# INDIRECT OPTICAL TRANSITIONS IN CROSSED ELECTRIC AND MAGNETIC FIELDS

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We calculate the probability of indirect optical transitions involving absorption or emission of phonons in crossed electric and magnetic fields in semiconductors in which the extrema of the valence band and the conductivity band are at different points in k-space. It is shown that measurement of changes of the absorption coefficient in an electric field considerably raises the sensitivity of magneto-optical methods for indirect transitions. These methods consequently can be used to determine the band structure of semiconductors.

## 1. INTRODUCTION

MEASUREMENT of the absorption coefficient of light, in the case of interband transitions in a quantizing magnetic field, is one of the effective methods of studying the band structure of a semiconductor, especially when observation of cyclotron resonance is impossible for lack of sufficiently high mobility, for in the measurements of magneto-optical effects the cyclotron frequency can be appreciably higher. Such measurements, however, make it possible to determine only the value of the reduced effective mass $m^* = m_c^* m_v^*/(m_c^* + m_v^*)$, where $m_c^*$ and $m_v^*$ are the corresponding effective masses of the electron and the hole [1-3].

As shown in [4], measurements in crossed electric and magnetic fields make it possible to determine the effective masses in each of the bands, both from the line shift in the electric field and from the frequency difference in the transitions that are allowed and forbidden in the absence of an electric field. Such measurements were recently made [5-7] and gave good agreement between theory and experiment [1)]. It was proposed in [5,6] to measure the change of the absorption coefficient in an alternating electric field, which appreciably increases the sensitivity of the method and makes it possible to carry out the measurements even at room temperature, when the cyclotron frequency $\omega_{v,0}$ does not exceed the reciprocal relaxation time. The use of these methods, however, is limited to those bands for which vertical direct transitions can be observed, that is, transitions without a change in the electron momentum. As is well known, magneto-optical effects are observed in indirect transitions [3,8,15], but because of their low resolution these methods are not used in practice in such cases. As follows from the calculations presented below, measurement of indirect transitions in crossed fields makes it possible to increase appreciably the sensitivity of the method, thus uncovering a possibility for employing the method to determine the structure of bands between which no direct transitions have been observed, and also for determining the frequencies of the corresponding phonons. Indirect transitions in crossed fields take place also in the case of vertical transitions. Apparently, however, the greatest interest is attached to just nonvertical transitions, when the direct transition is impossible. We shall therefore consider just this case.

## 2. COEFFICIENT OF ABSORPTION OF LIGHT IN CROSSED FIELDS

Let us consider for concreteness a cubic crystal in which both bands are not degenerate, the valence band has an extremum at the point $k = 0$, and the conduction band has, besides an extremum at $k = 0$, a lower minimum at the point $K_0$ and its equivalents. In this case the vertical distance between the bands at the point $k = K_0$, which is equal to $\epsilon_{K_0}$, greatly exceeds the distance between them at $k = 0$, which is $\epsilon_{g_0}$. Then, of all the possible types of transitions shown in Fig. 1, it is sufficient to take into account only transition I, since the probability of transition II is smaller by a factor $(\epsilon_{g_0}/\epsilon_{K_0})^2$ and the probability of transitions III and

[1)] It was correctly noted in [7] that the theory developed in [4] is valid when $cE/sH \ll 1$, where $s = (\epsilon_g/4m^*)^{1/2}$, and $\epsilon_g$ is the width of the forbidden band. This condition is assumed satisfied in this paper, too.

IV, which are connected with the intraband absorption of the photon, is smaller by a factor $(\hbar/L_M p_e)^2 \sim \hbar\omega_{v,0}/\epsilon_{g_0}$ than the probability of transition I.

Here $L_M = (\hbar c/eH)^{1/2}$ is the magnetic length and $p_e$ is given by (17) below.

