ANISOTROPY OF RADIATION FROM ATOMIC HYDROGEN IN AN ELECTRIC FIELD

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Interference effects in the radiative transitions of atomic hydrogen in an external electric field are studied. It is shown that even in comparatively weak fields the radiation for the $2P_{1/2} \rightarrow 1S_{1/2}$ and $2P_{3/2} \rightarrow 1S_{1/2}$ transitions is noticeably anisotropic and partially linearly polarized.

WE have previously investigated [1] the effect of mixing quantum levels by means of external fields on radiative transitions of atoms and nuclei. In particular, we discussed the interference effects which arise in the transitions of atoms located in a homogeneous electric field. It is well known that, to a first approximation, the electric field mixes only levels of opposite parity 1). This mixing leads to interference in the E1 and M1 transitions, or E1 and E2. Since E2 and M1 transitions are strongly forbidden relative to E1 transitions, the corresponding interference effects are quite small.

In the approximation quadratic in the field, levels of the same parity are mixed and interference of E1 transitions of the same type is possible. However, in the fields attainable in practice, level mixing will be insignificant for nearly all atoms. Interference effects are therefore also negligibly small.

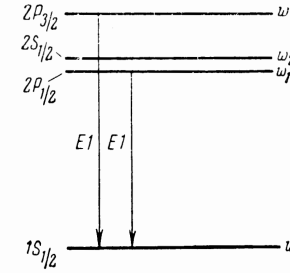
An exception is the hydrogen atom, for which levels with the same j but different l are separated by only the Lamb shift, and levels with different j by relativistic effects and the LS interaction (fine structure). Owing to the smallness of both the Lamb shift and the fine structure 2), noticeable interference effects arise in atomic hydrogen even with weak fields. The present paper is devoted to the study of these effects.

From the invariance of the electromagnetic interaction to spatial inversion and time reversal, it follows directly that one would expect interference terms in the expression for the angular distribution of radiation of the type

$$E_\alpha E_\beta (n_\alpha n_\beta - {}^1/_3\delta_{\alpha\beta}), \quad P_{\alpha\beta}^{(2)} E_\alpha E_\beta \text{ etc.} \tag{1}$$

Here the E_α are the components of the external electric field, **n** is a unit vector in the direction of the radiation, and $\{P_{\alpha\beta}^{(2)}\}$ is the quadrupole polarization tensor. In particular, the radiation of atoms in unpolarized excited states with any total angular momentum should be anisotropic. The radiation of these atoms will also be partially linearly polarized in the plane of the vectors **E** and **n**, or in the direction normal to this plane. The degree of polarization is proportional to $\sin^2\theta/w(\mathbf{n})$, where θ is the angle between **E** and **n**. It follows from general considerations that the coefficient in front of $\sin^2\theta/w(\mathbf{n})$ in the expression for the degree of polarization is the same as the coefficient in front of $-\cos^2\theta$ in the expression for the interference portion of the angular distribution of $w(\mathbf{n})$ (see below).

As an example we shall examine the transition between the first excited state and the ground state of hydrogen. A diagram of the n = 1 and n = 2 levels, including both the fine-structure and the Lamb shift, is shown in the figure. In a homogeneous electric field mixing will occur between all three levels $2P_{1/2}$, $2S_{1/2}$, and $2P_{3/2}$, and, because of this



Level scheme for n = 1 and n = 2; $w_3 - w_1 = 1.1\times10^4$ Mc, $w_2 - w_1 = 1058$ Mc; $w_1 - w_0 = 2.4\times10^9$ Mc.

1)For example, in the case of LS coupling, this will be the levels with $\Delta L = 1$.

2)Thus, for example, for the n = 2 states the Lamb shift between the $2S_{1/2}$ and $2P_{1/2}$ levels is equal to 1058 Mc, and fine-structure splitting between the $2P_{3/2}$ and $2P_{1/2}$ levels is 1.1×10^4 Mc.

mixing, interference will occur for the E1 transitions:

$$2P_{1/2} \to 1S_{1/2}, \quad 2P_{3/2} \to 1S_{1/2}.$$

Although the Stark shift of these levels is much less than the separation between them (for fields $E \ll 500$ v/cm), the $2P_{3/2}$ level is mixed with the $2P_{1/2}$ level in the approximation quadratic in the field:

$$|2P_{1/2}m\rangle' = \left[1 - \frac{3a_0^2e^2E^2}{2(w_2-w_1)^2}\right]|2P_{1/2}m\rangle - \frac{(-1)^{-1/2-m}\sqrt{3}\,a_0eE}{w_2-w_1}|2S_{1/2}m\rangle + \frac{(-1)^{-1/2-m}3\sqrt{2}\,a_0^2e^2E^2}{(w_2-w_1)(w_3-w_1)}|2P_{3/2}m\rangle, \tag{2}$$

