\omega_0$.

We make use of the well known formula for σ_{xx} (see, for example, [21] page 113 of the Russian translation):

$$\sigma_{xx} = \frac{Le^2 l_H^4}{(2\pi)^4 T} \sum_{n'} \int dk_y' \, dk_z' \, dk_y \, dk_z (k_y' - k_y)^2$$
$$\times \exp\left[\frac{\xi - \varepsilon_{0,\, s=-1}(k_z)}{T}\right] W(n', k_y', k_z', s = 1;$$
$$n = 0, k_y, k_z, s = -1), \qquad (2.1)$$

$$W = \frac{2\pi}{\hbar} \sum_{i,\, q} |M(n', k_y', k_z', s' = 1; n = 0, k_y, k_z, s = -1)|^2$$
$$\times \delta(\varepsilon_{\varkappa,\, s} - \varepsilon'_{\varkappa',\, s'} + \hbar\omega_0). \qquad (2.2)$$

Here $M(\kappa', s'; \kappa, s)$ is the matrix element of the transition, ξ the chemical potential, and L^3 the normalized volume. In (2.1), the contributions from scattering by longitudinal and transverse branches of the optical vibrations of the lattice are summed. They can be considered separately with account of the difference of the limiting frequencies of transverse and longitudinal optical phonons.

Making use of (1.6) and averaging over the directions of the polarization vector of the transverse optical phonons, which is connected with the substitution of the expressions

$$\overline{(e_z^{(-)})^2} = \frac{M}{2\overline{M}} \frac{q_\perp^2}{q^2}, \quad e_z^{(-)}(e_x^{(-)} q_x + e_y^{(-)} q_y) = -\frac{M}{2\overline{M}} \frac{q_z q_\perp^2}{q^2}$$

$$\overline{(e_x^{(-)})^2} + \overline{(e_y^{(-)})^2} = \frac{M}{2\overline{M}}\left(1 + \frac{q_z^2}{q^2}\right), \qquad (2.3)$$

we get the following expression for the contribution to the electrical conductivity from scattering by transverse optical phonons

$$\sigma_{xx\perp} = \sigma_0 \sum_{n'} \int \frac{d\varepsilon \, d\varepsilon' \, e^{-\varepsilon/T} \delta(\varepsilon - \varepsilon' + \hbar\omega_{0\perp})}{\varepsilon^{1/2}(\varepsilon' - n'\hbar\omega_c - |g|\mu_0 H)^{1/2}}, \qquad (2.4)^*$$

where

$$\sigma_0 = \frac{1}{16\sqrt{\pi}} \frac{n_0 e^2}{m} \frac{\hbar^2}{\tau_{\text{opt.sp}} T^2} \frac{\hbar\omega_c}{T} e^{-\hbar\omega_0/T}, \qquad (2.5)$$

$\tau_{\text{opt. sp}}$ is defined in (3.10) (see Sec. 3) and the relation (3.16) between the concentrations and the chemical potential is also used.

Equation (2.4) has logarithmic singularities for those values of the magnetic field at which the condition $\hbar\omega_0 = n'\hbar\omega_c + |g|\mu_0 H$ is satisfied, which leads to oscillations in σ_{xx} with the period

$$\Delta(1/H) = e/m\omega_{0\perp}c. \qquad (2.6)$$

The oscillating maxima of σ_{xx} are located at the inverse values of the magnetic field

$$\frac{1}{H} = \left(n' \frac{e\hbar}{mc} + |g|\mu_0\right) \frac{1}{\hbar\omega_{0\perp}}, \qquad (2.6a)$$

that is, for reciprocal values of the magnetic field larger than $|g|\mu_0/\hbar\omega_{0\perp}$, the corresponding magnetophonon maxima of σ_{xx} are impossible.

In the case of a nonzero population of the magnetic band $n = 0$ and $s = 1$, maxima of σ_{xx} must also appear at the values

$$\frac{1}{H} = \left(n' \frac{e\hbar}{mc} - |g|\mu_0\right) \frac{1}{\hbar\omega_{0\perp}}, \quad n' \geqslant 1, \qquad (2.6b)$$

which are associated with the transitions of the electron to the band with $n' \geq 1$ and $s' = -1$, and for values of the reciprocal of the magnetic field smaller than the value $|g|\mu_0/\hbar\omega_0$ corresponding to the magnetophonon maxima σ_{xx}.

It is easily seen from Eq. (1.6), in which the following expression is used for the longitudinal optical phonons:

$$e_z^{(-)} = (M/\overline{M})^{1/2} q_z / q, \qquad (2.6c)$$

that the square of the modulus of the matrix element for the transition of an electron from the state $n = 0$, k_y, k_z, $s = -1$ to the state $n' \geq 0$, k_y', k_z', $s = 1$ is proportional to k_z^2. This causes the logarithmic singularity which takes place in the case of scattering by transverse optical phonons to vanish, inasmuch as

$$\sigma_{xx\parallel} \sim \sum_{n'} \int d\varepsilon \, d\varepsilon' \, e^{-\varepsilon/T} \frac{(\varepsilon - \hbar\omega_c/2 + |g|\mu_0 H/2)^{1/2}}{[\varepsilon' - (n' + 1/2)\hbar\omega_c - |g|\mu_0 H/2]^{1/2}}$$
$$\times \delta(\varepsilon - \varepsilon' + \hbar\omega_{0\parallel}). \qquad (2.7)$$

Thus, to the oscillating maxima on the curve of σ_{xx}, which are associated with MPR, there should

*See Erratum, p. 1147.

be added maxima associated with SMPR in scattering by transverse optical phonons; these are located close to the former for reciprocal values of the magnetic field which differ by $|g|\mu_0/\hbar\omega_{0\perp}$. The oscillating maximum associated with transitions of electrons between spin subbands of the zero Landau magnetic band (upon satisfaction of the condition $\hbar\omega_{0\perp} = |g|\mu_0 H$) has no neighboring maximum brought about by MPR.