For the transitions indicated above, the absorption coefficient at the extremum $K_0$ is given by the expression [3]

$$\Gamma_{K_0} = \frac{4\pi^2\omega_0}{n_0 c E_0^2 V} \sum |H_{cv}|^2 \delta(\hbar\omega_0 \mp \hbar\omega_{ph} + \varepsilon_v - \varepsilon_c), \quad (1)$$

where $E_0$ is the light-wave field intensity, $\omega_0$ the light frequency, $n_0$ the refractive index, V the volume, $\hbar\omega_{ph}$ the energy of the absorbed or emitted phonon, and

$$H_{cv} = \sum_o \frac{\langle v|H_1|o\rangle\langle o|H_2|c\rangle}{\varepsilon_c - \varepsilon_o \pm \hbar\omega_{ph}}. \quad (2)$$

The index v here denotes the initial state of the system when an electron was situated in the valence band, and the index c denotes the final state, when the electron is in the conduction band near the extremum $K_0$, while o denotes an intermediate state, when the electron is in the conduction band near the extremum at the point k = 0.

The operator $H_1$, which describes dipole interaction with the light, is of the form [3]

$$H_1 = \frac{eE_0}{m\omega_0}[(\mathbf{pe})e^{i\omega_0 t} + \text{c.c.}] \quad (3)$$

(e is the light-wave polarization). The operator of the interaction with the lattice vibrations can be written in the general form [2)]

$$H_2 = \sum_{m,j} \mathbf{u}_{jm}\mathbf{W}_j(\mathbf{r} - \mathbf{R}_m). \quad (4)$$

Here $\mathbf{u}_{jm}$ is the displacement of the j-th atom of the m-th cell, the coordinate of the center of gravity of which is $\mathbf{R}_m$. The displacement $\mathbf{u}_{jm}$, as a rule, is expanded in normal modes [9].

According to [4,11], if the magnetic field H is directed along the z axis, the vector potential is chosen in the form (– Hy, 0, 0), and the electric field is directed along the y axis, then the wave functions and the eigenvalues of the electron energy are determined, in the effective mass approximation by the following expressions (all the energies are reckoned from the edge of the valence band at k = 0):

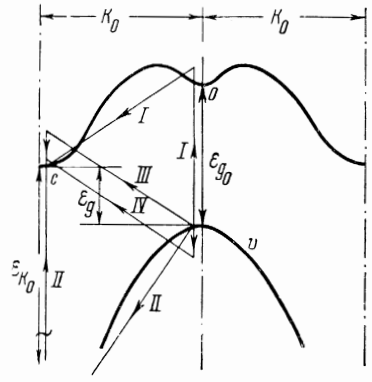


FIG. 1. Band scheme and possible types of indirect transitions I, II, III, IV.

A. For bands at extremum k = 0

$$\Psi_{v,o} = u_{v,o} f_{v,o}(\mathbf{r}), \quad (5)$$

were $u_{v,o}$ are the Bloch functions at k = 0 normalized to the unit cell;

$$f_{v,o} = (L_x L_z)^{-1/2} \exp[i(k_x x + k_z z)]\Phi_n(\xi - \xi_{v,o}),$$
$$\xi_{v,o} = \frac{y_{v,o}}{L_M} = L_M k_x \mp \frac{eEL_M}{\hbar\omega_{v,o}}. \quad (6)$$

Here $\Phi_n(\xi - \xi_{v,o})$ is the harmonic-oscillator function, normalized in analogy with (12), $\xi = y/L_M$, $\omega_{v,o} = eH/cm^*_{v,o}$ is the cyclotron frequency, and $m^*_c$ and $m^*_o$ are the effective masses of the hole and of the electron. Here

$$\varepsilon_v = -\frac{\hbar^2 k_z^2}{2m_v^*} - \hbar\omega_v\left(n + \frac{1}{2}\right) - eEy_v - \frac{1}{2}\frac{(eEL_M)^2}{\hbar\omega_v}, \quad (8)$$

$$\varepsilon_o = \varepsilon_{go} + \frac{\hbar^2 k_z^2}{2m_o^*} + \hbar\omega_o\left(n + \frac{1}{2}\right) - eEy_o + \frac{1}{2}\frac{(eEL_M)}{\hbar\omega_o}. \quad (9)$$

