INVESTIGATION OF THE HIGH-FREQUENCY JOSEPHSON CURRENT

I. M. DMITRENKO and I. K. YANSON

Physico-technical Institute for Low Temperatures, Academy of Sciences, Ukrainian S.S.R.

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The results are given of an experimental investigation of the dependence of the step-like structure of the current–voltage characteristics of Sn–I–Sn superconducting tunnel junctions on temperature, constant magnetic field, and the dimensions of the junctions. It is shown that the steps are due to the interaction between the density wave of the Josephson current and high-frequency resonance electromagnetic oscillations in the tunnel junction, which forms a strip resonator. Cases are investigated where the junction width is greater or less than twice the depth of penetration of the magnetic field into the junction. The experimental results are in qualitatively good agreement with the theory.

JOSEPHSON predicted theoretically[1] a new type of tunnel current flowing between two superconductors separated by a very thin (≈ 10 Å) dielectric layer. This current is not due to the motion of one-particle excitations (electrons) but involves the motion of electron pairs in the "condensed" superconducting states, i.e., the tunnel current has the same nature as the usual nondecaying current in superconductors. This form of tunnel current is due to the overlap of the wave functions of "superconducting" electrons in the region of a sufficiently thin barrier, which permits tunnel transitions of electrons through the barrier without the breakdown of the pair correlations (Cooper effect) between electrons with opposite momenta and spins. It is manifested by a finite current through the barrier without any potential difference across the barrier, i.e., a tunnel junction is "superconducting." Several papers have described experimental investigations of this current—which is also called the constant superconducting Josephson tunnel current—and the results of these investigations[2-4] are in good agreement with the theoretical conclusions.[5-6]

If the potential difference V across a barrier is not equal to zero, then, according to Josephson,[1] in addition to the usual current of quasiparticles between superconductors (which always flows at $T > 0$), there may be a superconducting tunnel current which will vary harmonically with time at a frequency $\omega = 2eV/\hbar$. The tunnel transitions of electrons through the barrier can, in this case, be represented as quantum transitions of Cooper electron pairs from a superconductor on one side of the barrier to a superconductor on the opposite side. Then, according to Josephson,[5] to obey the law of conservation of energy a photon[1)] should be emitted (or absorbed) at a frequency

$$\hbar\omega = 2eV. \tag{1}$$

Indirect confirmation of the existence of the alternating Josephson current is provided by the experiments carried out by Shapiro et al.[8] In these experiments, a tunnel structure exhibiting the Josephson effect was subjected to external electromagnetic radiation in the microwave range. Then, the current–voltage characteristic exhibited a number of regions with an almost vertical slope, which are called "steps" and represent current increases at almost constant voltages, the values of which are related to the frequency of the external radiation by the expression $V_n = n\hbar\omega/2e$, where $n = 1, 2, 3, \ldots$. The appearance of the "steps" in the current–voltage characteristics obtained by dc measurements was interpreted by Shapiro et al. as being a consequence of the modulation of the alternating Josephson current frequency by the external radiation frequency. The observed "steps" represented the constant components of such a frequency-modulated superconducting tunnel current.

Further indirect confirmation of the existence of the alternating Josephson current was provided by the observation of a maximum, of the resonance kind, in the current–voltage characteristics of Pb–I–Pb Josephson tunnel structures[9] (I denotes dielectric). The origin of this maximum and its

[1)]Processes involving phonon participation are also probable.[7]

dependence on a constant magnetic field H_0 could be explained satisfactorily on the basis of an interaction between the alternating Josephson current and a retarded electromagnetic wave propagated between lead films in the region of the junction.

Several investigators[3, 4, 10, 11] have reported the observation of the step-like structure in the current–voltage characteristics of Sn–I–Sn tunnel junctions in the absence of external radiation. It was suggested[3, 4, 12] that the presence of the steps in the current–voltage characteristics was due to the excitation of electromagnetic oscillations in the region of the junction and data were reported confirming this suggestion.[10, 11]

A direct experiment that we carried out with Svistunov[13] showed that the appearance of a step in the current–voltage characteristic was accompanied by the emission of photons at a frequency satisfying the Josephson frequency relationship of Eq. (1). The observation of electromagnetic radiation generated by a tunnel structure provided a direct proof of the existence of the alternating superconducting Josephson tunnel current.

The present paper reports the results of an experimental investigation of the dependence of the step-like structure of the current–voltage characteristics of Sn–I–Sn tunnel junctions on various parameters (dimensions of the junction, temperature, and a constant magnetic field). It was found that the observed features were in agreement with the idea of the existence of an interaction between the alternating Josephson current and resonance oscillation modes in the superconducting tunnel structure.