From (2.4), it is easy to find the shape of the oscillating peak: fall off takes place as the exponential $\exp[-(|g|\mu_0 H - \hbar\omega_0)/T]$ in the direction of stronger-than-resonant magnetic fields and as $[(\hbar\omega_0 - |g|\mu_0 H)/T]^{1/2}$ on the side of weaker magnetic fields.

Inasmuch as the experimentally measured values of the transverse magnetoresistance in the case of a quantizing magnetic field under consideration is associated with σ_{xx} by the relation $\rho_{xx} \equiv \sigma_{xx}/\sigma_{xy}^2$, in which $\sigma_{xy} = n_0 ec/H$ does not depend on the scattering, all that has been said above about the character of the oscillations of σ_{xx} applies equally to ρ_{xx}, in view of the direct proportionality $\rho_{xx} \sim \sigma_{xx}$.

3. OSCILLATIONS OF THE LONGITUDINAL CONDUCTIVITY

We shall consider the behavior of σ_{zz} close to the values of the magnetic field for which the transition of an electron from a lowest spin subband of the zero Landau magnetic band to the upper becomes possible, that is, in the region close to the resonance value of the magnetic field determined by the relation $|g|\mu_0 H = h\omega_0$. Here the basic scattering mechanism in the case of SMPR—scattering by optical phonons with spin flip—interferes with the "phonon" scattering mechanisms, of which we shall consider the following: scattering by acoustical phonons both with and without spin flip, and scattering by polarized vibrations of the lattice (without spin flip).

The value of the current density along the z axis

$$j_z = -e \sum_{n,s,k_y,k_z} \frac{\hbar k_z}{m} f_{n,s}(k_z) \tag{3.1}$$

is determined by the correction to the distribution function $f_{n,s}(k_z)$, which in turn can be found by solution of the kinetic equation

$$\frac{eE}{\hbar k_t}\frac{\partial F_0}{\partial z} = \hat{S}_{\rm opt} f_{n,s}(z) + \hat{S}_{\rm ac} f_{n,s}(z). \tag{3.2}$$

Here the following notation is used:[6)] $F_0 = e^{(\xi-\epsilon)/T}$ is the equilibrium distribution function for the electrons,

$$z = k_z/k_t, \quad k_t = \hbar^{-1}\sqrt{2mT}, \quad \varepsilon = 2\alpha(n+1/2) + z^2 + s\cdot 2\beta,$$
$$2\beta = |g|\mu_0 H/T, \qquad 2\alpha = \hbar\omega_c/T; \tag{3.2a}$$

$\hat{S}_{\rm opt}$ and $\hat{S}_{\rm ac}$ are the scattering operators for optical and acoustical phonons. (For economy, we do not write down the general expressions for them; they are given, for example, in [3].) The so-called "incoming" terms in (3.2) in the case of the extreme quantum limit under consideration can be omitted both for scattering by acoustical phonons (in terms of the small parameter $(mv_s^2\, 2\alpha/T)^{1/2}$), and for optical phonons (small parameter $\sqrt{2\beta/2\alpha}$).

We shall seek a solution of Eq. (3.2) in the form

$$f_{n,s}(z) = \frac{\hbar e k_t}{mT}\frac{\tau_a}{\alpha} E z \chi_{n,s}(|z|)$$
$$\times \exp\left[\frac{\xi}{T} - 2\alpha\left(n+\frac{1}{2}\right) - s\beta - z^2\right], \tag{3.3}$$

where $\tau_a = \pi\hbar^4\rho v_s^2/\sqrt{2}\,E_0^2(mT)^{3/2}$. Inasmuch as "arrival" is not taken into consideration, we immediately obtain an equation for the function $\chi_{n,s}(|z|)$ which is obtained by substitution of (3.3) in (3.2):

$$1 = \frac{1}{|z|}\nu_{n,s}(z^2)\chi_{n,s}(|z|). \tag{3.4}$$

Here we have the notation

$$\frac{1}{|z|}\nu_{n,s}(z^2) = \frac{\tau_a}{\alpha}(-S_{\rm ac} - S_{\rm opt} - S_{\rm ac.sp} - S_{\rm opt.sp}). \tag{3.5}$$

The terms in (3.5) associated with scattering by phonons without spin flip were computed with [3]

$$\frac{\tau_a}{\alpha}(-S_{\rm ac} - S_{\rm opt}) = \sum_{n'} \frac{1}{[z^2 + 2\alpha(n-n')]^{1/2}}$$
$$+ \frac{N_0}{\Gamma}\sum_{n'}\frac{G_{nn'}(y_1^+) + G_{nn'}(y_2^+)}{[z^2 - 2\alpha(n'-n-\omega_0/\omega_c)]^{1/2}}$$
$$+ \frac{N_0+1}{\Gamma}\sum_{n'}\frac{G_{nn'}(y_1^-) + G_{nn'}(y_2^-)}{[z^2 - 2\alpha(n'-n+\omega_0/\omega_c)]^{1/2}},$$
$$\frac{1}{\Gamma} = \frac{1}{4\alpha}\frac{\tau_a}{t_0}\left(\frac{\hbar\omega_0}{T}\right)^{1/2} \tag{3.6}$$

(the notation of [3] is preserved).

By using (1.5) and (1.6) with the substitution

[6)]The designations given here and below actually are not different from those used in [3] which can simplify the comparison of the results for MPR and SMPR in σ_{zz}.

