HIGH ENERGY SUCCESSIVE INTERACTIONS

E. L. FEĬNBERG

P. N. Lebedev Physics Institute, Academy of Sciences, U.S.S.R.

Submitted to JETP editor July 22, 1965

J. Exptl. Theoret. Phys. (U.S.S.R.) **50**, 202–214 (January, 1966)

The properties (interaction features) of a particle experiencing a sharp momentum change are investigated. The analysis is carried out for two successive bremsstrahlung acts of an electron on immobile centers within the framework of standard renormalized quantum electrodynamics. The difference between the present analysis and the usual one is that the time variation of the system functional (assumed initially as a packet) is calculated. It is shown that after the first interaction the particle stays in a state in which its subsequent interaction differs from the normal one for a long period of time (which is macroscopically long at very high energies and much greater than the time of passage of the packet). In this state the proper field of the particle differs from the stationary proper field of an ordinary particle with the given momentum. The effect is of a classical nature and is quite large because of the relativistic slowing down of the restoration time of the stationary state at high energies. It is characteristic not only of bremsstrahlung and not only of electrodynamics. The possibility of the existence of this effect for a nucleon strongly interacting with its meson field is discussed and some estimates are presented.

INTRODUCTION

FROM the point of view of classical electrodynamics it is obvious that an electrically charged body whose momentum changes abruptly will not establish instantaneously around itself a stationary co-moving electromagnetic field that corresponds to the new state of motion. In a coordinate frame fixed in the particle, the time of establishment of the stationary field at a given point should be the larger, the farther this point is from the axis of the motion or—in terms of the Fourier representation of the field—the smaller the wave number of the Fourier component. So long as the field of the charge is still in the transient state, the interaction between the body and other bodies (to the extent that it is due to the contribution of the given Fourier component) will differ from the interaction produced when the same body (charge) arrives with the same momentum from infinity surrounded by a stationary field (for the given body momentum).

In nonrenormalized quantum electrodynamics (for example, with a form factor) it is likewise possible to trace a similar delay in the "normal" state of the self-field of the electron, and to verify that there exists a time interval during which the charged particles differ from normal for the same particle velocity (see Sec. 4 below). Similar phenomena should take place also for any other field, at least in the weak-coupling model.

The same effect should appear also in renormalized quantum electrodynamics, although the self-field of the particle is described here in a singular fashion. In the present paper, we demonstrate this with bremsstrahlung as an example, but the described effect is not at all limited to the chosen concrete process.

The effects that arise at high energies are quantitatively appreciable. An example is the process described in the last paragraph in Sec. 2. Essentially, however, the phenomenon in question is the basis of many so-called diffractive inelastic (or coherent inelastic) processes that are investigated at high energies, transition radiation in layered media, etc.

The result of the present analysis reduces for the most part to proving that an electron can have a sufficiently long-lasting state in which it has a nonequilibrium self-field ("semi-bare electron"), and that this state can be registered separately from the first interaction event that generates it. Such a conclusion makes it possible to raise the question of the existence and possible role of such a nonequilibrium state for a particle that interacts strongly with its field (nucleon). This process cannot be solved by direct calculations. It is discussed in Sec. 5, where certain estimates are presented, showing that if this conclusion is valid for a nucleon, then appreciable effects can take place when superrelativistic nucleons interact with nuclei, etc.

1. BREMSSTRAHLUNG FROM ONE CENTER

Let us consider the bremsstrahlung of an electron at a stationary (for example, Coulomb) center located at the origin $x = 0$. Since we are interested in the time evolution of the process, the initial state of the electron must be specified in the form of a packet that is bounded in the direction of motion. When $t \to -\infty$ the functional of this state (in the interaction representation) is

$$\overline{\Phi}(-\infty) = \int f(p_1) a_{\nu_1}^{+}(\mathbf{p}_1) dp_1 |0\rangle \equiv \overline{\Phi}_0, \qquad (1.1)$$

where $p_1 = |\mathbf{p}_1|$, ν_1 is the spin index of the electron, and $\overset{+}{a}_{\nu_1}(\mathbf{p}_1)$ is the renormalized operator of its creation. The electron energy is $\epsilon_{p_1} = (p_1^2 + m^2)^{1/2}$ where m is the renormalized mass. The superior bar in (1.1) denotes averaging over the packet $f(p_1)$.

We stipulate that the center of the packet pass through the point $x = 0$ at the instant $t = 0$, i.e., that in x-space we have a wave

$$\exp\left[-i\left(\varepsilon_{q_1} t - \mathbf{q}_1 \mathbf{x}\right)\right] F\left(\frac{\mathbf{x} - \mathbf{v}t}{2L}\right), \qquad (1.2)$$

where $v = (\nabla_p \epsilon_p)_{p = q_1}$ is the velocity of the center of the packet, $2L$ the width, and $\mathbf{q}_1$ the average momentum. The function $F(\xi)$ has a maximum at $\xi = 0$ and $F(\xi) \to 0$ at $|\xi| \gg 1$. It is convenient to assume that the packet is Gaussian. Then, in p-space

$$f(p_1) = \frac{L}{\sqrt{\pi}} \exp\left[-L^2 (p_1 - q_1)^2\right] \qquad (1.3)$$

and when $L \to \infty$ we get

$$f(p_1) \to \delta(p_1 - q_1). \qquad (1.4)$$

The scattering is from a static internal field with 4-potential

$$A_\mu^e = a_\mu(\mathbf{x}) = \frac{1}{(2\pi)^{3/2}} \delta_{\mu 0} \int e^{-i\mathbf{q}\mathbf{x}} \varphi_\mu(\mathbf{q}) d^3 q; \qquad (1.5)$$

for a Coulomb center

$$\varphi_0(\mathbf{q}) = Ze / (2\pi)^{3/2} \mathbf{q}^2, \qquad A_\mu^e = Ze / 4\pi |\mathbf{x}|.$$