1. EXPERIMENTAL METHOD

We investigated the current–voltage characteristics of Sn–I–Sn tunnel structures at temperatures below T_c of tin ($\approx 3.8\,°K$). Films of tin of various widths and 1000–2000 Å were deposited in $\sim 10^{-6}$ torr vacuum onto glass slides on which indium had been deposited first; these tin films were oxidized in 30 min at an air pressure of ≈ 0.5 torr. Then, at right-angles to the oxidized films, additional films of tin of the same thickness were deposited. The film thickness was determined with a microinterferometer to within 10–20%. By suitably selecting such parameters as the rate of evaporation of the metal, the duration of oxidation and the air pressure during oxidation, and by selecting a suitable substrate temperature, we could control the thickness of the insulating layer between the tin films. The Josephson tunnel effect was observed in low-resistance tunnel structures

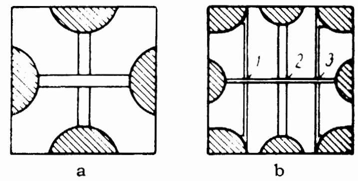

FIG. 1. Investigated samples of thin-film Sn–I–Sn tunnel structures. The shaded areas represent indium contacts. The numbers indicate separate tunnel junctions.

having $\sim 10^{-2}\ \Omega$.mm resistivity in the normal state. The investigated samples are shown in Fig. 1. Figure 2a shows the arrangement used for the automatic recording of the current–voltage characteristics. The current and potential leads were soldered to the tunnel structure using the standard four-probe layout. The tunnel structure was placed in a 3-cm waveguide. The waveguide was inside a small single-layer solenoid, 128 mm long and 38 mm in diameter, placed in a magnetic screen made of annealed Armco iron. The magnetic screen reduced the external magnetic fields (including the terrestrial field) by a factor of about 30. The solenoid was calibrated with a mi-

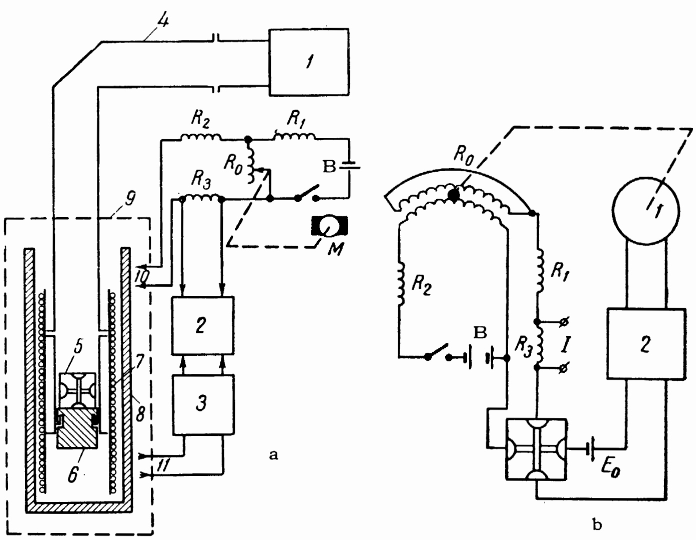

FIG. 2. a) Schematic representation of the experimental apparatus used for automatic recording of the current–voltage characteristics (during simultaneous investigation of the interaction of a tunnel structure with an electromagnetic field): 1) sensitive radiation receiver of the P5-10 type; 2) two-coordinate automatic potentiometer; 3) dc amplifier; 4) 23 × 10 mm waveguide; 5) tunnel structure; 6) short-circuiting piston; 7) single-layer solenoid; 8) magnetic screen; 9) liquid-helium bath; 10, 11) current and potential leads; R_0 is the slide wire; R_1, R_2, R_3 are resistors; B is the battery; M is the motor. b) Circuit for automatic recording of critical currents at the steps as a function of a magnetic field (the circuit supplying current to the solenoid is not shown): 1) motor of the RD-09 type; 2) dc amplifier; R_0 is the slide wire; R_1, R_2, R_3 are resistors; B is the battery; E_0 is the reference voltage source.

crowebermeter of the F-18 type. The tunnel structure was oriented in such a way that the lower film was parallel to the solenoid axis. The whole system was placed in a liquid-helium cryostat. The bath temperature was reduced by pumping helium vapor and it could be kept constant in the range 1.6–4.2°K to within $\sim 10^{-3}$ °K. A highly sensitive microwave receiver, used to pick up the radiation generated by the structure, or a microwave oscillator, used to study the influence of the external radiation on the current–voltage characteristics, could be connected to the output flange of the waveguide. Since the resistance of the connecting leads and the additional resistance R_2 were much higher than the resistance of the tunnel structure, the current through the junction was controlled with a slide wire whose slide was set in motion by a reversible motor of the RD-09 type fitted with a reduction gear. The current was increased automatically and smoothly from zero to some maximum value.