$e_q^{(-)} \to aq$, we obtain the result that the departure from the state n = 0, s = −1, k_y, k_z (as a consequence of the scattering of the electrons by the acoustical phonons with spin flip) is determined by the expression

$$S_{\text{ac.sp}} = -\frac{\alpha}{\tau_{\text{ac.sp}}}\sum_{n'}\frac{z^2}{(z^2-2\alpha n'-2\beta)^{1/2}}$$
$$\times\left[\mathcal{F}_{n'+1}\left(\frac{\tilde{q}_1^2}{2\alpha}\right)+\mathcal{F}_{n'+1}\left(\frac{\tilde{q}_2^2}{2\alpha}\right)\right],$$
$$\mathcal{F}_{n'+1}(x)=\frac{1}{n'!}\int_0^\infty du\,\frac{u^{n'+1}e^{-u}}{u+x},$$
$$\tilde{q}_{1,2}=-z\pm[z^2-2\alpha(n'-n)-2\beta]^{1/2}, \tag{3.7}$$

where ρ is the density of the crystal,

$$\frac{1}{\tau_{\text{ac.sp}}}=\frac{(2mT)^{5/2}a^2d^2}{2\pi\rho v_s{}^2\hbar^6}. \tag{3.8}$$

By using (1.5) and (1.6) and averaging over the directions of the polarization vector of the optical phonons, we obtain an expression including in itself the principal terms and characterizing the "departure" as a result of scattering with spin flip by the optical phonons:

$$-S_{\text{opt.sp}}=\frac{\alpha^2}{2\tau_{\text{opt.sp}}}\left\{\sum_{n'}\frac{N_0}{(z^2-2\alpha n'-2\beta+\hbar\omega_0/T)^{1/2}}\right.$$
$$\times\left[\mathcal{F}_{n'+2}\left(\frac{(\tilde{q}_1^+)^2}{2\alpha}\right)+\mathcal{F}_{n'+2}\left(\frac{(\tilde{q}_2^+)^2}{2\alpha}\right)\right]$$
$$+\sum_{n'}\frac{N_0+1}{(z^2-2\alpha n'+2\beta-\hbar\omega_0/T)^{1/2}}$$
$$\times\left[\mathcal{F}_{n'+2}\left(\frac{(\tilde{q}_1^-)^2}{2\alpha}\right)+\mathcal{F}_{n'+2}\left(\frac{(\tilde{q}_2^-)^2}{2\alpha}\right)\right], \tag{3.9}$$

where

$$\tilde{q}_{1,2}^{+}=-z\pm(z^2-2\alpha n'-2\beta+\hbar\omega_0/T)^{1/2},$$
$$\tilde{q}_{1,2}^{-}=z\pm(z^2-2\alpha n'-2\beta-\hbar\omega_0/T)^{1/2},$$
$$\bar{\rho}=\frac{\bar{M}}{V_{\text{cell}}},\quad \bar{M}=\frac{M_1M_2}{M_1+M_2},\quad \frac{1}{\tau_{\text{opt.sp}}}=\frac{md^2(mT)^{3/2}}{\sqrt{2}\pi\bar{\rho}\omega_0\hbar^5}, \tag{3.10}$$

$\bar{M}$ is the reduced mass of the cell.

By substituting (3.6), (3.7), (3.10) in (3.5), we obtain

$$\frac{1}{|z|}\nu_{0,-1}(z^2)=\sum_{n'}\frac{1}{(z^2-2\alpha n')^{1/2}}$$
$$+\frac{N_0}{\Gamma}\sum_{n'}\frac{G_{0n'}(y_1^+)+G_{0n'}(y_2^+)}{[z^2-2\alpha(n'-\omega_0/\omega_c)]^{1/2}}$$
$$+\frac{N_0+1}{\Gamma}\sum_{n'}\frac{G_{0n'}(y_1^-)+G_{0n'}(y_2^-)}{[z^2-2\alpha(n'+\omega_0/\omega_c)]^{1/2}}$$
$$+\frac{1}{\Gamma_{\text{ac.sp}}}\sum_{n'}\frac{z^2[\mathcal{F}_{n'+1}(\tilde{q}_1^2/2\alpha)+\mathcal{F}_{n'+1}(\tilde{q}_2^2/2\alpha)]}{(z^2-2\alpha n'-2\beta)^{1/2}}$$
$$+\frac{\alpha}{2\Gamma_{\text{opt.sp}}}\sum_{n'}\frac{N_0}{(z^2-2\alpha n'-2\beta+\hbar\omega_0/T)^{1/2}}$$
$$\times\left[\mathcal{F}_{n'+2}\left(\frac{(\tilde{q}_1^+)^2}{2\alpha}\right)+\mathcal{F}_{n'+2}\left(\frac{(\tilde{q}_2^+)^2}{2\alpha}\right)\right]$$
$$+\frac{\alpha}{2\Gamma_{\text{opt.sp}}}\sum_{n'}\frac{N_0+1}{(z^2-2\alpha n'+2\beta-\hbar\omega_0/T)^{1/2}}$$
$$\times\left[\mathcal{F}_{n'+2}\left(\frac{(\tilde{q}_1^-)^2}{2\alpha}\right)+\mathcal{F}_{n'+2}\left(\frac{(\tilde{q}_2^-)^2}{2\alpha}\right)\right], \tag{3.11}$$

where the following notation is used:

$$1/\Gamma_{\text{ac.sp}}=\tau_a/\tau_{\text{ac.sp}},\quad 1/\Gamma_{\text{opt.sp}}=\tau_a/\tau_{\text{opt.sp}}. \tag{3.12}$$

By knowing the correction to the distribution function, we obtain an expression for the longitudinal conductivity

$$\sigma_{zz}=\frac{2}{\sqrt{\pi}}\frac{n_0e^2\tau_a}{\alpha m}e^{-\alpha+\beta}\frac{\text{sh}\,\alpha}{\text{ch}\,\beta}J, \tag{3.13}*$$

where

$$J=\sum_{n,s}e^{-2\alpha n-s\beta}\int_{-\infty}^{\infty}dz\,z^2e^{-z^2}\chi_{n,s}(|z|). \tag{3.14}$$

Because of the condition of the extreme quantum limit $2\beta \gg 1$, we limit ourselves in (3.14) to terms with n = 0 and s = ±1:

$$J=\int_0^\infty dx\,\frac{xe^{-x}}{\nu_{0,-1}(x)}+e^{-2\beta}\int_0^\infty dx\,\frac{xe^{-x}}{\nu_{0,1}(x)},\qquad x=z^2. \tag{3.15}$$

In (3.13), we have made use of a relation between the concentration of free electrons in the crystal and the chemical potential

$$n_0=\frac{1}{L^3}\sum_{n,s,k_y,k_z}F_0=\frac{\sqrt{\pi}}{(2\pi)^2}k_t\,l_H^{-2}\,e^{\zeta/T}\frac{\text{ch}\,\beta}{\text{sh}\,\alpha}. \tag{3.16}$$

The oscillating part of (3.15), which is associated with scattering by optical phonons, is proportional to exp $(-\hbar\omega_0/T)$. In order to keep terms of lowest order in this main small parameter, we regard the integration as actually extending from 0-2β in the first component of (3.15), and in the limits 0-1 in the second. The degree of deviation of the value of the magnetic field from the resonance value will be characterized by the parameter

$$\delta=\frac{\hbar\omega_0}{|g|\mu_0H}-1. \tag{3.17}$$

*sh ≡ sinh, ch ≡ cosh.