B. For the band with extremum near $K_0$

$$\Psi_c = \exp(i\mathbf{K}_0\mathbf{r})u_{K_0} f_c(\mathbf{r}); \quad (10)$$

$$f_c(\mathbf{r}) = (L_x L_z)^{-1/2}\exp[i(k_x x + k_z z)]\Phi_{n k_x k_z}(y), \quad (11)$$

$$\Phi_{n k_x k_z}(y) = \frac{\beta^{1/4}}{(2^n n!\sqrt{\pi})^{1/2} L_M^{1/2}} \exp\left[ i\frac{\mu_{12}}{2\mu_{22}}(\xi - \xi_1)^2 - \frac{(\xi - \xi_c)^2}{2\beta}\right] H_n\left(\frac{\xi - \xi_c}{\beta^{1/2}}\right), \quad (12)$$

where $H_n(\xi)$ is a Hermite polynomial

$$\xi_1 = L_M\left(k_x + \frac{\mu_{23}}{\mu_{12}}k_z\right), \quad (13)$$

[2)] This form of the interaction operator was proposed by Peierls[9]. As indicated by Pavlov and Firsov[10] it includes both the short-range forces (deformation potential), and the long-range ones (polarization). Equation (4) is the most general expansion of the potential produced by the oscillations in powers of the displacement $\mathbf{u}_{jm}$. Indeed, let us consider a perturbation produced at the point r by the j-th atom of cell m. Then from the condition that this perturbation must not change when r and $\mathbf{R}_m$ are displaced by an integer number of lattice constants, it follows that the expansion coefficient $W_{jm}$ depends only on the difference $\mathbf{r} - \mathbf{R}_m$.

$$\xi_c = \frac{y_c}{L_M} = L_M\left[\left(k_x - \frac{M_{31}}{M_{33}}k_z\right) + \beta\frac{eE}{\hbar\omega_c}\right], \quad (14)$$

and the energy is

$$\varepsilon_c = \varepsilon_g + \frac{\hbar^2 k_z^2}{2m_z^*} + \hbar\omega_c\left(n+\frac{1}{2}\right) - eEy_c + \frac{\beta}{2}\frac{(eEL_M)^2}{\hbar\omega_c}, \quad (15)$$

where

$$m_z^* = \frac{1}{\mu_{22}M_{33}^2}(M_{11}M_{33} - M_{31}^2),$$

$$\omega_c = \frac{eH}{c}\sqrt{M_{33}}, \quad \beta = \frac{\mu_{22}}{\sqrt{M_{33}}}, \quad (16)$$

$\mu_{ik}$ are the components of the reciprocal-mass tensor in the coordinate frame connected with the **E** and **H**, while $M_{ik}$ are the cofactors of this tensor [3)].

When calculating the matrix element we first integrate, as usual, with respect to the rapid functions within the limits of one unit cell of volume $\Omega$ and put

$$p_e = \frac{1}{\Omega}\int_\Omega u_o^*(\mathbf{r})(\mathbf{ep})u_v(\mathbf{r})d^3r. \quad (17)$$

We proceed analogously in calculating $\langle H_2\rangle$ and put

$$b_{\mu q} = \sum_j \mathbf{e}_{q\mu}{}^j \frac{1}{\Omega}\int_\Omega u_{K_0}^*(\mathbf{r})\sum_m \mathbf{W}_j(\mathbf{r}-\mathbf{R}_m)e^{i\mathbf{qR}_m}u_0(\mathbf{r})d^3r, \quad (18)$$

where q is the wave vector of the phonon, $e^j_{q\mu}$ is the phonon polarization vector, and $\mu$ is the number of the oscillation branch.

When summing over the intermediate states in the denominator of (2), we can neglect the kinetic energies of the electrons and the phonon energy compared with the large energy difference $\epsilon_0 = \epsilon_{go} - \epsilon_g$ between the extrema of the conduction band at $k = 0$ and $k = K_0$. We can then take $\epsilon_0$ in (2) outside the summation sign and interchange the order of summation and integration. Further, recognizing that the functions $f_0(\mathbf{r})$ form a complete orthonormal system of slow functions and, consequently, when integrating over the smooth functions

$$\sum_o f_o(\mathbf{r})f_o(\mathbf{r}') = \delta(\mathbf{r}-\mathbf{r}'),$$

we can immediately carry out the summation and integration with respect to one of the coordinates. Then, substituting (2) in (1), and calculating the matrix element in terms of the phonon functions [9], we obtain for the transitions involving emission or absorption of a phonon of branch $\mu$:

