RESONANCE RADIATION FROM LIQUID HELIUM

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The effect of surrounding fermions and bosons on processes whose end products are the same fermions and bosons is discussed. For liquid He^3 this effect leads to a red shift of the upper boundary of the radiation spectrum of the excited atoms by an amount equal to twice the Fermi energy and, possibly, to an increase of the lifetime of atoms in the excited state. For liquid He^4, the lifetime in the excited state must decrease, but the transparency of the medium at the resonance line should increase.

THE probability of various processes in which fermions appear among the final products is proportional to the number of empty cells available in phase space. If all of phase space is already occupied by fermions of the same type as those resulting from the reaction, the process will be forbidden. As an example we can cite processes taking place with participation of a neutrino in the presence of a saturated current of neutrinos of the same energies.[1,2]

Another example, in which the presence of fermions with identical momenta in the final state shows up, is resonance radiation of liquid He^3. Let us consider this process qualitatively. As is well known, helium forms a Fermi liquid at low temperatures. By some method we excite an arbitrary atom of this liquid, and we consider the probability of inverse transitions to the ground state. (Strictly speaking, one should not talk about the excitation of an atom, but about the excitation of a quasiparticle; however, such a change does not essentially alter the following arguments.) For definiteness, we assume that the excited electron is in one of the first excited p-states. The excited atom differs substantially from the remaining indistinguishable atoms which form the Fermi liquid, and is therefore no longer part of them. If the number of excited atoms is not large, then their equilibrium distribution is not degenerate.

Let us estimate the energy ε_2, to which the excited atom is slowed down during a time $\tau_0 \sim 10^{-9}$ to 10^{-10} sec—the lifetime of an isolated He atom in the excited state.[3] Let the initial energy ε_1 of the excited atom be much greater than the thermal energy, $kT \ll \varepsilon_F$, where ε_F is the Fermi energy. When $\varepsilon_1 \gtrsim \varepsilon_F$ the excited atom can exchange excitation with an unperturbed atom, and the energy of the former becomes $\varepsilon_1' < \varepsilon_F$. This process takes place rapidly because all unexcited atoms may participate in such an interaction. When $\varepsilon_1 < \varepsilon_F$ the excited atom now cannot transmit the excitation to the unexcited atoms, for in this case it would go into the ground state with a momentum which lies inside the Fermi sphere, but the phase cells there are already occupied. When $\varepsilon_1 < \varepsilon_F$, the basic process will be elastic scattering by unexcited atoms. Thus the unexcited atoms will be ejected outside the Fermi surface.

One can estimate the time between two collisions of an excited atom by the same method which was used by Pomeranchuk[4] to estimate the mean free path of an excitation in liquid He^3. The number n of unexcited atoms which will participate in the scattering when $kT \ll \varepsilon_1 \ll \varepsilon_F$ is given by

$$n \sim N\frac{p_F^2\Delta p}{p_F^3} \sim N\frac{p_F - (p_F^2 - p_1^2)^{1/2}}{p_F} \sim N\frac{\varepsilon_1}{\varepsilon_F}, \qquad (1)$$

where N is the number of He^3 atoms in one cm^3. When $\varepsilon_1 \ll \varepsilon_F$ the collisions will be pair collisions. Since only a region of finite momenta lying in a layer of thickness ε_1 near the Fermi surface is available to the unexcited atom after collision, the effective scattering cross section is also reduced in comparison with the gas-kinetic cross section σ_1 by an amount $\sim\varepsilon_1/\varepsilon_F$. As a result, the average time τ_1 between two collisions turns out to be on the order of

$$\tau_1 \sim \frac{1}{\sigma_1 N v_F}\left(\frac{\varepsilon_F}{\varepsilon_1}\right)^2, \qquad (2)$$

where v_F is the velocity of an unexcited atom near the Fermi surface. An exact calculation of the average probability for a pair collision with s-scattering in the case of identical masses of ex-

cited and unexcited atoms and $T = 0$ gives a result in agreement with formula (2), to within numerical coefficients.

The time τ_1 between two collisions increases rapidly with decreasing ε_1. Therefore, one can assume that all of the slowing down time is determined by the time between the last two collisions. The final energy ε_2 to which the atom slows down in a time τ_0 can be determined from (2), having assumed $\tau_0 = \tau_1$:

$$\varepsilon_2 \sim \varepsilon_F (\sigma_1 N v_F \tau_0)^{1/2}. \quad (3)$$

Substituting the values $\sigma_1 \sim 10^{-15}$ cm^2, $N = 1.6 \times 10^{22}$ cm^{-3}, $\tau_0 \sim 10^{-9}$ sec, $v_F \sim 10^4$ cm/ sec into (3), we obtain $\varepsilon_2 \sim 0.1\varepsilon_F$.

For $kT \gtrsim \varepsilon_2$ the excited atoms are able to thermalize, and their distribution is close to equilibrium. In connection with the excitation of an indistinguishable atom, a hole is formed in its place in the distribution of the remaining atoms in momentum space. Because of the scattering of unexcited atoms by one another, a hole with momentum p decays into two with momenta p_1, $p_2 > p$. Here one of the atoms goes over into the state with momentum p, and the other is ejected outside the Fermi surface.