We limit ourselves to the case of approach to resonance from the side of higher magnetic fields, where $\delta < 0$. Close to resonance, $|\delta| \ll 1$. In the case $\delta > 0$ (the approach from the side of smaller magnetic fields) the results are similar.

Limiting ourselves to the fundamental terms for $|\delta| \ll 1$, and applying (3.11), we obtain the following expressions for the discontinuous functions $\nu_{0,-1}(x)$ and $\nu_{0,1}(x)$:

1) $\nu_{0,-1}(x) = 1, \quad x < 2\beta|\delta|;$

2) $\nu_{0,-1}(x) = 1 + \frac{2\alpha}{\Gamma_{\text{opt.sp}}} N_0 \left(\frac{x}{x - 2\beta|\delta|}\right)^{1/2},$

$2\beta|\delta| < x < 2\beta(1 - |\delta|);$

3) $\nu_{0,-1}(x) = 1 + \frac{b}{\Gamma}\left(\frac{x}{x - 2\beta(1 - |\delta|)}\right)^{1/2},$

$2\beta(1 - |\delta|) < x < 2\beta;$

4) $\nu_{0,-1}(x) = 1 + \frac{2x}{\Gamma_{\text{ac.sp}}}\left(\frac{x}{x - 2\beta}\right)^{1/2}$

$+ \frac{b}{\Gamma}\left(\frac{x}{x - 2\beta(1 - |\delta|)}\right)^{1/2}, \quad 2\beta < x;$

5) $\nu_{0,1}(x) = 1 + \frac{2x}{\Gamma_{\text{ac.sp}}}\left(\frac{x}{x + 2\beta}\right)^{1/2}$

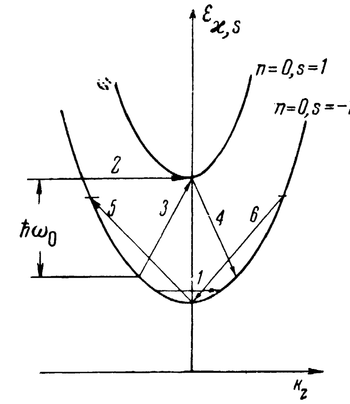
$+ \frac{2\alpha}{\Gamma_{\text{opt.sp}}}\left(\frac{x}{x + 2\beta(1 - |\delta|)}\right)^{1/2},$

$$x < 2\beta(1 - |\delta|); \qquad (3.18)$$

where $b \approx 1.2$; $N_0 = [\exp(\hbar\omega_0/T) + 1]^{-1}$.

The form of the function $\nu_{0,-1}(x)$ in region 1) is determined by the elastic scattering of electrons by acoustical phonons inside the magnetic band $n = 0$, $s = -1$. In region 2), the electron, absorbing an optical phonon and reversing its spin, can undergo a transition to the band $n = 0$, $s = 1$. In region 3), the scattering inside the band $n = 0$, $s = -1$ is possible with emission of an optical phonon. In region 4), the transition to the band $n = 0$, $s = 1$ can take place also under scattering by acoustical phonons. Finally, for the region 5), we give without derivation an expression for the function $\nu_{0,1}(x)$, which is obtained similar to $\nu_{0,-1}(x)$ and under the same fundamental assumptions regarding the "arriving" terms.

In the drawing, the possible transitions of an electron that are vital to the problem under consideration are shown by the arrows. The arrows 1 and 2 indicate transitions in the scattering of electrons by acoustical phonons without and with spin flip, respectively. Arrows 3 and 4 indicate electron transitions in scattering by optical phonons with spin flip (with absorption and emission of an optical phonon, respectively). The arrows 5 and 6 indicate scattering of electrons by optical phonons inside the band $n = 0$, $s = -1$ (with absorption and emission of an optical phonon, respectively).



Possible electron transitions in scattering by acoustical and optical phonons.

We proceed to analysis of the character of the extremum of the longitudinal magnetoresistance ρ_{zz} at the resonance point $H = \hbar\omega_0/|g|\mu_0$.

A. Limiting case of scattering by optical phonons. In this case, the fundamental contribution to the integral J (3.15) is given by two regions inside which the function $\nu_{0,-1}(x)$ is determined by the expression

$$\nu_{0,-1}(x) = \frac{bN_0}{\Gamma}\left(\frac{x}{2\alpha}\right)^{1/2}, \quad x < 2\beta|\delta|;$$

$$\nu_{0,-1}(x) = \frac{bN_0}{\Gamma}\left(\frac{x}{2\alpha}\right)^{1/2} + \frac{2\alpha}{\Gamma_{\text{opt.sp}}} N_0 \left(\frac{x}{x - 2\beta|\delta|}\right)^{1/2},$$

$$2\beta|\delta| < x < 2\beta(1 - |\delta|). \qquad (3.19)$$

Carrying out the necessary calculations, we obtain an expression for ρ_{zz}:

$$\rho_{zz} = \rho(0)\left(\frac{\omega_0}{\omega_c}\right)^{1/2} \frac{b}{4} \frac{1}{1 - F_{\text{sp}}(2\beta|\delta|)}, \qquad (3.20)$$

where

$$\rho(0) = \frac{m}{n_0 e^2 t_0} \exp\left(-\frac{\hbar\omega_0}{T}\right),$$

$$F_{\text{sp}}(x) = e^{-x} \frac{2}{\sqrt{\pi}} \int_0^\infty d\zeta\, e^{-\zeta} \sqrt{\zeta + x}$$

$$\times \left[1 + \frac{b\tau_{\text{opt.sp}}}{8\alpha^2 t_0}\left(\frac{\hbar\omega_0}{T}\right)^{1/2}\left(\frac{\zeta}{2\alpha}\right)^{1/2}\right]^{-1}. \qquad (3.21)$$

As $\delta \to 0$, the value of ρ_{zz} has a maximum.