$$\Gamma^{\pm}_{K_0\mu} = \frac{(2\pi e)^2\hbar p_e^2 b^2_{K_0\mu}(n_{K_0\mu} + 1/2 \pm 1/2)}{\varepsilon_0{}^2 m^2\omega_0 c n_0\rho\omega_{K_0\mu}V^2} \times \sum\left|\int_V f^*_{c\,k_x,k_z,n}(\mathbf{r})e^{i(\mathbf{q}-\mathbf{K}_0)\mathbf{r}}f_{v\,k_x',k_z',n'}(\mathbf{r})d^3r\right|^2 \times \delta(\hbar\omega_0 \pm \hbar\omega_{K_0\mu} + \varepsilon_v - \varepsilon_c); \quad (19)$$

the summation is over $k_X$, $k_X'$, $k_Z$, $k_Z'$; n, n', **q**. We allowed here for the fact that the values of q of importance in the integral (19) are those close to $K_0$, and took outside the summation sign the values of $b_{q\mu}$, $\omega_{q\mu}$, and the average occupation numbers $n_{q\mu}$ at $q = K_0$.

We go over in (19) from summation with respect to **q** and **k** to integration, and represent the square of the modulus as the product of two integrals with respect to **r** and **r'**. We then reverse the order of integration with respect to **r**, **r'** and **q**, **k**. Since

$$\sum_{\mathbf{q}}\exp[i(\mathbf{q}-\mathbf{K}_0)(\mathbf{r}-\mathbf{r}')] = \frac{V}{(2\pi)^3}\int\exp[i(\mathbf{q}-\mathbf{K}_0)(\mathbf{r}-\mathbf{r}')]d^3q = V\delta(\mathbf{r}-\mathbf{r}'),$$

there remains in (19), after integration with respect to **q** and **r'**, the integral

$$\int|f_c(\mathbf{r})|^2|f_v(\mathbf{r})|^2 d^3r,$$

in which the integrand depends only on y. We now use the Fourier representation of the $\delta$-function

$$\delta(\varepsilon) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}e^{is\varepsilon}ds \quad (20)$$

and integrate with respect to $k_Z$ and $k_Z'$, using the well known expression for Fresnel's integral

$$\int_{-\infty}^{+\infty}\exp\{-i\alpha k_z^2\}dk_z = \left(\frac{\pi}{2\alpha}\right)^{1/2}(1-i).$$

In integrating with $k_X$ and $k_X'$ in accordance with (6), (7), (12), and (14), we go over to the variables $\xi' = \xi - \xi_V$ and $\xi'' = \xi - \xi_C$ and use the following formula [12]

$$\frac{1}{2^n n!\sqrt{\pi}}\int_{-\infty}^{+\infty}dt\,e^{-\alpha t^2+ist}H_n^2(\alpha^{1/2}t) = \frac{1}{\sqrt{\alpha}}L_n\left(\frac{s^2}{2\alpha}\right)e^{-s^2/4\alpha}, \quad (21)$$

where $L_n(x)$ is a Laguerre polynomial. Then the integrand is in general independent of **r**, and after integrating with respect to r we obtain for $\Gamma^{\pm}_{K_0\mu}$

$$\Gamma^{\pm}_{K_0\mu} = \Gamma_0{}^{\pm}\sum_{nn'}J^{\pm}_{nn'}; \quad (22)$$

$$J^{\pm}_{nn'} = \frac{1}{2\pi i}\int_{-\infty}^{+\infty}\frac{ds}{s}\exp\left\{-is\varepsilon^{\pm}_{nn'} - \frac{s^2(eEL_M)^2}{2}(1+\beta)\right\}$$

[3)] It is assumed here that the g-factors in both bands are identical and that the transitions proceed with conservation of spin. Therefore the spin splitting in (5)-(15) need not be taken into account explicitly. Generalization to the case of different and anisotopic g-factors is trivial.

