SOVIET PHYSICS JETP VOLUME 22, NUMBER 6 JUNE, 1966

# *ANOMALOUS PENETRATION OF AN ELECTROMAGNETIC FIELD INTO A METAL*

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Submitted to JETP editor July 6, 1965

J. Exptl. Theoret. Phys. (U.S.S.R.) **49**, 1895–1903 (December, 1965)

A new mechanism is considered for the anomalous penetration of a radio-frequency field into a metal, leading to the appearance of a periodic system of narrow and slowly decaying peaks. The effect should occur in pure single crystals of metals in a magnetic field directed at an angle to the sample surface. The form, amplitude, and the law of decay of the peaks with distance are investigated.

1. Recently, several theoretical and experimental investigations have appeared[1-5] dealing with the anomalous penetration of a high-frequency electromagnetic field into a metal in the presence of a strong (constant and uniform) magnetic field **H**. In these investigations, it has been shown that slowly decaying high-frequency field and current peaks appear in the interior of a metal at large distances from the surface. Such an unusual distribution of an alternating field in a metal is due to the different mechanisms of the selection of electrons which interact effectively with the electromagnetic field near the surface of the metal and those which give rise to new "skin layers" in the interior of the sample. If the dispersion law of electrons differs strongly from the quadratic form, the cyclotron resonance plays the role of this mechanism when the magnetic field is strictly parallel to the metal surface and the field frequency is high.[1]

Low-frequency field peaks, predicted earlier,[2] are due to the fact that at low angles of inclination of the vector **H** to the surface the main contribution to the current is made by electrons near the central cross section of the Fermi surface. Having practically the same orbit diameter in a magnetic field, these electrons reproduce "skin layers" at depths which are multiples of the orbit diameter. The anomalous penetration of an electromagnetic field into a metal also takes place during the excitation of natural weakly damped electromagnetic oscillations by an external wave.[3]

Gantmakher and one of the present authors[4] have investigated the field distribution in a plane-parallel plate. The observed impedance singularities are due to the phenomenon of periodic focusing of "effective"[1)] electrons in a constant magnetic field, parallel or inclined to the metal surface. The "ineffective" electrons may be focused in a perpendicular magnetic field.[5] When electrons are "ineffective," the electromagnetic field distribution established by them in the interior of a metal obeys an harmonic law, in contrast to those cases when the anomalous penetration of a high-frequency field is due to the "effective" electrons and the field distribution is characterized by narrow quasi-periodic singularities. From the mathematical point of view, the appearance of field peaks periodic in space is explained by the presence of various singularities in the Fourier component of the conductivity $\sigma(\mathbf{k})$ expressed as a function of the wave vector **k**. Thus field peaks due to an "orbit chain"[1, 2] are caused by a sharp reduction in the conductivity $\sigma(k)$ at the values $k = k_n \equiv 2\pi n/D$. The singularities in the distribution of an alternating field are due to a periodic system of delta-shaped maxima of the conductivity $\sigma(k)$ expressed as a function of k.[4] Finally, the anomalous penetration of a field in the focusing of the "ineffective" electrons[5] is associated with the existence of a single branching point of the function $\sigma(k)$ near the real axis of k.

In the present study, we deal with a different mechanism of selection of the "effective" electrons, which also leads to an anomalous penetra-

[1)] The "effective" electrons are those which over some part of their trajectory move parallel to the surface of a metal. In the anomalous skin effect, these are the electrons which make the principal contribution to the high-frequency current.

tion of a radio-frequency field into a metal. In contrast to the cases investigated earlier, this mechanism is important at relatively large angles of inclination $\Phi$ of the magnetic field with respect to the surface. This mechanism can be described as follows. It is known that in a magnetic field the condition of resonance interaction with an external wave of frequency $\omega$ is satisfied by electrons for which

$$|k_\zeta v_\zeta| = |\omega - n\Omega|, \quad n = 0, 1, 2, \ldots, \tag{1.1}$$

where $v_\zeta$ is the projection of the electron velocity onto the direction $\mathbf{H}$, $\Omega$ is the cyclotron frequency, and $\mathbf{k}$ is the wave vector. For simplicity, we shall consider electrons with an isotropic quadratic dispersion law. In the anomalous skin effect ($kv/\Omega \gg 1$), the dominant role is played by the "effective" electrons, satisfying the condition