B. The case of the interference of the mechanisms for scattering by acoustical and optical phonons. Inasmuch as only the part of (3.15) associated with scattering by optical phonons is responsible for the oscillations, we separate it explicitly:

$$J = J_{ac} - \Delta J, \tag{3.22}$$

where

$$\Delta J = \int_0^\infty dx\, xe^{-x}\left(\frac{1}{\nu_{0,-1}^{ac}(x)} - \frac{1}{\nu_{0,-1}(x)}\right) + e^{-2\beta}\int_0^\infty dx\, xe^{-x}\left(\frac{1}{\nu_{0,1}^{ac}(x)} - \frac{1}{\nu_{0,1}(x)}\right) \tag{3.23}$$

or, with account of (3.18),

$$\begin{aligned}\Delta J = &\int_{2\beta|\delta|}^{\infty} dx\, xe^{-x}\left\{1 - \left[1 + \frac{2\alpha N_0}{\Gamma_{opt.sp}}\left(\frac{x}{x-2\beta|\delta|}\right)^{1/2}\right]^{-1}\right\} \\ &+ \int_{2\beta(1-|\delta|)}^{2\beta} dx\, xe^{-x}\left\{1 - \left[1 + \frac{b}{\Gamma}\left(\frac{x}{x-2\beta(1-|\delta|)}\right)^{1/2}\right]^{-1}\right\} \\ &+ \int_{2\beta}^{\infty} dx\, xe^{-x}\left\{\left[1 + \frac{2x}{\Gamma_{ac.sp}}\left(\frac{x}{x-2\beta}\right)^{1/2}\right]^{-1}\right. \\ &\left. - \left[1 + \frac{2x}{\Gamma_{ac.sp}}\left(\frac{x}{x-2\beta}\right)^{1/2} + \frac{b}{\Gamma}\left(\frac{x}{x-2\beta(1-|\delta|)}\right)^{1/2}\right]^{-1}\right\} \\ &+ e^{-2\beta}\int_0^\infty dx\, xe^{-x}\left\{\left[1 + \frac{2x}{\Gamma_{ac.sp}}\left(\frac{x}{x+2\beta}\right)^{1/2}\right]^{-1}\right. \\ &- \left[1 + \frac{2x}{\Gamma_{ac.sp}}\left(\frac{x}{x+2\beta}\right)^{1/2} \right. \\ &\left.\left. + \frac{2\alpha}{\Gamma_{opt.sp}}\left(\frac{x}{x+2\beta(1-|\delta|)}\right)^{1/2}\right]^{-1}\right\}.\end{aligned} \tag{3.24}$$

The character of the extremum of the oscillating part ΔJ at the resonant point $H = \hbar\omega_0/|g|\mu_0$ and its value relative to the "noise" are determined by the difference

$$\Delta J|_{2\beta|\delta|\ll 1} - \Delta J|_{2\beta|\delta|\gg 1} \tag{3.24a}$$

in the values of ΔJ computed in the resonance region ($2\beta|\delta| \ll 1$) and somewhat to one side of it ($2\beta|\delta| \gg 1$).

From (3.24), we find

$$\Delta J|_{2\beta|\delta|\gg 1} = N_0\int_0^\infty dx\, e^{-x}\left(x + \frac{\hbar\omega_0}{T}\right) \times\left[1 - \frac{1}{1 + b\Gamma^{-1}((x+\hbar\omega_0/T)/x)^{1/2}}\right] \tag{3.25}$$

$$= N_0\begin{cases}\sqrt{\pi}\dfrac{b}{\Gamma}\left(\dfrac{\hbar\omega_0}{T}\right)^{3/2}, & \dfrac{b}{\Gamma}\left(\dfrac{\hbar\omega_0}{T}\right)^{1/2} \ll 1 \\ 1 + \dfrac{\hbar\omega_0}{T} - \dfrac{\sqrt{\pi}}{2}\left(\dfrac{\hbar\omega_0}{T}\right)^{1/2}\dfrac{\Gamma}{b}, & \dfrac{b}{\Gamma}\left(\dfrac{\hbar\omega_0}{T}\right)^{1/2} \gg 1\end{cases}$$

In the case of simultaneous satisfaction of conditions a), we have

$$\frac{b}{\Gamma}\left(\frac{\hbar\omega_0}{T}\right)^{1/2} \ll 1, \quad \frac{2\alpha}{\Gamma_{opt.sp}} \ll 1, \quad \frac{2}{\Gamma_{ac.sp}}\left(\frac{\hbar\omega_0}{T}\right)^{3/2} \ll 1, \tag{3.26}$$

when the strongest scattering is that due to acoustical phonons:

$$\Delta J|_{2\beta|\delta|\ll 1} = N_0\left[\frac{b\sqrt{\pi}}{\Gamma}\left(\frac{\hbar\omega_0}{T}\right)^{3/2} + \frac{4\alpha}{\Gamma_{opt.sp}} + \frac{2\sqrt{\pi}}{\Gamma_{ac.sp}}(2\beta)^{5/2} + \frac{30\sqrt{\pi}/8}{(2\beta)^{1/2}\Gamma_{ac.sp}}\right]. \tag{3.27}$$

Inasmuch as the sum of the three latter components of (3.27), which are responsible for the oscillation, is positive, ρ_{zz} must be a maximum in this case, at a value of magnetic field $H = \hbar\omega_0/|g|\mu_0$.