A voltage, proportional to the current, was applied to one coordinate of a two-coordinate automatic potentiometer, and the amplified voltage developed at the tunnel structure was applied to the other coordinate. Thus, we could obtain automatically the current–voltage characteristic in a constant magnetic field (in the range 0–35 Oe) and at a constant temperature. Typical current–voltage characteristics are shown in Fig. 3. The dependence of the critical values of the current at the steps on the constant magnetic field was found both at individual points, by analyzing a large number of current–voltage characteristics recorded at different values of the constant magnetic field, and automatically using the circuit shown in Fig. 2b. The operating principle of the circuit was as follows. A slide wire R_0, coupled to the axle of the motor, RD-09, governed the current through the structure. When the voltage across the structure was equal to zero, the amplifier input received a constant reference voltage of such polarity that the motor moved the slide in the direction which increased the current through the tunnel junction. As soon as the current reached a critical value, the voltage across the junction increased suddenly. The direction of flow of the current through the junction was selected in such a way that the polarity of the voltage developed across the junction was opposite to the polarity of the reference voltage. If the voltage across the structure was greater than the reference voltage by a value of the order of several microvolts, the total voltage across the amplifier input changed its sign and the

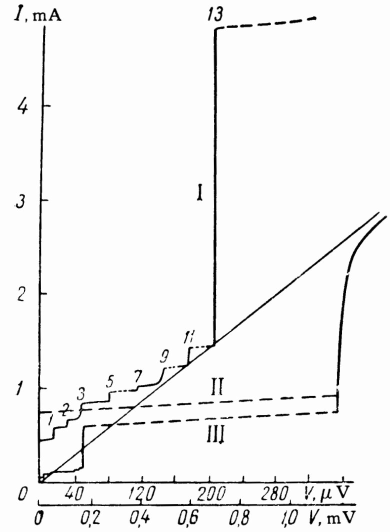

FIG. 3. Current–voltage characteristics of an Sn–I–Sn tunnel junction. Junction area S = 1.25 × 0.36 mm; T = 1.6°K. I) Initial part of the current–voltage characteristic, H = 1.12 Oe, numbers of steps are shown. The phase velocities [cf. Eq. (9)] are equal for the thirteenth step. II) Complete current–voltage characteristic, H = 0; III) the same, but for H = 1 Oe. For II and III, the current scale should be multiplied by 10; the scales for the voltage V are shown below.

motor began to move the slide in the opposite direction until the current reached a value such that a zero potential difference was again established across the junction. The process was then repeated. When the constant magnetic field in the solenoid was altered, the value of the critical current changed. When a voltage proportional to the current flowing through the solenoid was applied to one coordinate of a two-coordinate automatic potentiometer, a voltage proportional to the current flowing through the tunnel structure, was applied to the second coordinate and the current through the solenoid was varied smoothly with time, we obtained an oscillating curve whose envelope gave the required dependence of the critical current on the magnetic field. By selecting the reference voltage to be $V_p^{(n)} + \delta$, where $V_p^{(n)}$ is the value of the voltage corresponding to the n-th step (including n = 0, V = 0) and δ is of the order of several microvolts, we were able to record automatically the critical current $I_c^{(n)}(H)$ for any step, as long as the value of the critical current for a given step was higher than that for the steps with lower values of n. It was found, however, that automatic recording gave the critical current of the n-th step $I_c^{(n)}(H)$, even if that current was less than the critical current of any (n – m)-th step (here, m < n). This was obviously due to the strong hysteresis of the current–voltage characteristics of the tunnel structures when the current through the junction was increased or decreased.

2. PROPERTIES OF THE STEPS IN THE CURRENT-VOLTAGE CHARACTERISTICS OF Sn–I–Sn TUNNEL STRUCTURES

We shall consider a tunnel junction formed by two identical superconductors, I and II, separated by a thin dielectric layer of thickness $l \sim 10$ Å (Fig. 4). The superconductors are in the form of plane-parallel plates of width w_1, length w_2, and thicknesses t_1 and t_2. A constant magnetic field $H_0 = H_0\mathbf{y}_0$ is applied parallel to one side of the junction along the y axis. The magnetic field H_0 penetrates the tunneling region between the two films and the superconductors to a depth λ_L (the London penetration depth). The electric field in the junction is independent of the coordinate x,[9, 14] so that the voltage between the films is $V = E_x l$.