The perturbation operator is

$$H_1(t) = \int_{-\infty}^{+\infty} : j^\mu(x) \left(A_\mu(x) + A_\mu^e(x)\right) - \delta m \overline{\psi}(x) \psi(x) : e^{-\varepsilon |t|} d^3 x$$

$$= H_1^r(t) + V(t) - H_{\delta m}(t), \qquad (1.6)$$

where H_1^r is the Hamiltonian of the interaction with the radiation field and $V \sim e^2$—with the scattering center, and we have introduced explicitly the factor of adiabatic switching-on and switching-off the field at $t = \pm\infty$ (it is understood that at the end we must put $\epsilon = +0$).

Accurate to the lowest required order (third) in e, we have

$$\Phi(t) = T \exp\left[-i \int_{-\infty}^{t} H_1(t') dt'\right] \Phi_0$$

$$= \Big\{1 - i \int_{-\infty}^{t} H_1^r(t') dt' + \frac{(-i)^2}{2} T \int_{-\infty}^{t} dt'$$

$$\times \int_{-\infty}^{t} dt'' H_1^r(t') H_1^r(t'') - \int_{-\infty}^{t} dt' H_{\delta m}(t') + (-i) \int_{-\infty}^{t} V(t') dt'$$

$$+ \frac{(-i)^2}{2} \cdot 2 \int_{-\infty}^{t} dt' \int_{-\infty}^{t} dt'' \, T\left(H_1^r(t') V(t'')\right)\Big\} \Phi_0. \qquad (1.7)$$

The first four terms in the brackets described the free motion of the electron without scattering from the center; the next describes the Born scattering from the center without interaction with the radiation. The bremsstrahlung is given by the last term.

We are interested in the admixture of the state $a_{\nu_2}^{+}(\mathbf{p}_2 - \mathbf{k}) \alpha_j^{+}(\mathbf{k}) |0\rangle$, containing the electron $(\mathbf{p}_2 - \mathbf{k}, \nu_2)$ and the photon (k, j) ($\alpha_j^{+}(\mathbf{k})$—creation operator for such a photon with momentum $\mathbf{k}$ and polarization j):

$$M_{\mathbf{p}_1 \nu_1}^{\mathbf{p}_2 \nu_2 \mathbf{k} j}(t) = \langle 0 | \overset{*}{a}_{\nu_2}^{-}(\mathbf{p}_2 - \mathbf{k}) \alpha_j^{-}(\mathbf{k}) \Phi(t) = -\langle 0 | \overset{*}{a}_{\nu_2}^{-}(\mathbf{p}_2 - \mathbf{k})$$

$$\times \alpha_j^{-}(\mathbf{k}) \int_{-\infty}^{t} dt' \int_{-\infty}^{t} dt'' \, T\left(H_1^r(t') V(t'')\right) \Phi_0. \qquad (1.8)$$

This expression must be furthermore averaged over the packet. Calculation (in the notation of [1]) yields ($\epsilon_1 \to +0$, $\epsilon_2 \to +0$, $\epsilon_1 \neq \epsilon_2$):

$$M_{\mathbf{p}_1 \nu_1}^{\mathbf{p}_2 \nu_2 \mathbf{k} j}(t) = \frac{e^2}{8\pi^3} \varphi_0(\mathbf{p}_2 - \mathbf{p}_1) \int_{-\infty}^{t} dt_1 \int_{-\infty}^{t} dt_2 \exp\left[-\varepsilon_1 |t_1|\right.$$

$$\left. - \varepsilon_2 |t_2|\right] \{\,\}, \qquad (1.9)$$

$$\{\,\} = \theta(t_2 - t_1) \{R_2^{+} \exp\left[-i(\varepsilon_{\mathbf{p}_2} + \varepsilon_{\mathbf{p}_1}) t_2\right.$$

$$\left. + i(\varepsilon_{\mathbf{p}_2} + \varepsilon_{\mathbf{p}_2 - \mathbf{k}} + k) t_1\right] - R_1^{-} \exp\left[-i(\varepsilon_{\mathbf{p}_1 - \mathbf{k}} - \varepsilon_{\mathbf{p}_2 - \mathbf{k}}) t_2\right.$$

$$\left. - i(\varepsilon_{\mathbf{p}_1} - \varepsilon_{\mathbf{p}_1 - \mathbf{k}} - k) t_1\right]\} + \theta(t_1 - t_2) \{- R_2^{-} \exp\left[-i(\varepsilon_{\mathbf{p}_2}\right.$$