In case b)

$$\frac{b}{\Gamma}\left(\frac{\hbar\omega_0}{T}\right)^{1/2} \gg 1, \quad \frac{2\alpha}{\Gamma_{opt.sp}} \ll 1, \quad \frac{2}{\Gamma_{ac.sp}}\left(\frac{\hbar\omega_0}{T}\right)^{3/2} \ll 1 \tag{3.28}$$

(but $N_0\Gamma^{-1}b(\hbar\omega_0/T)^{1/2} \ll 1$, that is, the background is determined by the acoustic scattering as before) in the region of resonance we have

$$\Delta J|_{2\beta|\delta|\ll 1} = N_0\left[1 + \frac{\hbar\omega_0}{T} - \frac{2\alpha\sqrt{\pi}}{\Gamma_{ac.sp}}\left(\frac{\hbar\omega_0}{T}\right)^{3/2} + \frac{4\alpha}{\Gamma_{opt.sp}}\right], \tag{3.29}$$

and the sum of the last two components can be negative upon satisfaction of an additional condition imposed on the relation between the acoustical scattering with spin flip and scattering by optical phonons also with spin flip, viz.,

$$\frac{\sqrt{\pi}\,\Gamma_{opt.sp}}{2\,\Gamma_{ac.sp}}\left(\frac{\hbar\omega_0}{T}\right)^{3/2} > 1. \tag{3.30}$$

If the conditions (3.28) and (3.30) are satisfied, then ρ_{zz} must be a minimum at the resonance value of the magnetic field $H = \hbar\omega_0/|g|\mu_0$.

4. DISCUSSION OF THE RESULTS

A theoretical analysis shows that SMPR in semiconductors leads to spin-magnetophonon oscillations of the magnetoresistance.

If the electrons are in both spin subbands of the zero Landau magnetic band, the magnetophonon maxima on the curve will be bracketed by spin-magnetophonon maxima. In the extreme quantum limiting case, when the electrons remain only in the lowest spin subband $n = 0$, $s = -1$, the SMP maxima vanish at values of the reciprocal magnetic field.

$$H^{-1} = \left(K\frac{e\hbar}{mc} - |g|\mu_0\right)/\hbar\omega_0, \quad K \geqslant 1 \tag{4a}$$

leaving only the SMP maxima at the values

$$H^{-1}=\left(K\frac{e\hbar}{mc}+|g|\mu_0\right)\Big/\hbar\omega_0,\qquad K\geqslant 0. \quad (4b)$$

The SMP maxima on the curve must have a sharply asymmetric form: the fall off in the direction of larger magnetic fields is exponential, viz.,

$$\exp\left(-\frac{|g|\mu_0 H-\hbar\omega_0}{T}\right), \quad (4c)$$

while in the direction of weaker magnetic fields, it behaves as

$$\left(\frac{\hbar\omega_0-|g|\mu_0 H}{T}\right)^{-1/2}. \quad (4d)$$

In those cases in which the optical vibrations separate into purely longitudinal and purely transverse, the SMP oscillations of ρ_{xx} are brought about by scattering from transverse optical phonons. In correspondence with this, there will enter into all resonance conditions the quantity $\omega_{0\perp}$—the limiting frequency of long wave transverse optical vibrations of the lattice.

Close to the value of the magnetic field $H = \hbar\omega_0/|g|\mu_0$ (we neglect the difference in the limiting frequencies of longitudinal and transverse optical phonons) ρ_{zz} can have either a maximum (in the limiting case of scattering only by optical phonons and for Raman scattering, for example, in the case (3.26)) or a minimum (see conditions (3.28), (3.30)).

In case a) (see 3.26)) a maximum of ρ_{zz} appears, and its ratio to the background is

$$\left[\frac{4\alpha}{\Gamma_{\text{opt.sp}}}+\frac{2\sqrt{\pi}}{\Gamma_{\text{ac.sp}}}(2\beta)^{5/2}\right]e^{-\hbar\omega_0/T} \quad (4.1)$$

(we recall that here T is the temperature in energy units, that is, $T = k_B T°K$).

In case b) (see (3.28), (3.30)) the value of the oscillating part of ρ_{zz} relative to the background is

$$\left[-\frac{\sqrt{\pi}}{\Gamma_{\text{ac.sp}}}\left(\frac{\hbar\omega_0}{T}\right)^{3/2}\frac{\hbar\omega_c}{T}+\frac{2}{\Gamma_{\text{opt.sp}}}\frac{\hbar\omega_c}{T}\right]e^{-\hbar\omega_0/T}. \quad (4.2)$$

Inasmuch as

$$\Gamma_{\text{ac.sp}}/\Gamma_{\text{opt.sp}}=\frac{\tau_{\text{ac.sp}}}{\tau_{\text{opt.sp}}}=\frac{T_0^2}{4T\,\hbar\omega_0} \quad (4.2a)$$

(where $T_0 = \hbar v_s a^{-1} \approx 10^{-2}$ eV), the ratio of the first component in (4.2) to the second is equal to

$$\frac{T_0^2 T^{1/2}}{(\hbar\omega_0)^{5/2}\sqrt{\pi}}<1 \quad \text{when } T\approx 10^{-2}\,\text{eV}, \quad (4.3)$$

that is, in case b) ρ_{zz} has a minimum when the magnetic field is $H = \hbar\omega_0/|g|\mu_0$.