$$\times L_{n'}\left(\frac{(eEL_M s)^2}{2}\right) L_n\left(\frac{(eEL_M s)^2}{2}\beta\right), \quad (23)$$

$$\varepsilon^{\pm}_{nn'} = \varepsilon_g - \hbar\omega_0 \pm \hbar\omega_{K_0\mu} + \hbar\omega_v(n' + 1/2) + \hbar\omega_c(n + 1/2)$$
$$+ 1/2(eEL_M)^2(1/\hbar\omega_v + \beta/\hbar\omega_c), \quad (24)$$

$$\Gamma_0^{\pm} = \frac{1}{4\pi}\frac{e^2(m_v^* m_z^*)^{1/2} p_e^2 b_{K_0\mu}^2 (n_{K_0\mu} + 1/2 \pm 1/2)}{\varepsilon_0^2 m^2 \omega_0 c n_0 \rho \omega_{K_0\mu} \hbar L_M^4}. \quad (25)$$

To calculate the integral $J_{nn'}$ we use the integral representation for the Laguerre polynomials [12]:

$$L_n(z) = \frac{1}{2\pi i}\int_c (1-t)^{-1}\exp\left(-\frac{zt}{1-t}\right) t^{-n-1}\,dt. \quad (26)$$

The integration is over the contour $|t| < 1$, which encloses the point $t = 0$. We change over to the variables

$$x = \frac{1}{1-t}, \quad y = \frac{\beta}{1-\tau}, \quad s' = eEL_M s\frac{(x+y)^{1/2}}{\sqrt{2}},$$
$$\lambda_{nn'} = \frac{\varepsilon_{nn'}\sqrt{2}}{eEL_M(x+y)^{1/2}}$$

and obtain

$$J_{nn'} = \frac{1}{(2\pi i)^3}\int_c \frac{x^n dx}{(x-1)^{n+1}}\int_{c'}\frac{y^{n'}dy}{(y-\beta)^{n'+1}}\int_{-\infty}^{+\infty}\frac{ds'}{s'}$$
$$\times \exp(-s'^2 - is'\lambda_{nn'}). \quad (27)$$

We readily see that the derivative of the latter integral in (27) is

$$dJ/d\lambda = -i\sqrt{\pi}e^{-\lambda^2/4}$$

and consequently this integral is equal to

$$J = 2\pi i\Phi(-\lambda/2),$$

where

$$\Phi(t) = \frac{1}{\sqrt{\pi}}\int_{-t}^{\infty} e^{-t^2}\,dt$$

is the probability integral. We took account here of the fact that $J_{nn'}$ should vanish when $\lambda_{nn'} \to \infty$. The indicated "boundary condition" corresponds to circling the singular point $s = 0$ in (22) from below.

Integrating with respect to x and y in (27) by residues, and taking account of the fact that the integrand has poles at $x = 1$ and $y = \beta$, we obtain as a final expression for $\Gamma^{\pm}_{K_0\mu}$:

$$\Gamma^{\pm}_{K_0\mu} = \Gamma_0^{\pm}\sum_{nn'}\frac{d^n}{dx^n}\frac{d^{n'}}{dy^{n'}}\left[\frac{x^n y^{n'}}{n!n'!}\right. \quad (28)$$
$$\left.\times\Phi\left(-\frac{\varepsilon^{\pm}_{nn'}}{\sqrt{2}\,eEL_M(x+y)^{1/2}}\right)\right]\Big|_{x=1,\ y=\beta}.$$

When $E \to 0$ the terms in (28) go over into $\theta$-functions of $\epsilon^{\pm}_{nn'}$, and the spectrum consists, as shown in [3], of individual steps which occur when $\epsilon^{\pm}_{nn'}$ vanishes, that is, when the quantum energy becomes sufficient to excite the transition between the given Landau levels.