where a_0 is the Bohr radius and e is the electron charge. When the Stark splitting is comparable to $w_2 - w_1$, but still much less than $w_3 - w_1$, we must use the more general equations:

$$|2P_{1/2}m\rangle' = \frac{\sqrt{3}\,a_0eE}{[3a_0^2e^2E^2+(w_1-\mathscr{E}_1)^2]^{1/2}}|2P_{1/2}m\rangle - \frac{(-1)^{-1/2-m}(w_1-\mathscr{E}_1)}{[3a_0^2e^2E^2+(w_1-\mathscr{E}_1)^2]^{1/2}}|2S_{1/2}m\rangle + \frac{(-1)^{-1/2-m}(w_1-\mathscr{E}_1)}{[3a_0^2e^2E^2+(w_1-\mathscr{E}_1)^2]^{1/2}}\frac{\sqrt{6}\,a_0eE}{w_3-\mathscr{E}_1}|2P_{3/2}m\rangle,$$

$$\mathscr{E}_1 = \frac{w_2+w_1}{2} - \left[\left(\frac{w_2-w_1}{2}\right)^2 + 3a_0^2e^2E^2\right]^{1/2}. \tag{3}$$

Whereas in the former case we have two parameters of smallness, $a_0eE/(w_2 - w_1)$ and $a_0eE/(w_3 - w_1)$, only the second parameter remains when $a_0eE \sim (w_2 - w_1)$, and Eq. (3) for $|2P_{1/2}m\rangle'$ is written in the approximation that is linear in this parameter. We consider just such fields in what follows.

If the atom is in a state with energy $\mathscr{E}_1$ prior to de-excitation, the radiation can be described by using the amplitudes

$$\langle 1S_{1/2}m_2, \mathbf{n}\alpha|T|2P_{1/2}m_1\rangle' = \frac{\sqrt{3}\,a_0eE}{[3a_0^2e^2E^2+(w_1-\mathscr{E}_1)^2]^{1/2}}\langle 1S_{1/2}m_2, \mathbf{n}\alpha|T|2P_{1/2}m_1\rangle + \langle 1S_{1/2}m_2, \mathbf{n}\alpha|T|2P_{3/2}m_1\rangle \times \frac{(-1)^{1/2-m_1}\sqrt{6}a_0\,eE(w_1-\mathscr{E}_1)}{[3e^2a_0^2E^2+(w_1-\mathscr{E}_1)^2]^{1/2}(w_3-\mathscr{E}_1)}, \tag{4}$$

where the unprimed amplitudes describe the transitions with no field, and $|1S_{1/2}m_2, \mathbf{n}\alpha\rangle$ denotes the final state with a photon emitted in the direction **n** with polarization α. From (4) it follows immediately that a contribution from the interference of E1 transitions $2P_{1/2} \to 1S_{1/2}$ and $2P_{3/2} \to 1S_{1/2}$ will be added to the probability of transitions $|2P_{1/2}m_1\rangle' \to |1S_{1/2}m_2\rangle$. The equations for these probabilities, summed over all hyperfine splitting components of the final state and averaged over all components of the initial state, will also contain interference terms. As a result of these terms, the Stokes parameter

$$\xi_3(\mathbf{n}) = \frac{Q(\mathbf{n},\mathbf{e}_1)-Q(\mathbf{n},\mathbf{e}_2)}{Q(\mathbf{n},\mathbf{e}_1)+Q(\mathbf{n},\mathbf{e}_2)} = \frac{w_1-\mathscr{E}_1}{w_3-\mathscr{E}_1} \times 3\sin^2\theta\left[1+\frac{w_1-\mathscr{E}_1}{w_3-\mathscr{E}_1}(1-3\cos^2\theta)\right]^{-1}, \tag{5}$$

which describes the linear polarization of the radiation, will be non-zero. The quantity $Q(\mathbf{n}, \mathbf{e}_1)$ in (5) denotes the probability of finding the photon linearly polarized in the plane of the vectors **E** and **n**, and $Q(\mathbf{n}, \mathbf{e}_2)$ is the normal to this plane.

Note that photons for the transitions $|2P_{1/2}m\rangle' \to 1S_{1/2}$ are found to be partially linearly polarized predominantly in the **E**, **n** plane. The angular distribution is anisotropic and is given by the equation

$$w(\mathbf{n}) = \frac{1}{4\pi} + \frac{1}{4\pi}\frac{w_1-\mathscr{E}_1}{w_3-\mathscr{E}_1}(1-3\cos^2\theta). \tag{6}$$

The total transition probability, however, contains no interference terms and is given by

$$Q = \frac{3a_0^2e^2E^2}{3a_0^2e^2E^2+(w_1-\mathscr{E}_1)^2}Q_0, \tag{7}$$

where Q_0 is the total transition probability with zero field.