$$\mathbf{kv} = \omega. \tag{1.2}$$

The relationships (1.1) and (1.2) define those electron states on the Fermi surface which make the main contribution to the conductivity. Figure 1 shows these states schematically; the horizontal line (y axis) is the projection of the curve $\mathbf{k} \cdot \mathbf{v} = \omega$ onto the yz plane. The small vertical marks on this line represent the states (1.1), which satisfy the condition (1.2). A change in the wave number or the magnetic field alters the number of possible states (1.1) [i.e., the number of different values of n, for which there are solutions of Eq. (1.1)]. In other words, a group of resonance electrons appears (or disappears). Consequently, the conductivity $\sigma(k)$ should exhibit sharp discontinuities at those values of $\Omega$ or $k_\zeta$, which satisfy the relationship

$$|k_\zeta v_\zeta|_{\text{ex}} = |\omega - n\Omega|. \tag{1.3}$$

The subscript "ex" denotes that the value of $v_\zeta$ should be taken in one of the cross sections $\pm p_{\zeta\text{ex}}$. Such discontinuities have been predicted[6] for the absorption of ultrasound and have been discovered recently in antimony by Korolyuk and Matsakov.[7] The presence of periodic conductivity discontinuities leads to the anomalous penetration into a metal of individual harmonics of a wave packet, whose wave numbers k satisfy the condition (1.3). Since, in a skin layer, all the harmonics are excited in phase, they interfere in the interior of the metal and give rise to a periodic system of narrow peaks with a space period equal to $(2\pi v_\zeta/\Omega) \sin\Phi$. The width of the peaks is governed by the number of interfering components, i.e., in the final analysis, by the depth of the skin layer.

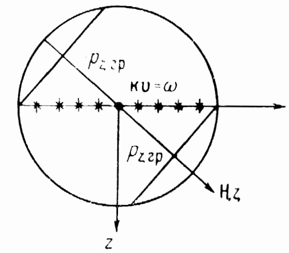


FIG. 1. Projection of the Fermi sphere onto the yz plane.

The decay is governed by the electron "lifetime," i.e., by the mean free path. The effect is strongest for an electromagnetic field $\mathbf{E}$ linearly polarized along the y axis. In a magnetic field perpendicular to the surface, the peaks disappear because the two conditions (1.1) and (1.2) are compatible only when n = 0.[2)] The angular criterion, for small values of $\Phi$, has the form

$$\Phi \gg \delta / R, \tag{1.4}$$

where $\delta$ is the effective depth of the skin layer, $R = v/\Omega$ is the characteristic dimension of the electron orbit in the magnetic field. As the angle of inclination $\Phi$ is decreased, the extremal cross section approaches the limit point. The discontinuities of the conductivity $\sigma(k)$ are replaced by delta-shaped maxima.[4] For large values of $\Phi$, the condition has the following form:

$$\Phi < (\delta / R)^{1/3}. \tag{1.5}$$

This inequality follows from the requirement that electrons near the special point should be "effective." The characteristic angular dimensions $\psi$ in the vicinity of the limit point should be greater than the angle of inclination of the magnetic field $\Phi$. The quantity $\psi$ is of the order of $(\delta/R\Phi)^{1/2}$,[4] from which follows the relationship (1.5).

In the next section, we shall develop a detailed theory of this effect.

**2.** To find the field distribution in the interior of a metal, it is necessary to solve Maxwell's equation

$$\frac{\partial^2 E(z)}{\partial z^2} = -\frac{4\pi i\omega}{c^2} j(z), \tag{2.1}$$

where E(z) and j(z) are the y components of the electric field and current in the metal. The z axis is selected along the normal to the surface, and the y axis along the projection of the magnetic field on the surface of the metal.

We shall consider only one equation in (2.1)

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2) In the case of a complex dispersion law for electrons, when curve (1.2) does not lie in one plane, the effect should be observed also in normal fields.

since, as pointed out already, the effect occurs only in the presence of the y component of the field $\mathbf{E}(z)$.