The ratio $\Delta\rho_{xx\,\text{osc}}/\rho_{xx}$ of the oscillating part of the transverse magnetoresistance, brought about by scattering from optical phonons with spin flip, to the background brought about by acoustical scattering,[1] is equal to

$$\frac{\Delta\rho_{xx\,\text{osc}}}{\rho_{xx}}=\frac{1}{\Gamma_{\text{opt.sp}}}\frac{\hbar\omega_c}{T}e^{-\hbar\omega_0/T} \quad (4.4)$$

On the other hand, if the background is brought about by scattering from optical phonons without spin flip, as is the case when ρ_{xx} satisfies the condition

$$\frac{1}{\alpha}\frac{\tau_a}{t_0}\left(\frac{\hbar\omega_0}{T}\right)^{1/2}e^{-\hbar\omega_0/T}>1, \quad (4.4a)$$

and ρ_{zz} satisfies the condition

$$\frac{2}{\Gamma}e^{-\hbar\omega_0/T}=\frac{1}{2\alpha}\frac{\tau_a}{t_0}\left(\frac{\hbar\omega_0}{T}\right)^{1/2}e^{-\hbar\omega_0/T}>1, \quad (4.4b)$$

this ratio is equal to

$$\frac{\Delta\rho_{xx\,\text{osc}}}{\rho_{xx}}=\frac{1}{2}\left(\frac{\hbar\omega_0}{T}\right)^{1/2}\left(\frac{\hbar\omega_c}{T}\right)^2\frac{t_0}{\tau_{\text{opt.sp}}}. \quad (4.5)$$

In the last case one can measure $t_0/\tau_{\text{opt. sp}}$. Substituting the limiting value $t_0/\tau_{\text{opt. sp}}$, we find that the maximum value of the ratio (4.5) is

$$\frac{1}{\Gamma_{\text{opt.sp}}}\frac{\hbar\omega_c}{T}\left(\frac{\hbar\omega_0}{T}\right)^{1/2}e^{-\hbar\omega_0/T}$$

that is, it is of the same order as (4.4) when background is brought about by acoustical scattering.

For the longitudinal effect, the ratio of the oscillating part to the background is

$$\frac{\Delta\rho_{zz\,\text{osc}}}{\rho_{zz}}=\frac{1.8}{\Gamma_{\text{opt.sp}}}\frac{t_0}{\tau_a}\left(\frac{\hbar\omega_c}{T}\right)^2\left(\frac{\omega_c}{\omega_0}\right)^{1/2}, \quad (4.6)$$

that is, it is of the same order as for the transverse effects.

Under the conditions of experiment [17] for InSb, the background of the magnetoresistance is brought about by scattering by acoustical phonons ($\Gamma^{-1} > 1$, $\Gamma^{-1}e^{-\hbar\omega_0/T} \ll 1$) and case b) is clearly realized, although the question as to why the magnetophonon oscillation is a minimum and not a maximum in this case is not very clear. On the experimental curve, the ratio of the oscillation to the noise is about 20%. The value of $\Delta\rho_{zz}$ consists of a slowly changing part $\overline{\Delta\rho_{zz}} \approx \rho_0$ and an oscillating part $\Delta\rho_{zz\,\text{osc.}}$. Inasmuch as the real noise $\overline{\rho_{zz}} = \rho_0 + \overline{\Delta\rho} \approx 2\rho_0$, we have $\overline{\Delta\rho_{zz\,\text{osc.}}}/\rho_{zz} = 10\%$. Substituting in (4.2)

$$\hbar\omega_c/T\approx 9,\quad T\approx 10^{-2}\,\text{eV},\quad \hbar\omega_0/T\approx 3, \quad (4.6a)$$

we get

$$1/\Gamma_{\text{ac.sp}}=\Delta\rho_{zz\,\text{osc}}/5\rho_{zz}=2\cdot 10^{-2}. \quad (4.6b)$$

Inasmuch as

$$\frac{1}{\Gamma_{ac.sp}}=\left(\frac{d}{E_0}\right)^2\frac{T}{\hbar^2/2ma^2}, \qquad (4.6c)$$

it follows from the experimental data that $d/E_0 \approx 10$, which is in excellent agreement with our estimates of the constant d in [13]. We note that the possibility of obtaining $\tau_{ac.\ sp}$ from experiment is brought about by the subtle interference effect in the simultaneous scattering by acoustical and optical phonons. The value of

$$\frac{1}{\Gamma_{opt.sc}}\sim\frac{1}{10\,\Gamma_{ac.sp}}\approx 2\cdot 10^{-3} \qquad (4.6d)$$

and, in accord with (4.4), $\Delta\rho_{xx\ osc}/\rho_{xx}$ under the conditions of [17] is a quantity of the order of several per cent.[7)]

We note that in accordance with the results of our theory, under the conditions in which the background is brought about by acoustical scattering, the smallness of the transverse effects in comparison with the longitudinal is parametric and not numerical, that is, relative to the parameter

$$\left[\left(\frac{\hbar\omega_0}{T}\right)^{3/2}\frac{\tau_{opt.\ sc}}{\tau_{ac.\ sp}}\right]^{-1}$$

The value of the g factor, which is determined from the minimum on the ρ_{zz} curve, is 56.

Thus, from experiments on measurement of SMP oscillations of the magnetoresistance, it is possible to obtain $\tau_{ac.\ sp}$, $\tau_{opt.\ sp}$ and the g factor. In principle, the g factor can be determined from data on the splitting of the oscillation maxima in the Shubnikov-deHaas effect. However, under experimental conditions (see, for example, [23]), the diffuseness of the Landau levels as the result of scattering causes the uncertainty in the determination of the g factor to be ~ 50%.

Thus it must be admitted that at the present time SMPR is apparently the most convenient and simplest method of determining a number of spin characteristics of a solid.

Among these yet unexplained factors is why $\bar{\rho}(H) = 2\rho_0$, and not $3\rho_0$ as follows from [3]. It is likewise not clear why, under the conditions when $1/\Gamma > 1$, ρ_{zz} has a minimum and not a maximum under conditions of magnetophonon resonance. It is possible that this is connected with the approximate character of the estimates in [3]

7)We have learned that I. M. Tsidil'kovskiĭ and his colleagues succeeded in measuring the oscillation of ρ_{xx} in regions of magnetic field $H = \hbar\omega_0 / |g|\mu_0$. The amount of the oscillation was ≈ 1% of the background [22].

Finally, it should be noted that the present theory gives the correct order of magnitude of the quantities measured experimentally and in principle contains in itself the necessary data for the determination of the shape of the oscillations of ρ_{zz}.