$$\left. - \varepsilon_{\mathbf{p}_2 - \mathbf{k}} - k) t_1 - i(\varepsilon_{\mathbf{p}_1} - \varepsilon_{\mathbf{p}_2}) t_2\right]$$

$$+ R_1^{+} \exp\left[-i(\varepsilon_{\mathbf{p}_1 - \mathbf{k}} + \varepsilon_{\mathbf{p}_1} - k) t_1 + i(\varepsilon_{\mathbf{p}_1 - \mathbf{k}} + \varepsilon_{\mathbf{p}_2 - \mathbf{k}}) t_2\right]\}, \qquad (1.10)$$

$$R_1^{\pm} = \frac{1}{2\varepsilon_{\mathbf{p}_1 - \mathbf{k}}} \sum_\mu \overline{v}^{\nu_1 -}(\mathbf{p}_1) \gamma^\mu (\varepsilon_{\mathbf{p}_1 - \mathbf{k}} \gamma^0 \pm (\mathbf{p}_1 - \mathbf{k}) \boldsymbol{\gamma} \pm m) \gamma^0 v^{\nu_2 +}$$

$$\times (\mathbf{p}_2 - \mathbf{k}) \frac{e_\mu^j}{\sqrt{2k}},$$

$$R_2^{\pm} = \frac{1}{2\varepsilon_{p_2}} \sum_{\mu} \bar{v}^{v_1-}(p_1)\gamma^0(\varepsilon_{p_2}\gamma^0 \pm p_2\gamma \pm m)\gamma^{\mu}v^{v_2+} \times (p_2 - k)\frac{e_\mu^j}{\sqrt{2k}}. \quad (1.11)$$

The integration with respect to t_1 and t_2 is carried out simultaneously with averaging over p_1. The terms ϵ_{p_1} and ϵ_{p_1-k} in the exponents are then expanded in powers of the deviation from the center of the packet, for example

$$\varepsilon_{p_1} = \varepsilon_{q_1} + (\partial\varepsilon_p/\partial p)_{p=q_1}(p_1 - q_1) = \varepsilon_{q_1} + v_{q_1}(p_1 - q_1), \quad (1.12)$$

where $v_{q_1} \approx 1$ is the velocity of the center of the packet. When we encounter the expression

$$(v_{q_1-k} - v_{q_1})\, t_i \approx {}^1/_2 m^2 (\varepsilon_{q_1}^{-2} - \varepsilon_{q_1-k}^{-2})\, t_i, \quad (1.13)$$

we can discard it as being relatively small for all the time intervals of interest to us (see below). The spreading of the packet is of the same order as given by (1.13). According to the same estimate, this spreading is small over the times of interest to us.

Substitution of (1.10) in (1.9) gives four integrals with respect to time (and with respect to p_1). Thus, for example, if (1.12) is used, we get for the first of them (we replace $\mathbf{p}_1$ by $\mathbf{q}_1$ in $R_1^{\pm}$)

$$\begin{aligned}\bar{J}_1 = &\int_{-\infty}^{t} dt_2 \int_{-\infty}^{t} dt_1 \int_{-\infty}^{\infty} \frac{L}{\sqrt{\pi}} \exp[-L^2(p_1 - q_1)^2]\, dp_1 \\ &\times \exp[i(\varepsilon_{p_2} + \varepsilon_{p_2-k} + k)t_1 - i(\varepsilon_{p_2} + \varepsilon_{p_1})t_2 \\ &- \varepsilon_1|t_1| - \varepsilon_2|t_2|] = \frac{1}{i(\varepsilon_{p_2} + \varepsilon_{q_1})}\left\{\frac{2L}{v_{q_1}} \exp\left[-(\varepsilon_{q_1} \right.\right. \\ &\left. - \varepsilon_{p_2-k} - k)^2 \frac{L^2}{v_{q_1}^2}\right] \sqrt{\pi}\left(1 - \frac{1}{\sqrt{\pi}}\int_a^{\infty} e^{-\xi^2} d\xi\right) \\ &- \frac{1}{i(\varepsilon_{p_2} + \varepsilon_{p_2-k} + k)} \exp\left[-i(\varepsilon_{q_1} - \varepsilon_{p_2-k} - k)\, t \right. \\ &\left.\left. - \frac{v_{q_1}^2 t^2}{4L^2}\right]\right\}, \end{aligned} \quad (1.14)$$

$$a = v_{q_1}t/2L + i(\varepsilon_{q_1} - \varepsilon_{p_2-k} - k)\, L/v_{q_1}. \quad (1.14a)$$