The current density $j(z)$ should be calculated using the transport equation for the electron distribution function:

$$j = -\frac{2|e|}{h^3}\int d^3p v_y f, \tag{2.2}$$

$$-i\omega f + v_z\frac{\partial f}{\partial z} + \Omega\frac{\partial f}{\partial \tau} + \nu f = |e|\,E(z)\,v_y\frac{\partial f_0}{\partial \varepsilon}. \tag{2.3}$$

Here, f is the nonequilibrium correction to the Fermi distribution function $f_0(\varepsilon - \mu)$; $-|e|$ is the electronic charge; h is Planck's constant; $\nu$ is the frequency of collisions between electrons and scatterers, which we shall assume to be constant; $\mathbf{v}$ is the velocity, $\mathbf{p}$ is the momentum, and $\varepsilon$ is the energy of electrons; $\mu$ is the chemical potential (Fermi energy); $\tau$ is the dimensionless time (phase) of the electron motion along an orbit in a magnetic field.

A boundary condition must be specified for Eq. (2.3) at the metal surface z = 0. Usually, the diffuse reflection condition is employed. However, since the main contribution to the current density is made by electrons which move parallel to the metal surface, collisions with the surface need not be allowed for. The allowance for such collisions gives only an unimportant (in our case) numerical multiplier of the order of unity.[8,1] Therefore, to an accuracy within this multiplier, the nature of electron scattering at the boundary is of no importance and we can use the solution of the transport equation for an infinite metal.

We shall go over to the Fourier components along the coordinates in Eqs. (2.1)–(2.3). Expanding the field E(z) and the current j(z) as even functions to the region z < 0, we obtain

$$[k^2 - i4\pi\omega c^{-2}\sigma(k)]\,\mathscr{E}(k) = -2E'(0),$$

$$\mathscr{E}(k) = 2\int_0^\infty E(z)\cos kz\,dz;\quad E(z) = \frac{1}{\pi}\int_0^\infty \mathscr{E}(k)\cos kz\,dk. \tag{2.4}$$

The Fourier component of the conductivity is given by the formula[2]

$$\sigma(k) = \frac{3Ne^2}{4\pi m\Omega}\int_0^\pi d\theta\sin\theta\int_0^{2\pi} d\tau n_y(\tau,\theta)\int_{-\infty}^{\tau} d\tau' n_y(\tau',\theta)$$
$$\times\exp\left[\frac{\nu - i\omega}{\Omega}(\tau' - \tau)\right]\cos\left(\frac{k}{\Omega}\int_\tau^{\tau'} v_z(\tau'')\,d\tau''\right), \tag{2.5}$$

where $N = 8\pi p_0^3/3h^3$ is the electron density, $p_0$ is the Fermi momentum, $\mathbf{n}(\tau,\theta) = \mathbf{v}/v_0$ is a unit vector along the direction of the electron velocity vector on the Fermi sphere;

$$n_x = \sin\theta\cos\tau,$$
$$n_y = \cos\Phi\cos\theta - \sin\theta\sin\tau\sin\Phi,$$
$$n_z = \sin\Phi\cos\theta + \sin\theta\sin\tau\cos\Phi,$$

$\theta$ is the polar angle and $\tau$ is the azimuthal angle in the velocity space with the polar axis along the magnetic field $\mathbf{H}$.

We shall consider low frequencies, when

$$\omega \ll \nu. \tag{2.6}$$

We can then neglect the quantity $\omega$ in the exponent of the exponential function in Eq. (2.5). Under the anomalous skin effect conditions

$$kR \gg 1. \tag{2.7}$$

Therefore, the integrals with respect to $\tau$ and $\tau'$ can be calculated using the constant-phase method. The constant-phase points in Eq. (2.5) are given by Eq. (1.2), in which the quantity $\omega$ must be taken as equal to zero in view of the condition (2.6). The line of the constant phase points $\mathbf{k}\cdot\mathbf{v} = 0$ lies in the range of angles $\Phi \le \theta \le \pi - \Phi$.