In the Josephson tunnel effect, a superconducting tunnel current is produced whose density is

$$j_x = j_s \sin \varphi, \tag{2}$$

where φ is the phase difference between wave functions for the superconductors I and II. If $H_0 \neq 0$ and $V \neq 0$, the current density is in the form of a traveling wave:[9]

$$j_x = j_s \sin(\omega t - kz),$$

where $\omega = 2eV/\hbar$ and $k = 2edH_0/\hbar c$; $d = 2\lambda_L + l$. The density wave of the current generates an alternating magnetic field of the same frequency and this field interacts with the current. Depending on the relationship between phases, we can have energy exchange between the electromagnetic wave and the current wave in either direction. The simultaneous solution of Maxwell's equations for the field and Josephson's equations for the current gives a complex expression for the current as a function of the voltage and the constant magnetic field.[10, 14]

According to [5], in a system of two superconductors separated by a dielectric layer (cf. Fig. 4), retarded electromagnetic waves may be propagated at the phase velocity:

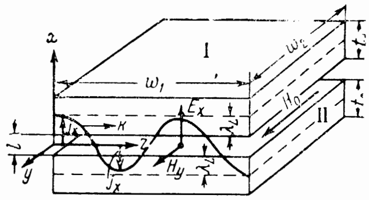

FIG. 4. Schematic representation of a tunnel junction.

$$\bar{c} = \frac{c}{\sqrt{\varepsilon\mu}}\left\{1 + \frac{\lambda_L}{l}\left[\coth\frac{t_1}{\lambda_L} + \coth\frac{t_2}{\lambda_L}\right]\right\}^{-1/2} \tag{3}$$

or, in the case of sufficiently thick films (t_1, $t_2 \gg \lambda_0$):

$$\bar{c} = c\sqrt{l/\mu\varepsilon d}. \tag{4}$$

Here, ε and μ are the permittivity and permeability of the dielectric, $\kappa \approx (l/2\varepsilon\lambda_L)^{1/2}$ is the damping factor of the electromagnetic wave.

If we consider electromagnetic waves propagated along the same direction as the current density wave, we can see that in the dielectric the non-zero components of the field E_x and H_y are related by

$$H_y = (a^2 - k^2)E_x / a\mu\omega,$$

where a is the wave vector of the electromagnetic wave, which is equal to $\omega/\bar{c}$, and $k^2 = a^2 - \omega^2\varepsilon\mu$. Since the wave impedance of such a strip transmission line is very low ($Z_0 = V/I \approx E_x l/H_y W_2 \sim 10^{-3}\,\Omega$), the wave propagated in the system will be almost completely reflected at the boundary, which is at the edge of the lower film, and the wave propagated along the z axis will have a number of discrete frequencies $\omega_m^{(n)}$ (n is the order of the resonance), at which a system of constant waves may be established along the width w_1. Then, there will be an antinode of the electric field and a node of the magnetic field at the boundary. Naturally, if the constant magnetic field is directed along the z axis, the current wave will be propagated along the y axis and it will excite electromagnetic waves along the same direction. In this case, we obtain a different system of resonance frequencies associated with the dimension w_2. If we allow for the fact that, at resonance, an integral number of half waves must be equal to the resonator length parallel to the direction of propagation of the wave, we can obtain from Eqs. (1) and (4) the complete system of steps along the voltage axis of the current–voltage characteristic of a tunnel junction:[11]

$$V_{p^{(n)}} \approx \frac{\hbar}{2e}\sqrt{\frac{l}{\varepsilon d}}\frac{\pi c n}{w_1}; \qquad n = 1, 2, 3, \ldots \tag{5}$$