We are interested in times t that are long compared with the time of motion τ_L of the packet past the scattering center:

$$t \gg \tau_L, \qquad \tau_L = 2L/v. \quad (1.15)$$

Under these conditions $|a| \gg 1$. When $t < -\tau_L$, i.e., so long as the packet has not yet reached the scattering center, the integral in (1.14) becomes equal to $\sqrt{\pi}$ and $\bar{J}_1 = 0$. On the other hand, if $t > \tau_L$ then the integral is exponentially small and we have approximately

$$\bar{J}_1 = \frac{1}{i(\varepsilon_{p_2} + \varepsilon_{q_1})} 2\pi\Delta_L(\varepsilon_{q_1} - \varepsilon_{p_2-k} - k), \quad (1.16)$$

$$\Delta_L(\xi) = Le^{-L^2\xi^2}/\sqrt{\pi}. \quad (1.17)$$

The δ-function Δ_L smeared over the packet guarantees an approximate satisfaction (within the limits imposed directly by the energy in the packet) of the energy conservation law. When $L \to \infty$

$$\Delta_L(\xi) \to \delta_L(\xi).$$

The integration in the remaining three integrals is similar. We assume throughout that $t \gg \tau_L$, discard the exponentially small terms which are essential only during the time of motion past the scattering center, and retain only the terms with "quasi-resonance" denominators $\epsilon_{q_1} - \epsilon_{q_1-k} - k$ and $\epsilon_{p_2} - \epsilon_{p_2-k} - k$, which do not vanish but are anomalously small at high energies and when the $\mathbf{k}$ make a small angle with $\mathbf{q}_1$ or $\mathbf{p}_2$, respectively (compared, for example, with $\epsilon_{p_2} + \epsilon_{p_2-k} + k$). We then obtain ($t \gg \tau_L$):

$$M_{q_1v_1}^{p_2v_2kj}(t) = \frac{ie^2}{4\pi^2}\varphi_0(q_1 - p_2)\left\{\frac{R_1^-}{\varepsilon_{q_1} - \varepsilon_{q_1-k} - k} - \frac{R_2^-(1 - \exp[-i(\varepsilon_{p_2} - \varepsilon_{p_2-k} - k - i\varepsilon)t])}{\varepsilon_{p_2} - \varepsilon_{p_2-k} - k}\right\} \times \Delta_L(\varepsilon_{q_1} - \varepsilon_{p_2-k} - k) \quad (\varepsilon \to +0). \quad (1.18)$$

In one case it is necessary to retain explicitly the switching-off factor $e^{-\epsilon t}$ which makes it possible to go to the limit as $t \to +\infty$. In such a transition the exponential drops out, and we obtain the usual matrix element of the bremsstrahlung (in particular, we can put $L = \infty$).[1)] It consists of two cones directed respectively along the initial momentum $\mathbf{q}_1$ and along the electron momentum after scattering $\mathbf{p}_2$. The denominators in (1.18) contain the usual expressions:

$$\Omega_1 = \varepsilon_{q_1} - \varepsilon_{q_1-k} - k = -\frac{2\varepsilon_{q_1}k}{\varepsilon_p + \varepsilon_{q_1-k}}(1 - v_0\cos\theta_0), \quad (1.18a)$$

$$\Omega_2 = \varepsilon_{p_2} - \varepsilon_{p_2-k} - k = -\frac{2\varepsilon_p k}{\varepsilon_{q_1} + \varepsilon_{p+k}}(1 - v\cos\theta), \quad (1.18b)$$

where $v_0 = q_1/\epsilon_{q_1}$, $v = p/\epsilon_p$; θ_0 and θ are the angles of the vector $\mathbf{k}$ with $\mathbf{q}_1$ and $\mathbf{p} \equiv \mathbf{p}_2 - \mathbf{k}$, respectively.

However, whereas the first cone is produced immediately at $t \geq \tau_L$, the second differs from

1)The exponential (1.18) is preceded in fact also by a factor G, which under the conditions of interest to us is close to unity:

$$G = \frac{\Delta_L(\varepsilon_{q_1} - \varepsilon_{p_2})}{\Delta_L(\varepsilon_{q_1} - \varepsilon_{p_2-k} - k)} = \exp\left[-L^2\Omega_2\left(\Omega_2 + O\left(\frac{1}{L}\right)\right)\right].$$

We shall henceforth impose on L the condition $L\Omega_2 \ll 1$ (see (2.1)), so that $G \approx 1 + O(L\Omega_2) \approx 1$.

zero only at times which are not small compared with the time interval

$$T_{\mathbf{k}} = -\frac{1}{\varepsilon_{p_2} - \varepsilon_{p_2-k} - k} = \frac{\varepsilon_{q_1} + \varepsilon_{p_2}}{2\varepsilon_{p_2-k} k (1 - v\cos\theta)}. \quad (1.19)$$

For reasons which will become clear later, we shall call this the regeneration time for the photons of momentum $\mathbf{k}$. For a nonrelativistic particle, $p_2 \ll m$, or simply in its rest system, we have putting for this case $\mathbf{k} = \mathbf{k}^0$,

$$T_{\mathbf{k}} \approx 1/k = 1/k^0. \quad (1.19a)$$

However, at high energies $p_2 \gg m$, $|\mathbf{p}_2 - \mathbf{k}| \gg m$, assuming $k_\perp \ll k_\| \lesssim p_2$, where $k_\perp$ and $k_\|$ are components of the vector $\mathbf{k}$ perpendicular and parallel to $\mathbf{p}_2$, we have

$$T_{\mathbf{k}} \sim \frac{2p_2 k_\| (p_2 - k_\|)}{m^2 k_\|^2 + k_\perp^2 p_2^2} = 2\frac{\varepsilon_{p_2}}{m}\frac{k_\|^0 (1 - k_\|^0/m)}{k_\|^{02} + k_\perp^{02}} \sim \frac{1}{k^0}\frac{\varepsilon_{p_2}}{m}, \quad (1.19b)$$

where $\mathbf{k}$ is expressed in terms of its value $\mathbf{k}^0$ in the rest system of the electron, $k_\| = \epsilon_{p_2} k_\|^0/m$, $k_\perp = k_\perp^0$.