Simple calculations give the contribution of the constant-phase points in the form

$$\sigma(k) = \frac{3Ne^2}{4\pi m\Omega}\operatorname{Re}\int_\Phi^{\pi-\Phi} d\theta\sin\theta\sum_{\tau_\alpha}\Bigg\{ n_y^2(\tau_\alpha,\theta)\,|J_\alpha(\theta)|^2$$
$$\times\frac{e^{2\pi(\gamma+i\bar{q}(\theta))}+1}{2\,[e^{2\pi(\gamma+i\bar{q}(\theta))}-1]}$$
$$+\sum_{\tau_\alpha-2\pi<\tau_\beta<\tau_\alpha}\Big[n_y(\tau_\alpha,\theta)\,n_y(\tau_\beta,\theta)\,J_\alpha(\theta)\,J_\beta^*(\theta)$$
$$\times\exp\int_{\tau_\alpha}^{\tau_\beta}(\gamma + iq)\,d\tau\Big]\,[1 - e^{-2\pi(\gamma+i\bar{q})}]^{-1}\Bigg\}, \tag{2.8}$$

$$\gamma = \frac{\nu}{\Omega},\qquad q(\tau,\theta) = \frac{kv_z(\tau,\theta)}{\Omega},$$

$$\bar{q}(\theta) = \frac{k\bar{v}_z}{\Omega} \equiv \frac{kv\sin\Phi}{\Omega}\cos\theta. \tag{2.9}$$

The constant-phase points $\tau_\alpha(\theta)$ are the solutions of the equation

$$n_z(\tau,\theta) \equiv \sin\theta\sin\tau\cos\Phi + \sin\Phi\cos\theta = 0;$$

$$\sin\tau_\alpha(\theta) = -\operatorname{tg}\Phi\,/\operatorname{tg}\theta. \tag{2.10*}$$

The quantity $J_\alpha(\theta)$ has the form

$$J_\alpha(\theta) = \int_{-\infty}^{\infty} d\tau\exp\left(-i\frac{q_\alpha'}{2}\tau^2 - i\frac{q_\alpha''}{6}\tau^3\right), \tag{2.11}$$

*tg ≡ tan.

$$q_\alpha' = \left.\frac{\partial q(\tau,\theta)}{\partial\tau}\right|_{\tau=\tau_\alpha(\theta)}; \quad q_\alpha'' = \left.\frac{\partial^2 q(\tau,\theta)}{\partial\tau^2}\right|_{\tau=\tau_\alpha(\theta)}.$$

The formula (2.11) gives, in the limiting cases,

$$J_\alpha(\theta) = (2\pi / |q_\alpha'|)^{1/2} \exp(-{}^1\!/_4\pi i \operatorname{sign} q_\alpha'),$$

$$|q_\alpha'|^3 \gg |q_\alpha''|^2, \qquad (2.12)$$

$$J_\alpha(\theta) = (6/|q_\alpha''|)^{1/3}3^{1/2}\Gamma(1/3), \quad |q_\alpha'|^3 \ll |q_\alpha''|^2. \quad (2.13)$$

In the formula (2.8) the main contribution to the conductivity $\sigma(k)$ is made by those ranges of the values of $\theta$, which satisfy the condition of resonance interaction (1.1):

$$\bar{q}(\theta) \equiv k\bar{v}_z(\theta) / \Omega = n. \qquad (2.14)$$

At high values of the quantity $|kv \sin \Phi/\Omega|$, the majority of the solutions of Eq. (2.14), $\theta = \theta_n$, fall within the interval $\Phi < \theta < (\pi - \Phi)$. Only at certain intervals of the wave vector **k** and the magnetic field **H** do solutions exist of Eq. (2.14) in the form $\theta = \Phi$, $(\pi - \Phi)$, which coincide with the limits of the range of integration in Eq. (2.8).