We see from (28) that the electric field smears out the steps and, in principle, a given frequency can excite transitions between arbitrary Landau levels. However, when $eEL_M/\hbar\omega_V \ll 1$ and $eEL_M/\hbar\omega_C \ll 1$ the steps are different. Here, as seen from (24), the center of the step shifts from higher frequencies by an amount

$$\Delta\varepsilon_E = \frac{(eEL_M)^2}{2}\left(\frac{1}{\hbar\omega_v} + \frac{\beta}{\hbar\omega_c}\right). \quad (29)$$

We recall that in direct transitions the line has shifted with the electric field by the same amount, but towards lower frequencies [4]. The difference is due to the fact that the direct transitions proceed with conservation of the wave vector $k_X$, and then the center of the oscillator $y_0$ shifts during the transition over the field, and accordingly, as seen from (7), (8), (14), and (15), the electron acquires from the field an energy which is just double the value of the terms quadratic in E in (8) and (15). To the contrary, in indirect transitions, the probability has a maximum with $y_0$ conserved, when the overlap of the wave functions is largest. Therefore, $y_0$ does not change during the transition, and the energy difference increases with increasing E.

It also follows from (28) that the electric field leads to the appearance of absorption in the long-wave region lying beyond the edge of the first step. Here the absorption coefficient attenuates with decreasing frequency like

$$\Gamma^{\pm} = \Gamma_0\frac{1}{2\pi^{1/2}Z}e^{-Z^2}, \quad Z = \frac{\varepsilon_g(E,H) - \hbar\omega_0 \pm \hbar\omega_{K_0}}{\sqrt{2}(1+\beta)^{1/2}eEL_M}, \quad (30)$$

where $\epsilon_g(E, H)$ is the width of the forbidden band with account of the shifts produced by the electric and magnetic fields.

As is well known, the long-wave absorption of light in semiconductors in strong electric fields (the Franz-Keldysh effect) is connected with the indeterminacy of the potential energy of the electron, since its motion in the electric field is infinite [13]. In crossed fields, when $cE/sH < 1$ the motion of the electron is finite, and its energy is fixed. Therefore the shift indicated above is connected with another effect, namely, with the fact that the electron can go over from band to band, shifting over the electric field and giving up an excess momentum $k_X$ to a phonon. Accordingly, the function $\Gamma(E)$ has far from the edge, in accordance with (29), the form

$$\ln \Gamma(E) \approx -(\varepsilon_g - \hbar\omega_0)^{3/2}/E$$

in place of the form

$$\ln \Gamma(E) \sim -(\varepsilon_g - \hbar\omega_0)^{3/2}/E$$

given in [13].

One might think that measurements in an electric field would have no advantages at all compared with the ordinary magneto-optical effect, owing to the smearing of the steps. However, if we measure, as proposed in [5], a change of the absorption coefficient in an alternating electric field at the second harmonic, then sharp peaks will be observed in place of the steps on the $d\Gamma/dE^2$ curve. As seen from (28) and Fig. 2 which shows a plot of $\Gamma(\omega_0)$ for the first lines, $d\Gamma/dE^2$ increases without limit near the steps. However, since the steps become smeared out even for E = 0 owing to the finite relaxation time of the electron and of the hole, it is necessary to take explicit account of this smearing in the calculation of $d\Gamma/dE^2$.

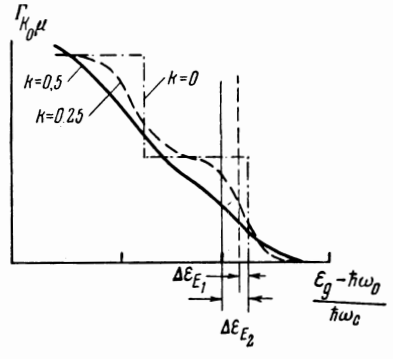


FIG. 2. Absorption coefficient vs. frequency for different values of the electric field, for the first two lines $k = eEL_M/\hbar\omega_c$ (it is assumed here that $\omega_c = \omega_v$ and $\beta = 1$).

## 3. VARIATION OF THE ABSORPTION COEFFICIENT IN WEAK ELECTRIC FIELDS

Let us calculate the variation of $\Gamma$ in the case when the smearing of the step by the electric field is smaller than the smearing connected with finite relaxation time $\tau$. As shown by Korovin and Kharitonov [14], the relaxation time of the electron in a magnetic field depends, generally speaking, on n and $k_z$. However, for simplicity we can introduce the average time $\tau_n$, which depends only on n and correspondingly include into the $\delta$-function a damping $\gamma_{nn'} = \hbar(1/\tau_n + 1/\tau_{n'})$, that is, we can write in lieu of (20)

$$\delta(\varepsilon_{nn'}) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} ds \exp(is\varepsilon_{nn'} - \gamma_{nn'}|s|). \quad (31)$$