Interference effects will also be observed with the de-excitation of the $|2S_{1/2} \pm 1/2\rangle'$ and $|2P_{3/2} \pm 1/2\rangle'$ states, which prior to the perturbation are the states $|2S_{1/2} \pm 1/2\rangle$ and $|2P_{3/2} \pm 1/2\rangle$ respectively. Thus, for example, the radiation for the transition $|2S_{1/2} \pm 1/2\rangle' \to 1S_{1/2}$ is anisotropic, and the photons are partially linearly polarized primarily normal to the **E**, **n** plane. As in the case of the $|2P_{1/2}\rangle' \to 1S_{1/2}$ transition, the interference effects are linear in the parameter $a_0eE/(w_3 - w_1)$:

$$\xi_3(\mathbf{n}) = \frac{3e^2a_0^2E^2}{(w_2-\mathscr{E}_2)(w_3-\mathscr{E}_2)} \times 3\sin^2\theta\left[1+\frac{3a_0^2e^2E^2}{(w_2-\mathscr{E}_2)(w_3-\mathscr{E}_2)}(1-3\cos^2\theta)\right]^{-1}, \tag{8}$$

$$w(\mathbf{n}) = \frac{1}{4\pi} + \frac{1}{4\pi}\frac{3a_0^2e^2E^2}{(w_2-\mathscr{E}_2)(w_3-\mathscr{E}_2)}(1-3\cos^2\theta), \tag{9}$$

where

$$\mathscr{E}_2 = \frac{w_2 + w_1}{2} + \left[\left(\frac{w_1 - w_2}{2}\right)^2 + 3a_0^2e^2E^2\right]^{1/2}.$$

We note that, owing to the mixing of the $2S_{1/2}$ and $2P_{1/2}$ states, the total transition probability is non-zero [2], but again does not contain interference terms:

$$Q = \frac{(w_2 - \mathscr{E}_2)^2}{3a_0^2e^2E^2 + (w_2 - \mathscr{E}_2)^2} Q_0. \qquad (10)$$

Interference effects in the $|2P_{3/2} \pm 1/2\rangle \to 1S_{1/2}$ transitions are already quadratic in $a_0eE/(w_3 - w_1)$. Again the photons are seen to be partially linearly polarized primarily in the **E**, **n** plane. In this case

$$\xi_3(\mathbf{n}) = \frac{3a_0^2e^2E^2}{(w_3 - \mathscr{E}_1)(w_3 - \mathscr{E}_2)} \times 3\sin^2\theta\left[1 + \frac{3a_0^2e^2E^2}{(w_3 - \mathscr{E}_2)(w_3 - \mathscr{E}_1)}(1 - 3\cos^2\theta)\right]^{-1}, \qquad (11)$$

$$w(\mathbf{n}) = \frac{1}{4\pi} + \frac{1}{4\pi}\frac{3a_0^2e^2E^2}{(w_3 - \mathscr{E}_2)(w_3 - \mathscr{E}_1)}(1 - 3\cos^2\theta). \qquad (12)$$

The total probabilities of the $|2P_{3/2} \pm 1/2\rangle' \to 1S_{1/2}$ transitions are independent of the magnitude of the field in this approximation: $Q = Q_0$.

It can be shown that when one adds together the probabilities of the three transitions

$$|2P_{1/2}m\rangle' \to 1S_{1/2}, \quad |2S_{1/2}m\rangle' \to 1S_{1/2}, \quad |2P_{3/2}m\rangle' \to 1S_{1/2}$$

the interference terms cancel each other completely (accurate to terms that depend upon the mixing of levels from different shells). Therefore, in order to observe this effect, it is necessary to separate one particular line, or a group of two lines.

Note also that the external field has no effect upon the transitions $|2P_{3/2} \pm 3/2\rangle \to 1S_{1/2}$.

Similar effects also occur for transitions between levels with larger quantum numbers n. However, these become apparent only at considerably weaker fields. Thus, for example, if the anisotropy in the angular distribution of radiation for the transition $2P'_{1/2} \to 1S_{1/2}$ has a magnitude about 0.1 in a field of about 500 V/cm, it will have the same value for the $3S'_{1/2} \to 2P'_{1/2}$ transition in a field of only about 50 V/cm.

[1] V. L. Lyuboshitz, V. A. Onishchuk, and M. I. Podgoretskiĭ, JINR Preprint R-2248, Dubna (1965).

[2] G. Bethe and E. Salpeter, Quantum Mechanics of One- and Two-electron Atoms (Russ. Transl.), Fizmatgiz (1960).

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249