We shall consider first the contribution of all the "internal" points $\theta = \theta_n$ $[\Phi < \theta_n < (\pi - \Phi)]$. Using the expression (2.12) for $J_\alpha$ and the formula*

$$\frac{1}{2i}\operatorname{ctg}\pi(\bar{q} - i\gamma) = \frac{1}{2\pi i}\sum_{n=-\infty}^{\infty}\frac{1}{\bar{q} - i\gamma - n},$$

we obtain

$$\sigma_0(k) = \frac{3}{4\pi}\frac{Ne^2}{m\Omega}\sum_{n=-\infty}^{\infty}\int_{\Phi}^{\pi-\Phi} d\theta \sin\theta \frac{\gamma}{\gamma^2 + (\bar{q} - n)^2}\sum_\alpha \frac{n_y^2(\tau_\alpha)}{|q_\alpha'|}.$$

Replacing the resonance multipliers with $\delta$-functions and using them to calculate the integral, we obtain

$$\sigma_0(k) = \frac{3}{2}\frac{Ne^2}{mkv}\sum_{-n_{max}}^{n_{max}}\left(\frac{n}{M}\right)^2\frac{1}{(M^2 - n^2)^{1/2}},$$

where $n_{max}$ represents the largest integer contained in $M = |kv \sin \Phi \cos \Phi/\Omega|$. Replacing the summation with respect to n by an integral ($n_{max} \gg 1$), we obtain

$$\sigma_0(k) = \frac{3\pi}{4}\frac{Ne^2}{mkv}. \qquad (2.15)$$

The formula (2.15) represents an asymptotic expression for the Fourier component of the conductivity in the anomalous skin effect in the absence of a magnetic field.

The singularities (sudden jumps) of the function $\sigma(k)$ are due to the contribution of the limits of the range of integration to the conductivity $\sigma(k)$ because the quantity $\bar{q} = k\bar{v}_z/\Omega$, considered as a function of $\bar{v}_z$, has no extremum within the range of integration. A singularity is obtained at

$$\bar{q}_{ex} \approx n; \quad \gamma \ll 1. \qquad (2.16)$$

In integration near the limits for $J_\alpha(\theta)$, we have to use the expression (2.13) (for an estimate, see [6]). Near the singularity, we have

$$\Delta\sigma(k) = \frac{A}{|n|^{5/3}}\left[\frac{\pi}{2} - \operatorname{arctg}\left(\frac{ku - n}{\gamma}\right)\right], \qquad (2.17)^*$$

$$A = \frac{9}{\pi^2}6^{2/3}\Gamma^2\left(\frac{1}{3}\right)\frac{Ne^2}{m\Omega}|\cos\Phi|;$$

$$u = \frac{v}{\Omega}|\sin\Phi\cos\Phi|. \qquad (2.18)$$

From the formula (2.17), it follows that $\Delta\sigma(k)$ does indeed have a discontinuity when ku is varied near integral values. At high values of n, we have the ratio

$$\sigma_0(n/u) / \Delta\sigma(n/u) \sim n^{2/3} \gg 1.$$

Knowing the nature of the function $\sigma(k)$, we shall find the field distribution in a semi-infinite metal at large distances from the surface.

From the formula (2.4), we have

$$-\frac{E(z)}{E'(0)} = \frac{2}{\pi}\int_0^\infty dk \cos kz\left[k^2 - i\frac{4\pi\omega}{c^2}\sigma(k)\right]^{-1}. \qquad (2.19)$$

We shall integrate the expression (2.19) by parts. The results are as follows:

$$-\frac{E(z)}{E'(0)} = \frac{2}{\pi z}\int_0^\infty dk \sin kz\left\{\frac{d}{dk}\left[k^2 - i\frac{4\pi\omega}{c^2}\sigma(k)\right]\right\}$$
$$\times\left[\left(k^2 - i\frac{4\pi\omega}{c^2}\sigma(k)\right)^2\right]^{-1}. \qquad (2.20)$$