Assuming the usual relationship for the temperature dependence of λ_L

$$\lambda_L(T) = \lambda_L(0)\left[1 - (T/T_c)^4\right]^{-1/2},$$

we can obtain the temperature dependence of the

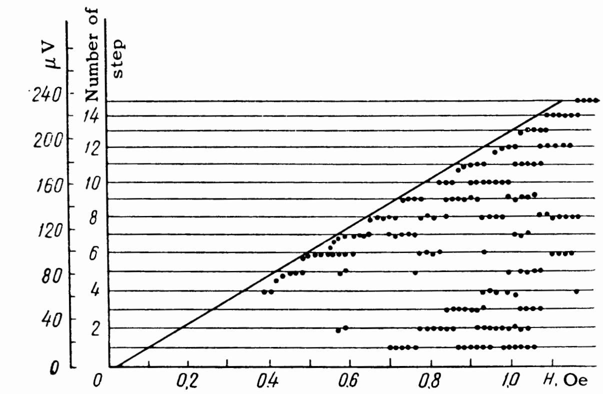

FIG. 5. Dependence, on a constant magnetic field, of the positions of the steps along the voltage axis of the current–voltage characteristics for a junction with w_1 = 1.25 mm. The steps are classified (horizontal lines), each step being given a number n; T = 1.6°K.

system of steps

$$V_{p^{(n)}}(T) = V_{p^{(n)}}(0)\,[1 - (T/T_c)^4]^{1/4}. \qquad (6)$$

Figure 5 shows the dependence of the positions of the steps in the current–voltage characteristics of a junction, of width w_1 = 1.25 mm, on a constant magnetic field. These results were obtained by recording automatically the current–voltage characteristics for various fixed values of the magnetic field. The positions of the steps could be classified by ascribing to each step a definite number n. The position of a step was found to be practically independent of the magnetic field. After making such a classification, we found that each step was separated by equal distances from its neighboring steps: $V_p^{(n)} = nV_p^{(1)}$ [cf. Eq. (5)], but some reduction in the interval was noticeable at higher values of n. Depending on the value of the constant magnetic field, the current–voltage characteristics exhibited a particular set of steps; the steps with higher numbers appeared in stronger magnetic fields. The interval between the positions of the steps on the voltage axis, equal to the position of the first step $V_p^{(1)}$, depended on the dimension w along which resonance occurred. To investigate the dependence of the position of the first step (and, therefore, of the interval between the steps) on the dimension w, we used a complex tunnel structure consisting of three tunnel junctions having lower films of different widths (Fig. 1b). Since all three tunnel junctions were prepared at the same time under the same conditions, they had the same tunnel resistance, found from the asymptotic behavior of the current–voltage characteristic at V > 2Δ. Consequently, the properties of the barrier in the three junctions were the same. Figure 6 shows the experimental de-

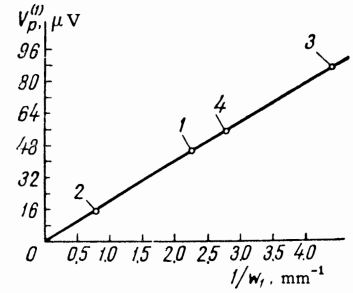

FIG. 6. Dependence of the voltage at the first step on the reciprocal of the junction width. The numbers represent the junctions shown in Fig. 1b; T = 1.6°K.

pendence of the position of the first step on the reciprocal of the film width. Three points (1, 2, 3) represent the widths of the corresponding lower films (Fig. 1b). Point 4 was obtained by investigating the system of steps of the junction 2 in a magnetic field directed along the plane of the junction at right-angles to the lower film. The magnetic field was established by passing an additional current through the lower film (cf. [4]). In this case, resonance occurred along the dimension w_2 (i.e., along the width of the upper film). All the points in Fig. 6 fit well a straight line, confirming Eq. (5). The tangent of the angle of slope of the straight line is $\approx 2.0 \times 10^{-6}$ V/cm, which gives the value $\kappa \approx 1/15.2$ for the retardation factor of the electromagnetic wave. If we assume the value 10 Å for l, the permittivity of the insulating layer is found to have a reasonable value, ε = 2.3. It should be remembered that the value of the retardation factor depends on the method of preparation of the tunnel structure.

The temperature dependence of the position of the first step on the voltage axis is shown in Fig. 7. It is evident that the temperature dependence is important only in the direct vicinity of T_c. This is the reason why the temperature dependence of the positions of the steps was not noticed in the earlier investigations.[4] The experimental points in Fig. 7 fit quite well the theoretical de-

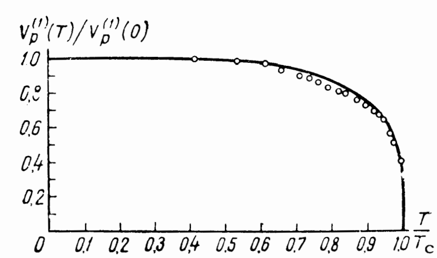

FIG. 7. Temperature dependence of the voltage at the first step, plotted in reduced units, T_c = 3.8°K. The continuous line represents the theoretical dependence of Eq. (6).

pendence of Eq. (6). Better agreement between the theory and experiment can obviously be obtained by using the more complicated expression of Eq. (3) for the retardation factor, which allows for the finite thickness of the superconducting films. As $T \to T_c$, the depth $\lambda_L \to \infty$ and the approximation $t_1, t_2 \gg \lambda_L$ ceases to be valid. Thus, from the reported experimental data we may conclude that the positions of the steps along the voltage axis correspond to the resonance frequencies for a strip resonator with open walls.