We include the indicated damping in (23), in which we expand all the factors in powers of $E^2$. Then, recognizing that $L_n(x)|_{x\to 0} = 1 - nx$ and confining ourselves to first-order terms in $E^2$ we obtain

$$J_{nn'} = \frac{1}{2\pi i}\int_{-\infty}^{+\infty} ds \exp(is\varepsilon^0_{nn'} - \gamma_{nn'}|s|)[s^{-1} - i\Delta\varepsilon_E - {}^1/_2 s(eEL_M)^2][(n' + {}^1/_2) + \beta(n + {}^1/_2)], \quad (32)$$

where $\epsilon^0_{nn'} = \epsilon_{nn'}$ (E = 0), and $\Delta\epsilon_E$ is defined by (29). After evaluating the integrals (32) in accord with (18), we obtain

$$\Gamma^{\pm}_{K_0\mu} = \Gamma_0{}^{\pm}\sum_{nn'}\left\{\left(\frac{1}{2} - \frac{1}{\pi}\tan^{-1}\frac{\varepsilon^{0\pm}_{nn'}}{\gamma_{nn'}}\right) - \frac{(eEL_M)^2}{\pi}\frac{\gamma_{nn'}}{(\varepsilon^{0\pm}_{nn'})^2 + \gamma^2_{nn'}}\left\{\frac{1}{2}\left(\frac{1}{\hbar\omega_v} + \frac{\beta}{\hbar\omega_c}\right) - \frac{\varepsilon^{0\pm}_{nn'}}{(\varepsilon^{0\pm}_{nn'})^2 + \gamma^2_{nn}}\left[\left(n' + \frac{1}{2}\right) + \beta\left(n + \frac{1}{2}\right)\right]\right\}\right\}. \quad (33)$$

It is seen from (33) and from Fig. 3, which shows a plot of $d\Gamma(\omega_0)/dE^2$ for the first lines, that near each step where the corresponding $\epsilon^{0\pm}_{nn'}$ vanishes, the $d\Gamma(\omega_0)/dE^2$ curve has two peaks, one positive and one negative, to the right and to the left of the point $\epsilon^{0\pm}_{nn'} = 0$. Measurement of the distances between the lines corresponding to different n and n′ makes it possible to determine the corresponding frequencies $\omega_c$ and $\omega_v$, and also the frequencies of the phonons of the different branches with wave vector $K_0$. The experiment should yield in this case a set of lines corresponding to different nonequivalent extrema, and measurement of $\omega_c$ as functions of the field orientation, just as in experiments on cyclotron resonance, makes it possible to determine the components of the effective-mass tensor in both bands. Explicit expressions for the angular variation of $\omega_c$ in terms of these components are given, for example, in [11]. The long-wave "tail" connected with the indirect transitions defined by (29) should be observed also for vertical transitions at those frequencies for which direct transitions are still impossible. Apparently, the

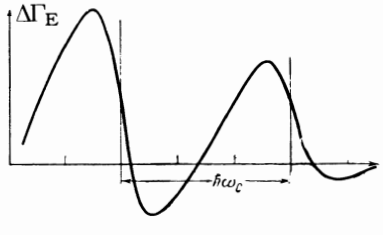


FIG. 3. $\Delta\Gamma_E$ vs. $(\epsilon_g - \hbar\omega_0)/\gamma$ for the first two transitions and $\omega_c\tau = 3$. The solid vertical lines correspond to the points at which $\epsilon^{\pm}_{nn'} = 0$.

absorption observed in [6] near the direct transitions is connected just with this effect, whereas the absorption of still longer wavelengths observed in [6] can be related to the nonvertical indirect transitions considered above. However, the data in [6] are insufficient for a detailed comparison of theory with experiment.

In conclusion the authors note the part played by A. I. Ansel'm, G. L. Bir, V. L. Gurevich, E. K. Kudinov, V. D. Laĭkhtman, S. T. Pavlov and Yu. A. Firsov in a discussion of the present work, and are grateful to them for useful advice and remarks.

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