At large distances from the surface of a metal, the terms in Eq. (2.20), which contain derivatives, with respect to k, of smoothly varying functions of k, make an exponentially decreasing contribution. The anomalous penetration of the field to great depths into a metal is due to that term which contains the derivative $d\Delta\sigma(k)/dk$. Retaining only such terms, we obtain

$$-\frac{E(z)}{E'(0)} = \frac{u}{z}\,\frac{8i\omega A}{c^2}\sum_{n=1}^{\infty}\frac{1}{n^{5/3}}$$
$$\times\int_0^\infty \frac{dk \sin kz}{(k^2 - i4\pi\omega c^{-2}\sigma_0(k))^2}\,\frac{\gamma}{\gamma^2 + (ku - n)^2} \qquad (2.21)$$

Since, at low values of $\gamma$, the function $\gamma/[\gamma^2 + (ku - n)^2]$ behaves as a $\delta$-function, the formula (2.21) may be written as follows:

*ctg ≡ cot.

*arctg ≡ $\tan^{-1}$.

$$-\frac{E(z)}{E'(0)} = \frac{8\pi i\omega A u^4}{zc^2}\exp\left(-\frac{\gamma z}{u}\right)\sum_{n=1}^{\infty} B_n \sin n\frac{z}{u}, \qquad (2.22)$$

$$B_n = n^{-5/3}[n^2 - 4\pi i\omega c^{-2}u^2\sigma_0(n/u)]^{-2}. \qquad (2.23)$$

From the formula (2.22), it follows that the sum with respect to n is a periodic function of z with the space period

$$\Delta z = 2\pi u = \frac{2\pi v}{\Omega}|\sin\Phi\cos\Phi|. \qquad (2.24)$$

We shall investigate the behavior of the function

$$S(x) = \sum_{n=1}^{\infty} B_n \sin nx \qquad (2.25)$$

within the limits of one period. Since

$$\frac{4\pi\omega u^2\sigma_0}{c^2} \gg 1, \qquad (2.26)$$

only the large values of n are important in the sum with respect to n and therefore this sum can be replaced by an integral. Then, instead of $\sigma_0(n/u)$, we can use the asymptotic expression (2.15):

$$S(x) = \int_0^\infty \frac{dn\sin(nx)\,n^{1/3}}{(n^3 - iM^3)^2} = \frac{1}{M^{14/3}}\int_0^\infty \frac{dt\sin(txM)\,t^{1/3}}{(t^3 - i)^2}, \qquad (2.27)$$

where

$$M = u/\delta_0 \gg 1; \qquad \delta_0 = (c^2 p_0 / 3\pi^2\omega N e^2)^{1/3}. \qquad (2.28)$$

The quantity $\delta_0$ is the effective depth of a skin layer in the anomalous skin effect, and $\delta_0 \ll u$.

Finally, at large distances from the metal surface

$$-\frac{E(z)}{\delta_0 E'(0)} = ia\frac{1}{M^{2/3}\sin\Phi}\exp\left(\frac{-z}{l|\sin\Phi\cos\Phi|}\right)\frac{2\pi u}{z}S\left(\frac{z}{u}\right),$$

$$a = \frac{12}{\pi^4}6^{2/3}\Gamma^2\left(\frac{1}{3}\right) \approx 2.8; \qquad l = \frac{v}{\nu}. \qquad (2.29)$$

Figure 2 shows the real and imaginary components of the function

$$P(t) = \int_0^\infty \frac{dx\sin(tx)\,x^{1/3}}{(x^3 - i)^2}, \qquad (2.30)$$

found by numerical integration. Figure 3 shows schematically the form of the peaks in the interior of a metal. The width of the peaks is of the order of $\delta_0$ and their decrease with distance (number) is represented by the function

$$\exp\left(-\frac{z}{l\sin\Phi\cos\Phi}\right)\frac{2\pi u}{z}. \qquad (2.31)$$

The amplitude of the first peaks is $M^{2/3}$ times smaller than the amplitude of the field near the metal surface.

Our treatment applies to the case of an isotropic quadratic dispersion law for electrons. The effect will obviously occur also in the case of a complex nonquadratic electron spectrum. Then the space period of the peaks will be given by the formula

$$\Delta z = \frac{c\sin\Phi}{|e|H}\left|\frac{\partial S(\mu, p_{H,\,ex})}{\partial p_H}\right|, \qquad (2.32)$$

where $S(\mu, p_{H,\,ex})$ is the area of the extremal cross section of the Fermi surface. The amplitude of the peaks is maximal for an external electric field polarized linearly along the electron velocity vector at the point $\mathbf{kv} = 0$, $|p_H| = p_{H,\,ex}$ on the Fermi surface.