The lowest resonance frequency represents the fundamental (dominant) mode of such a resonator. It cannot be made much less than 10^3 Mc since this mode would then be unstable on moving to the position with V = 0 across the structure.[16] Moreover, the preparation of tunnel junctions of large dimensions meets with considerable technological difficulties. The highest resonance frequency of oscillations excited by the alternating Josephson current is limited by the value of the energy gap of the superconductors used in the junction. When the frequency becomes greater than $2\Delta/\hbar$, the electromagnetic energy losses in a superconductor increase rapidly due to the excitation of electrons across the gap. This reduces the Q of the resonator. In the experiments, the height of the steps decreases and they become strongly "diffuse" along the voltage axis when $V_p^{(n)} \gtrsim \Delta$. A rise in temperature also increases the losses in the resonator, which produces noticeable finite slopes in the steps. If the films are very narrow, the Q of the resonator is markedly reduced by the edge effects.[15]

It was shown in [14] that the constant component of the Josephson current should be in the form of sharp maxima, located at the voltages $V_p^{(n)}$. The step-like nature of the experimentally observed current–voltage characteristics is due to the fact that a current is caused to flow across a junction and the resultant voltage is recorded. According to [14], the slope of the step may then be related to the value of the Q of the resonator. However, it should be remembered that, due to the approximate nature of the theory, this can be done only for sufficiently low values of Q_n. The dependence of the height of the n-th step on the magnetic field is given by the formula[14]

$$\bar{j}_n^{max} = j_s\left(\frac{w_1}{2\pi n\lambda_j}\right)^2 Q_n F_n^2\left(\frac{\Phi}{\Phi_0}\right), \tag{7}$$

where $\Phi = 2H_0\lambda_L w_1$ is the flux penetrating the junction, and $\Phi_0 = \hbar c/2e$. The function F_n is given by

$$F_n(X) = \frac{2}{\pi}\frac{X}{|X^2-(n/2)^2|}\begin{cases}|\cos\pi X|; & n=1,3,5\ldots\\ |\sin\pi X|; & n=2,4,6\ldots\end{cases},$$

where Q_n is the Q-factor for the n-th oscillation mode, and $\lambda_j^2 = \hbar c^2/16\pi e\lambda_L j_s$ is the square of the so-called "Josephson penetration depth."

Equation (7) is valid only in the case $j_n(H) \ll j_s$, or when

$$\left(\frac{w_1}{2\pi n\lambda_j}\right)^2 Q_n \ll 1. \tag{8}$$

It is clear that there will always be steps with a number n sufficiently high to satisfy Eq. (8) (Q_n also increases at higher values of n), but the theory is not valid for steps with low values of n even if Q_n is sufficiently high.

All the experimentally investigated junctions can be divided arbitrarily into two large groups. The first group includes junctions in which the dimension w_1 is less than or equal to twice the penetration depth of the magnetic field H_0 into a junction—$2\lambda_j$. Such junctions are characterized by the fact that the magnetic field in them can be assumed to be approximately uniform. The dependence of the critical constant Josephson current (height of the zeroth step) on the magnetic field obeys a simple formula $\alpha|(\sin \pi X/\pi X|$. Figure 8a shows the dependence of the height of the steps on the field for a junction with $w_1 = 2\lambda_j$. The results in Fig. 8a were obtained by analyzing a large number of the current–voltage characteristics recorded using different values of the field. Figure 8a shows clearly that where the step n = 0 has maxima, the step n = 1 has minima. The height of the step oscillates with the field at the same period as the constant current at V = 0. We can see also the principal maxima of the steps n = 4, 5, 6, 7, 8, 9, whose height decreases monotonically. The features described are accounted for satisfactorily by Eq. (7). For such a structure the inequality of Eq. (8) is not satisfied by the steps n < 4; this is why there are no principal maxima for these steps. Oscillations of the steps n > 4 cannot be seen because their heights are less than the heights of the steps n = 0 and n = 1 and they are masked by the latter. Figure 8b shows the dependence $I_c^{(n)}(H)$ (n = 0, 1, 2, 3) for a tunnel junction with $w_1 < 2\lambda_j$. In this case, the inequality (8) is satisfied by the fundamental oscillation mode n = 1. Again, we can observe satisfactory agreement between the experimental curves and the theoretical dependences (cf. [14]). The curves in Fig. 8b were obtained by automatic recording; this explains why we can see a dependence of the step height on the field even when $I_c^{(n)} < I_c^{(n-m)}$.