Thus, T_k can be large if $\epsilon_p \gg m$. Even if we are interested in photons whose energy in the electron rest system is of the order of m, $k_\perp^0 \sim k_\|^0 \sim m$, then an electron of energy $\sim 10^{16}$ eV should traverse a path of approximately 1 cm before such a photon will be emitted. With decreasing k, this time increases. At low energies ϵ_{p_2}, on the other hand, the regeneration time for this electron is of the order of $1/m$. The obvious reason for the difference between the times (1.19a) and (1.19b) is the relativistic retardation of the time on going over to the laboratory frame, where the same photon has $k_\| \sim \epsilon$.

2. SUCCEEDING INTERACTIONS

It is now appropriate to formulate more precisely the limitation and the width of the packet L. We shall assume that for the values of k of interest to us

$$\frac{1}{|\mathbf{p}_2 - \mathbf{q}_1|}, \frac{1}{\varepsilon_{p_2}}, \frac{1}{\varepsilon_{q_1}}, \frac{1}{k} \ll L \ll T_{\mathbf{k}}. \quad (2.1)$$

Therefore, within the limits of the time $t \sim T_k$ the electron is sufficiently well localized. Under these conditions, as assumed above,

$$\Delta_L(\varepsilon_{q_1} - \varepsilon_{p_2}) \approx \Delta_L(\varepsilon_{q_1} - \varepsilon_{p_2-k} - k).$$

Let us consider a particular case. Assume that an electron, without emitting a photon in a direction close to $\mathbf{q}_1$, is scattered through an angle which is much larger (for the given $\mathbf{k}$) than the apex angle of each of the bremsstrahlung cones. We can consider separately the corresponding contribution $\Delta\Phi(t)$ to the functional—it is obtained from the second term in the curly bracket of (1.18) (allowance for the first term changes nothing in principle). In addition, the elastically scattered neutron will move (without radiation) in this direction and its contribution to the functional is given by the next to last term in (1.7) (Born scattering):

$$\Delta\Phi(t) = -i\int_{-\infty}^{t} V(t')\,dt'\,\Phi_0 + \sum_{j,\,\nu_2}\iint d^3k\,d^3p_2\,M(t)\,a_{\nu_2}^+(\mathbf{p}_2 - \mathbf{k})\,\alpha_j^+(\mathbf{k})\,|0\rangle, \quad (2.2)$$

$$M = \frac{ie^2}{4\pi^2}\varphi_0(\mathbf{q}_1 - \mathbf{p}_2)\frac{R_2^-(\exp[-i(\varepsilon_{p_2} - \varepsilon_{p_2-k} - k - i\varepsilon)t] - 1)}{\varepsilon_{p_2} - \varepsilon_{p_2-k} - k} \times \Delta_L(\varepsilon_{q_1} - \varepsilon_{p_2-k} - k), \quad (2.2a)$$

$$-i\int_{-\infty}^{t} V(t')\,dt'\,\Phi_0 = \sum_{\nu_2}\int M_0^{\nu_1\nu_2}\Delta_L(\varepsilon_{p_2} - \varepsilon_{q_1})\,a_{\nu_2}^+ \times (\mathbf{p}_2)|0\rangle\,d^3p_2, \quad (2.3)$$

$$M_0^{\nu_1\nu_2} = \frac{-ie}{(2\pi)^{1/2}}\varphi_0(\mathbf{q}_1 - \mathbf{p}_2)R_0^{\nu_1\nu_2},$$

$$R_0^{\nu_1\nu_2} = (\bar{v}^{\nu_1-}(\mathbf{q}_1)\gamma^0 v^{\nu_2+}(\mathbf{p}_2)). \quad (2.3a)$$

Recognizing that by virtue of (2.1) the ratio of the Δ_L-functions is close to unity and substituting in R_2 (1.11)

$$(\hat{p}_2 - m)_{\alpha\beta} = 2\varepsilon_{p_2}\sum_\nu v_\alpha^{\nu+}(\mathbf{p}_2)\,\bar{v}_\beta^{\nu-}(\mathbf{p}_2),$$

we obtain

$$\overline{\Delta\Phi(t)} = \int d^3p_2\sum_{\nu_2} M_0^{\nu_1\nu}\Big\{\delta_{\nu_2\nu}a_\nu^+(\mathbf{p}_2)|0\rangle - \frac{e}{(2\pi)^{3/2}}\sum_j\int d^3k\,\frac{\exp[i(t/T_{\mathbf{k}} - i\varepsilon t)] - 1}{i/T_{\mathbf{k}}}M' a_{\nu_2}^+ \times (\mathbf{p}_2 - \mathbf{k})\,\alpha_j^+(\mathbf{k})\,|0\rangle\Big\}\,\Delta_L(\varepsilon_{p_2} - \varepsilon_{q_1})\,d^3p_2, \quad (2.4)$$

$$M' = \sum_\mu \frac{e_\mu^j}{\sqrt{2k}}(\bar{v}^{\nu-}(\mathbf{p}_2)\gamma^\mu v^{\nu_2+}(\mathbf{p}_2 - \mathbf{k})). \quad (2.4a)$$