Such anomalous penetration of an external radio

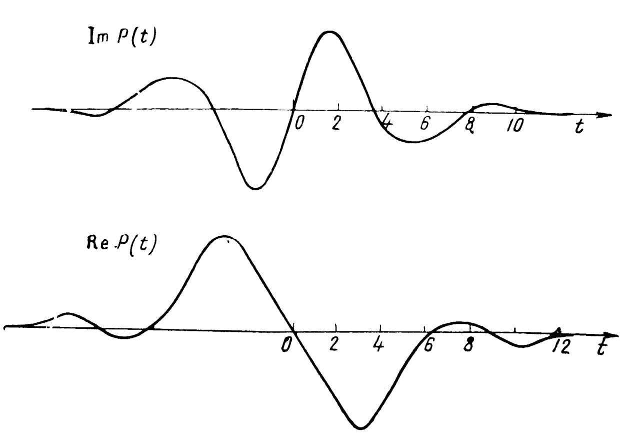


FIG. 2. Dependence of the real and imaginary components of the functions P(t) on t.

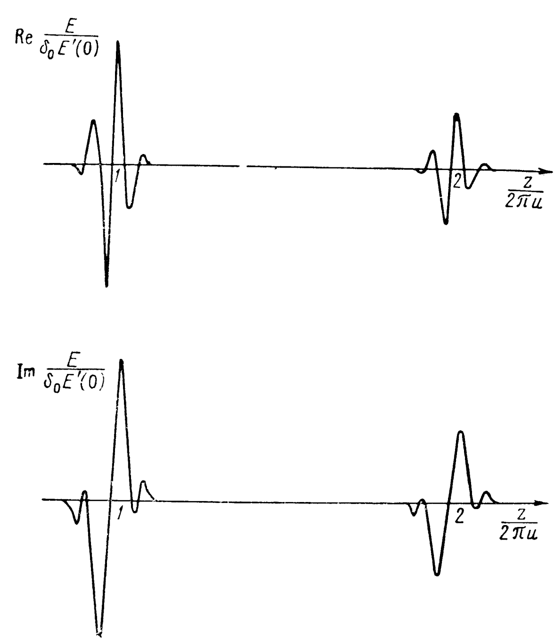


FIG. 3. Distribution of the real and imaginary components of the field $[E(z)/\delta_0 E'(0)]$ in the interior of a metal (the ordinate gives the field in arbitrary units; M = 100).

wave into a metal may be detected by observing the high-frequency size effect in a plane parallel plate.[4, 5] The emergence (or absence) of the next peak on the other side of the plate will give rise to corresponding singularities in the experimentally measured surface impedance.


[1] M. Ya. Azbel', JETP **39**, 400 (1960), Soviet Phys. JETP **12**, 283 (1961).

[2] É. A. Kaner, JETP **44**, 1036 (1963), Soviet Phys. JETP **17**, 700 (1963).

[3] É. A. Kaner and V. G. Skobov, JETP **45**, 610 (1963), Soviet Phys. JETP **18**, 419 (1964).

[4] V. F. Gantmakher and É. A. Kaner, JETP **45**, 1430 (1963), Soviet Phys. JETP **18**, 988 (1964).

[5] V. F. Gantmakher and I. P. Krylov, JETP **49**, 1054 (1965), Soviet Phys. JETP **22**, 704 (1966).

[6] É. A. Kaner, V. G. Peschanskiĭ, and I. A. Privorotskiĭ, JETP **40**, 214 (1961), Soviet Phys. JETP **13**, 147 (1961).

[7] A. P. Korolyuk and L. Ya. Matsakov, JETP Letters **2**, 30 (1965), transl. p. 18.

[8] M. Ya. Azbel' and É. A. Kaner, JETP **32**, 896 (1957), Soviet Phys. JETP **5**, 730 (1957).


Translated by A. Tybulewicz
243