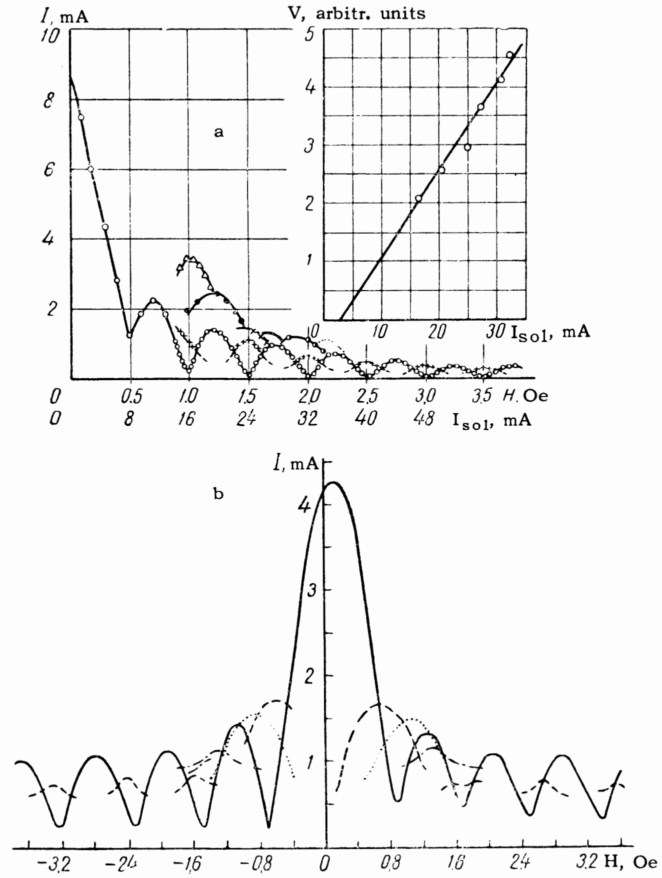

FIG. 8. Dependence of the critical current of the n-th step on a constant magnetic field: a) for a junction with $w_1 = 2\lambda_j$: ○ n = 0; + n = 1; △ n = 4; ● n = 5; × n = 6; – n = 7; ◑ n = 8; ... n = 9. The inset shows the dependence of the voltage at the n-th step (n = 4-9) on the magnetic field at which the relationship (9) is satisfied; b) for a junction with $w_1 < 2\lambda_j$: continuous curve represents n = 0; dashed curve represents n = 1; dotted curve represents n = 2; chain curve represents n = 3; T = 1.6°K.

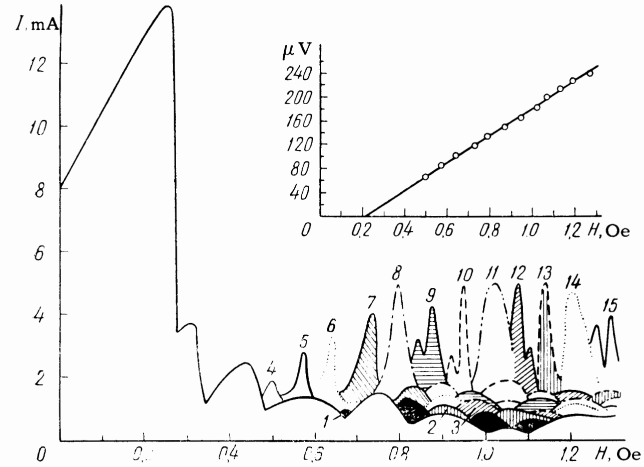

FIG. 9. Dependence of the critical current at the n-th step on a constant magnetic field for a junction with $w_1 > 2\lambda_j$. The appropriate number of the step is shown near each maximum. The auxiliary maxima for the same step have the same type of shading and the same type of curve as the principal maximum. The inset shows the dependence of the voltage at the n-th step (n = 4–15) on the constant magnetic field at which the step height is maximal [cf. Eq. (9)].