This expression has a simple meaning. Let us consider an electron ($\mathbf{p}_2$, ν_2) emitted at the instant $t = 0$ (this is equivalent to an electron having at the instant $t = 0$ an asymptotic state $\Phi_0 = a_{\nu_2}^+(p_2)|0\rangle$), and let us turn on instantaneously (non-adiabatically) its interaction with the field. Then for $t > 0$

$$\Phi_{p_2\nu_2}(t) \approx \Big(1 - i\int_0^t H^r(t')\,dt'\Big)a_{\nu_2}^+(\mathbf{p}_2)\,|0\rangle. \quad (2.5)$$

This expression (when averaged over the packet) coincides with the curly bracket in (2.4). Thus $\overline{\Delta\Phi(t)}$ is obtained by multiplying the amplitude of the Born scattering $(\mathbf{q}_1, \nu_1) \to (\mathbf{p}_2, \nu_2)$ by the functional of the state of the electron $(\mathbf{p}_2, \nu_2)$, calculated with allowance for the interaction with the field that is turned on at $t = 0$. The latter yields in (2.4) a nonvanishing integral (for a given $\mathbf{k}$) only if $t \gtrsim T_k$.

The functional $\Phi_{p_2\nu_2}(t)$ in (2.5), which enters in (2.4), can be broken up into two parts:

$$\overline{\Phi_{\mathbf{p}_2\nu_2}(t)} = \overline{\Phi^e_{\mathbf{p}_2\nu_2}(t)} + \overline{\Phi^f_{\mathbf{p}_2\nu_2}(t)}, \qquad (2.5a)$$

$$\overline{\Phi^e_{\mathbf{p}_2\nu_2}(t)} = a_{\nu_2}^+(\mathbf{p}_2)\,|\,0\rangle - \frac{ie}{(2\pi)^{3/2}}\int\sum_j d^3k\, \frac{\exp\,[it/T_{\mathbf{k}} - \varepsilon t]}{i/T_{\mathbf{k}}}$$

$$M' a_{\nu_2}^+(\mathbf{p}_2 - \mathbf{k})\, \alpha_j^+(\mathbf{k})\,|\,0\rangle\,, \qquad (2.6)$$

$$\overline{\Phi^f_{\mathbf{p}_2\nu_2}(t)} = \frac{e}{(2\pi)^{3/2}}\int\sum_j d^3k\, T_{\mathbf{k}} M' a_{\nu_2}^+(\mathbf{p}_2 - \mathbf{k})\, \alpha_j^+(\mathbf{k})\,|\,0\rangle. \quad (2.7)$$

The first part (2.6) describes a stationary free electron with its field and goes over as $t \to \infty$ into an asymptotic functional $a^+_{\nu_2}(\mathbf{p}_2)\,|0\rangle$, in which the self-field is taken into account in the renormalized mass. The second, (2.7), describes the really radiated field and the recoil electrons.[2)] A superposition of both parts for $t \ll T_k$ yields approximately

$$\overline{\Phi_{\mathbf{p}_2\nu_2}(t)} \approx \Phi_0 = a_{\nu_2}^+(\mathbf{p}_2)\,|\,0\rangle, \quad 0 < t \ll T_{\mathbf{k}} \qquad (2.8)$$

(more accurately, $\tau_L \ll t \ll T_k$). Within the limits of this time there are no photon states in the functional at all. Therefore under new interactions the system exhibits properties which, as we shall soon show, differ from the properties of an ordinary free electron.

Assume that in the direction of the vector $\mathbf{p}_2$ there is located a new scattering center at a distance l_0 from the first, and at the frequencies of interest to us

$$L \ll l_0 \ll T_{\mathbf{k}}. \qquad (2.9)$$

The result of the interaction with the second center can be obtained as follows

$$\Phi'(t) = T\exp\left[-i\int_{t_1}^{t} H_1(t')\,dt'\right]\overline{\Phi_{\mathbf{p}_2\nu_2}(t_1)}, \qquad (2.10)$$

where t_1 can be chosen to lie between zero (more accurately, between a quantity that greatly exceeds the dimension of the packet) and t. The calculations, of course, duplicate the preceding ones, the only difference being that the integration with respect to t begins not with $-\infty$ but with t_1, which by virtue of (2.1) can be assumed equal to zero. This yields in place of (1.18) for the second interaction, which deflects the electron and gives it a momentum $\mathbf{p}_3$, a single-term formula (the counterpart of the term proportional to R_1^- in (1.18) turns out to be reduced here in the ratio l_0/T_k and L/T_k):