Reasoning and calculations similar to the previous ones enable us to write down an estimate for the decay time τ_2 of a hole:

$$\tau_2 \sim \frac{1}{\sigma_0 N v_F} \left(\frac{\varepsilon_F}{\varepsilon_F - \varepsilon_1} \right)^2, \quad (4)$$

where σ_0 is the gas-kinetic cross section for the scattering of indistinguishable atoms by one another, and ε_1 is the initial energy of an excited atom. During a time $\tau_0 >> \tau_2$, the initial hole manages to decay into a collection of other holes concentrated in a narrow layer near the Fermi surface. Thus, the excited atom and the hole left behind diffuse in opposite directions in momentum space, and after a time τ_0 they turn out to be separated by an energy which is a little smaller than ε_F.

Up to now it has tacitly been assumed that the excitation is localized on an individual atom. This assumption is apparently valid, since there is no exchange interaction for an excited atom located inside the Fermi sphere. An exchange interaction is possible only for excited atoms located on the surface of the Fermi sphere. However, according to the reasons set forth above, an excited atom does not exist very long on the surface. Thus, in the energy region $E_0 \sim 20$ eV broadening of the atomic level because of collective motion in the Fermi liquid does not take place, and the situation is different from what happens during optical excitation of molecular crystals, when an exciton is formed.[5]

If the interaction of an excited atom with the walls of the container and with the surrounding atoms is turned off, then the radiation turns out to be strongly forbidden. Actually, because of the smallness of the photon's recoil energy, $\Delta E_0 \sim 10^{-7}$ eV, the kinetic energy of the radiating atom does not change and remains of the order of ε_2. The number of empty phase cells in the final state is proportional to $\exp[(\varepsilon_2 - \varepsilon_F)/kT]$.[6] Therefore, the probability of radiation is decreased by the same factor. For $\varepsilon_F - \varepsilon_2 \gg kT$ the excitation turns out to be practically "frozen" in the atom.

Interaction with surrounding atoms removes this prohibition. If the energy of the interaction is transmitted to two or to several atoms, then radiation becomes possible. The radiating atom must therefore have in the final state an energy exceeding the Fermi energy. Because of the law of momentum conservation, the second atom must acquire the same energy (one can neglect the photon momentum). Thus, the radiation spectrum turns out to be shifted with respect to the absorption line, by an amount $\sim 2\varepsilon_F$ (to within $\varepsilon_2 \ll \varepsilon_F$) towards the red side.

It is difficult to estimate the actual lifetime τ in an excited state because of the complicated nature of the interaction with surrounding atoms. However, one would think that the transfer of energy from the internal degrees of freedom to external will be difficult for $\tau > \tau_0$. In this case, one should substitute τ instead of τ_0 in formula (3), and the value of ε_2 decreases. For sufficiently large τ, this leads to the interesting possibility of storing excited atoms and removing the excitation by heating the liquid.

Because of strong absorption of the resonance line, it is difficult to experimentally investigate the spectrum of the transmitted radiation, since multiple scattering is possible in thick layers of liquid. The resonance line is absorbed by the surface of the liquid. During the subsequent migration, the excited atom is displaced through a distance $l \sim v\tau$, where v is its velocity. If $\tau \sim \tau_0$, then the excited atom is able to penetrate to a depth $l \sim 3 \times 10^{-6}$ cm. Since the incoherent radiation is shifted in energy by an amount not less than $2\varepsilon_F$, the incoherent scattering cross section is decreased by an amount $(\Gamma/2\varepsilon_F)^2$ (where $\Gamma \sim \hbar/\tau_0$) in comparison with the resonance. One can estimate the mean free path of the secondarily radia-

ted quantum from the formula

$$\Lambda^{-1} \lesssim N\sigma_\gamma(2\varepsilon_F) \sim N\lambda^2 \left(\frac{\Gamma}{2\varepsilon_F}\right)^2, \qquad (5)$$

where $\sigma_\nu(2\varepsilon_F)$ is the cross section for resonance scattering of quanta with energy $E_0 - 2\varepsilon_F$. The value of Λ turns out to be on the order of 2×10^{-5} cm.

Since $\Lambda \gg \lambda \sim \hbar c/E_0$, one can use kinetic theory to consider the subsequent propagation of the secondary radiation. For liquid He^3 films of thickness on the order of 10^{-5} cm, the secondary scattering will be small. Thus, an experimental study of the energy shift and of the spectrum of the radiation line is possible in thin films of liquid He^3, and such a study may yield additional information about the nature of the interaction in the helium Fermi liquid.

A completely different picture of the resonance radiation should be expected in liquid He^4 at temperatures below the λ-point. The probability of transition of an excited atom to the ground state will be increased because of the surrounding quasi-particles—bosons. As for induced electromagnetic radiation, the additional term in the transition probability is proportional to the density of quasiparticles in the final state. Thus, the presence of a condensate must lead to broadening of the resonance line. If the quasiparticle was in the bose-condensate before excitation and was returned there after radiation, then the momentum transferred to the particle is equal to zero, and the radiated quantum has the same frequency and direction as the incident one.

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[1] S. Weinberg, Phys. Rev. **128**, 1457 (1962).

[2] Ya. B. Zel'dovich, JETP **43**, 1561 (1962), Soviet Phys. JETP **16**, 1102 (1963).

[3] C. W. Allen, Astrophysical Quantities, Oxford, 1957 (Russ. Transl., IIL, 1960).

[4] I. Ya. Pomeranchuk, JETP **20**, 919 (1950).

[5] A. S. Davydov, UFN **82**, 393 (1964), Soviet Phys. Uspekhi **7**, 145 (1964).

[6] P. M. Richards, Phys. Rev. **132**, 1867 (1963).

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246