The second group includes junctions in which the dimension w_1 is greater than twice the penetration depth of the field into a junction.[17] The magnetic field in such a junction is nonuniform and, when increased above a certain value H'_c, it penetrates the junction in the form of isolated quantized filaments.[5] In this case, the field penetration pattern is the same as in superconductors of the second type. We are not aware of any published data on the field dependence of the critical values of the constant Josephson current or the steps in this case. Nevertheless, such tunnel junctions are of considerable interest in the investigation of the electromagnetic radiation generated by tunnel structures, since they have fairly low ($\sim 10^4$ Mc) frequencies of the fundamental oscillation modes. Figure 9 shows the dependence $I_c^{(n)}(H)$ for a tunnel structure with $w_1 > 2\lambda_j$ ($w_1 \approx 5\lambda_j$). The field dependence of the constant Josephson current resembles only remotely the correct periodic dependence. The envelope of the maxima is not of a monotonically decreasing nature.

The pattern of the field dependence and height of steps $I_c^{(n)}$ is now considerably different. The amplitudes of the principal maxima increase at first, then remain approximately constant, and begin to decrease only at sufficiently high values of n. The widths of the principal maxima are considerably less than the value predicted by Eq. (7). Nevertheless, it is very clear that the even-numbered steps have their maxima at the same positions as the constant current, while the odd-numbered steps behave conversely, as predicted by Eq. (7). These characteristics are of great interest and cannot be understood on the basis of the approximate solution (7).

The shift of the principal maximum of the constant critical current with respect to H = 0 and the asymmetry of the curve are associated with intrinsic magnetic fields excited by the passage of currents in the films. When the direction of the current flowing through a junction is reversed, the pattern suffers reflection with respect to the y axis.

From an analysis of the reported experimental data, we can obtain a general relationship, which is independent of that between w_1 and $2\lambda_j$. This relationship states that the height of a step has its maximum value in a field proportional to the voltage at the step[11] (cf. Figs. 9 and 8a):

$$V_{p^{(n)}} \approx dH_{max}\sqrt{l/\varepsilon d}. \qquad (9)$$

Departures from the relationship are observed only for the first numbers of the steps. These two conclusions follow from Eq. (7) and have a simple physical meaning: in a field H_{max}, the phase velocity of the density wave of the Josephson current ($v_f = \omega/k$) is equal to the phase velocity of the electromagnetic wave ($\bar{c} = \omega/a$). Since these frequencies are equal, it follows that the wavelengths of the current and the field are also equal. Thus, a constant magnetic field controls the phase velocity of the current density wave. When $v_f = \bar{c}$, the motion of the current density wave and of the electromagnetic wave in the junction is always in phase, and the interaction between them is maximal.

A detailed comparison of the dependence of the amplitudes of the principal maxima on the number n is difficult, since we do not know the variation of Q_n with the step number. For junctions satisfying the inequality (8) (cf., for example, Fig. 8b), we can say that the amplitudes of the principal maxima of the dependence $I_c^{(n)}(H)$ decrease as the number n gets higher much more slowly than would follow from the theory ($\sim 1/n^2$). The widths of the principal maxima are also in very approximate agreement with the theory.[14] One of the reasons for the discrepancy between the theory and experiment is possibly the fact that the experimental results were obtained at relatively high temperatures ($T/T_c \geq 0.4$), while the theory was developed for $T = 0$. The usual quasiparticle tunnel current makes a contribution to the damping of electromagnetic resonances in a tunnel structure. Its influence not only may reduce the Q-factor of the corresponding oscillation modes, but also may give rise to additional phase differences between the electromagnetic wave and the current density wave.

In summarizing our results, we can draw the following conclusions:

1. The so-called "steps" in the current–voltage characteristics of Sn–I–Sn superconducting tunnel structures are due to the excitation, by the density wave of the Josephson current, of resonant electromagnetic oscillations in a strip resonator.

2. The positions of the steps on the voltage axis V and the dependence of the amplitudes of the steps on a constant magnetic field are, in general, described well by the available theory.[10, 11, 14] There are quantitative discrepancies between the theory and the experiment, which are probably due to the approximate nature of the theory.

3. The experimental investigations carried out confirm the existence of the alternating Josephson current. The radiation generated by a tunnel structure is obviously coherent, since it appears as a result of stimulated emission of photons in quantum transitions of electrons through a barrier.[5]

The investigation of the spectral composition of the Josephson radiation, carried out by us earlier,[18] shows that the band of the radiated frequencies is much narrower than would follow from estimates of the Q of the resonator, and that, under certain conditions this band may become even narrower. The preliminary results of a direct experimental investigation of the Josephson radiation will be published separately.[18]

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