$$M^{\mathbf{p}_3\nu_3\mathbf{k}'j'}_{\mathbf{p}_2\nu_2}(t) = -\frac{ie^2}{4\pi^2}\varphi_0(\mathbf{p}_3 - \mathbf{q}_2)$$

$$\times \frac{R_2^-\left(1 - \exp\,[-i(\varepsilon_{\mathbf{p}_3} - \varepsilon_{\mathbf{p}_3-\mathbf{k}'} - k' - i\varepsilon)\,t]\right)}{\varepsilon_{\mathbf{p}_3} - \varepsilon_{\mathbf{p}_3-\mathbf{k}'} - k'}$$

$$\times\ \Delta_L(\varepsilon_{\mathbf{q}_2} - \varepsilon_{\mathbf{p}_3-\mathbf{k}'} - k'), \quad 2L/v \ll t \ll T_{\mathbf{k}}, \qquad (2.11)$$

($\mathbf{q}_2$ is the momentum of the center of the packet that moves along $\mathbf{p}_2$).

Here, thus, there is no cone of emitted photons directed along the "initial" electron motion $\mathbf{p}_2$. There is only a cone around the new direction of motion of the rescattered electron $\mathbf{p}_3$, a cone obtained upon decay of the new equilibrium state (2.11) with momentum $\mathbf{p}_3$. This radiation appears only when

$$t \gtrsim T_{\mathbf{k}'} = -1/(\varepsilon_{\mathbf{p}_3} - \varepsilon_{\mathbf{p}_3-\mathbf{k}'} - k').$$

A concrete example can illustrate the foregoing result. Assume that an electron with momentum $\mathbf{p}_1$ and energy of the order of 10^{16} eV produces bremsstrahlung in a thin layer of matter and is scattered through a large angle. The photon is emitted in a direction close either to the direction of its initial momentum $\mathbf{p}_1$ or to the direction of its new momentum $\mathbf{p}_2$. The second case, however, may not be realized: if at a distance $\ll 1$ cm there is a new layer of matter then the electron can experience a new scattering and acquire a momentum $\mathbf{p}_3$. However, the new bremsstrahlung quantum emitted to this case has a very low probability of being emitted in a direction close to $\mathbf{p}_2$, the radiation being directed predominantly along $\mathbf{p}_3$. Accordingly, the cross section of the second act will be appreciably smaller than normal.

3. DISCUSSION OF RESULTS

We have calculated above essentially the S matrix for finite times t. In place of this we could have considered immediately scattering by two centers and calculated $S(+\infty, -\infty)$ (this would have yielded in principle a more accurate result). The replacement of this quantity by the product $S(+\infty, t)\,S(t, -\infty)$ (which is equivalent to the approach in Secs. 2 and 3) signifies discarding higher-order diagrams, in which, for example, the internal photon lines connect the incoming electron (in our case $\mathbf{q}_1$) with the outgoing one (in our case $\mathbf{p}_3$), etc., and discard in general the diagrams with additional lines joining $S(+\infty, t)$ with $S(t, -\infty)$. The

[2)]If we go over to the Schrödinger representation, then the time dependence is expressed by the single factor $\exp(-i\epsilon_{p_2}t)$ in (2.6) and by $\exp[-i(\epsilon_{p_2-k} + k)]$ under the integral sign.

admissibility of the employed approximation follows, first, from the smoothness of the interaction constant, because of which the higher approximations should influence the result little (although they do perhaps contain a divergence); second, from the fact that, as already verified, the succeeding interaction events can be separated by distances that are much larger than the dimensions of the packet and separated by times much longer than the travel time of the packet. Therefore, the time sequence of the interaction between the particle and each of the scattering centers can be objectively fixed. We are thus fully justified in ascribing a relative independence to the state of the electron-plus-field system described by the functional (2.4)–(2.8) during the time interval $L \ll t \ll T_k$. During this time we can subject the system to any desired interaction.

The second interaction act could have a character different from bremsstrahlung. Therefore the examination of the S matrix for a finite time t has considerable heuristic value. By confining ourselves to examination of the S matrix between infinite limits we lose much information on the quasiequilibrium systems, that is, those relatively autonomous, and existing for a long time, although generally speaking they are not in equilibrium. Thus if a fast proton enters into a layer of matter and we are confining ourselves to a study of only the equilibrium products of the interaction at $t \to +\infty$, then we register only the electrons, photons, and neutrinos, and forego knowledge of the very fact of the existence and the properties of the intermediate unstable formations such as the pion and other unstable particles.

The superposition of the equilibrium state of the electron with a packet of free photons has in general much in common with the unstable particle states, for example, with resonances. However, there is no preferred resonant value of the energy here. The spectrum of the photons emitted during the decay of the state is a continuous bremsstrahlung spectrum with a maximum at $k \to 0$. The spin, on the other hand, and the other characteristics are the same as for the electron, since they are specified by the initial state when $t = 0$, $\Phi(0) = a^+_{\nu_2}(\mathbf{p}_2)\,|0\rangle$. The quantity T_k can be regarded as a measure of the lifetime of the system.