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1 V. L. Gurevich and Yu. A. Firsov, JETP **40**, 198 (1961), Soviet Phys. JETP **13**, 137 (1961).

2 Yu. A. Firsov and V. L. Gurevich, JETP **41**, 512 (1961), Soviet Phys. JETP **14**, 367 (1962).

3 V. L. Gurevich and Yu. A. Firsov, JETP **47**, 734 (1964), Soviet Phys. JETP **20**, 489 (1965).

4 A. L. Éfros, FTT **3**, 2848 (1961), Soviet Phys. Solid State **3**, 2079 (1962).

5 V. L. Gurevich, Yu. A. Firsov and A. L. Éfros, FTT **4**, 1813 (1962), Soviet Phys. Solid State **4**, 1331 (1963).

6 S. T. Pavlov and Yu. A. Firsov, FTT **6**, 3608 (1964), Soviet Phys. Solid State **6**, 2887 (1965).

7 Yu. A. Firsov, V. L. Gurevich, R. V. Parfen'ev and S. S. Shalyt, Phys. Rev. Lett. **12**, 660 (1964).

8 V. L. Gurevich, Yu. A. Firsov, R. V. Parfen'ev and S. S. Shalyt, Proc. Inter. Conf. on Semiconductors Phys., Paris, 1964, p. 653.

9 S. M. Pury and T. H. Geballe, Bull. Am. Phys. Soc. **8**, 309 (1963).

10 S. S. Shalyt, R. V. Parfen'ev, and V. M. Muzhdaba, FTT **6**, 647 (1964), Soviet Phys. Solid State **6**, 508 (1964).

11 R. V. Parfen'ev, S. S. Shalyt and V. M. Muzhdaba, JETP **47**, 444 (1964), Soviet Phys. JETP **20**, 294 (1965).

12 V. M. Muzhdaba, R. V. Parfen'ev and S. S. Shalyt, FTT **6**, 3194 (1964), Soviet Phys. Solid State **6**, 2564 (1965).

13 S. T. Pavlov and Yu. A. Firsov, FTT **7**, 2634 (1965), Soviet Phys. Solid State **7**, 2131 (1966).

14 E. O. Kane, J. Phys. Chem. Sol. **1**, 82, 249 (1956).

15 G. Benski, Phys. Rev. Lett. **4**, 62 (1960).

16 L. M. Roth, B. Lax and S. Zwerdling, Phys. Rev. **114**, 90 (1959).

17 D. V. Mashovets, R. V. Parfen'ev and S. S. Shalyt, JETP **47**, 2007 (1964), Soviet Phys. JETP **20**, 1348 (1965). JETP letters **1**, No. 3, 2 (1965), transl. **1**, 77 (1965).

18 I. M. Tsidil'kovskiĭ, M. M. Aksel'rod and V. I. Sokolov, FTT **7**, 316 (1965), Soviet Phys. Solid State **7**, 253 (1965).

19 M. M. Aksel'rod and I. M. Tsidil'kovskiĭ, Phys. Stat. Solidi **8**, K15 (1965), **9**, K163 (1965).

[20] L. E. Gurevich and S. T. Pavlov, ZhTF **30**, 41 (1960), Soviet Phys. Tech. Phys. **5**, 37 (1960).
[21] R. Kubo, H. Hasegawa and N. Hashitsumo, J. Phys. Soc. Japan **14**, 56 (1959).
[22] I. M. Tsidil'kovskiĭ, M. M. Aksel'rod and Z. I. Uritskii, Phys. Stat. Solidi (in press).
[23] S. T. Pavlov, R. V. Parfen'ev, Yu. A. Firsov and S. S. Shalyt, JETP **48**, 1565 (1965), Soviet Phys. JETP **21**,

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Formula (2.4) for the case $n' = 0$ should read

$$\sigma_{xx\perp} = \sigma_0 \left\{ \int \frac{d\varepsilon\, d\varepsilon'\, e^{-\varepsilon/T}\, \delta\left(\varepsilon - \varepsilon' + \hbar\omega_{0\perp}\right)}{[\varepsilon(\varepsilon' - |g|\mu_0 H)]^{1/2}} + e^{\hbar\omega_0/T} \int \frac{d\varepsilon\, d\varepsilon'\, e^{-\varepsilon/T}\, \delta(\varepsilon - \varepsilon' - \hbar\omega_{0\perp})}{[\varepsilon'(\varepsilon - |g|\mu_0 H)]^{1/2}} \right\},$$

so that the oscillating increment $\sigma_{xx\perp}$ doubles in value.

A more detailed analysis of formulas (3.22)—(3.24) shows that in the case of the mixed scattering mechanism the maximum on the $\rho_{zz}(H)$ curve (case a) appears under more stringent conditions than (3.24), namely

$$\frac{b}{\Gamma}\left(\frac{\hbar\omega_0}{T}\right)^{1/2} \gg 1, \qquad \frac{2\alpha}{\Gamma_{\text{opt.sp}}} > 1, \qquad \frac{2\alpha}{\Gamma_{\text{ac.sp}}}\left(\frac{\hbar\omega_0}{T}\right)^{1/2} < 1.$$

The minimum of $\rho_{zz}(H)$ at resonance occurs if the sign of the middle inequality is reversed and in addition

$$\frac{1}{\Gamma_{\text{opt.sp}}} < \frac{1}{\Gamma_{\text{ac.sp}}}\left(\frac{\hbar\omega_0}{T}\right)^{3/2}.$$

The ratio of the SMPR oscillation to the last MPR oscillation at $\Gamma \approx 1$ is of the order of $2\alpha(\hbar\omega_0/T)^{1/2}\Gamma_{\text{ac.sp}}$. Experiment with InSb[17] gave for this ratio a value of the order of unity, as would be the case if $1/\Gamma_{\text{ac.sp.}} \approx 10^{-1}$. This value of $1/\Gamma_{\text{ac.sp.}}$ seems somewhat too high to us.

The results do not depend on the sign of the g-factor.