4. PHYSICAL INTERPRETATION OF THE EFFECT

We know how intuitive is the interpretation of the formation of two bremsstrahlung cones. When the electron momentum changes abruptly from $\mathbf{p}_1$ to $\mathbf{p}_2$, the self-field carried by it must rearrange itself, part of the equilibrium field, corresponding to the motion with momentum $\mathbf{p}_1$, breaks away, propagates along the direction of $\mathbf{p}_1$, and gives the cone expressed by the first term in the curly bracket of (1.18). On the other hand, establishment of the field corresponding to the new momentum $\mathbf{p}_2$ occurs in exactly the same manner as if the initially-resting electron were to acquire abruptly this momentum (within a time short in comparison with the reciprocal frequency of the component of the field $\mathbf{k}$ of interest to us). This is the situation both in classical and in quantum electrodynamics.[2] Understandably, the establishment of an equilibrium field component with frequency ω_k at low electron velocities should occur within a time $\omega_k^{-1} = k^{-1}$. But for an electron with $\epsilon^p \gg m$ the time slows down, and the regeneration of the field occurs after a longer time than given by formulas (1.19a)–(1.19b).

Within the framework of nonrenormalized quantum electrodynamics (for example, with a cut-off form factor), the additional term in (2.5) clearly stands for the self-field of the electron, and the operators $a^+(\mathbf{p})$ pertain to the electron with nonrenormalized mass, $\epsilon^p = (\mathbf{p}^2 + m_0^2)^{1/2}$. Here (2.8) describes a "bare" or "undressed" electron, and (2.5) a "dressed" one. It can be stated that as a result of the first interaction the electron with momentum $\mathbf{p}_2$ is emitted in the nonequilibrium—bare—state. After a time $\sim T_k$ this state breaks up into an equilibrium "dressed" electron with functional Φ^e (2.6) (in this case, of course, without the factor $e^{-\epsilon t}$) or (2.5), and a packet of real photons with recoil electrons Φ^f (2.7).

For field frequencies $k \lesssim m$ under the assumption that $\delta m = m_0 - m \ll m_0$, the difference between the results of the two approaches—consecutively normalized and nonrenormalized—should be quantitatively insignificant. The formation of an equilibrium cloud around the undressed nonrelativistic electron in the nonrenormalized electrodynamics was considered long ago.[3] It was shown that the initially undressed electron not only produces around itself a nonequilibrium dragged field, but should furthermore radiate free photons, the energies of the two fields being equal.

In renormalized electrodynamics, however, neither this entire terminology nor the simple approach itself can be employed literally. One might think that only "renormalized" particles of mass m, in which the sought field of the particle is already taken into account, are involved in this case, and no components of this field apparently can be separated, investigated separately, etc. On the other hand, it is clear that the delay in the formation of the equilibrium field in the remote regions of space should in some manner be reflected in the formalism of the theory. The answer, in our opinion, reduces to the following.

Mass renormalization is carried out in the theory in an infinite time interval, and reflects an integral characteristic of the self-field of the particle—its energy. In particular, it describes completely the influence of the self-field on the passive behavior of the electron in external fields that change at infinitesimally slow speed. On the other hand, if the electron experiences acceleration, then the reconstruction of its self-field—especially in remote regions (that is, for small k), can be only gradual. In the formalism of quantum field theory this is ensured by additional adiabatic turning on (at $t \to -\infty$) and turning off (at $t \to +\infty$) of the interaction with the field. Because of this, the state of the electron at the final instant of time t is described by the functional $\Phi(t)$ (2.5), which differs from $\Phi(-\infty) = \Phi_0$ (2.8). The additional term contains entirely that part of the electromagnetic field, which is "thrown off" when the momentum is suddenly changed in any given concrete process.

In contrast, as seen from (2.4)–(2.8), during the initial time after the scattering, the electron is described by the functional $a_{\nu_2}(\mathbf{p}_2)\,|0\rangle$, that is, it does not contain additional terms that reflect the dynamic behavior of the self field. They have had no time to form and therefore the electron has nothing to "throw off" during the second scattering process (and this is why no corresponding cone is produced). During this time the self-field is taken into account only in that respect that the electron mass contains its energy. Accordingly, in an arbitrary and limited sense we can say that when $0 < t < T_k$ the electron is not uniformly "dressed" with respect to the given components $\mathbf{k}$ of the field, or more accurately speaking, the electromagnetic field is not yet in that equilibrium state which ensures during subsequent interactions the results that are customary for the free electron. The time that the electron stays in this nonequilibrium state at high energies is so long that it can be experimentally registered.

Thus, the effect under consideration has essentially a purely classical nature—retardation during establishment of the equilibrium field (especially in the remote regions). The additional stretching out of the process in time at high energies has a purely kinematic nature—the relativistic slowing down of time on going over from the electron's proper system to the laboratory system. Therefore the effect under consideration should be manifest also in other processes. Indeed, if we consider, for example, the simple Compton effect on an electron, then the recoil electron also is emitted in a nonequilibrium state $a^{+}_{\nu_2}(\mathbf{p}_2)\,|0\rangle$. Only after a time of the order of T_k does this state go over into the equilibrium state of (2.5) and a set of additionally radiated photons. (In fact, the calculation reduces to a calculation of the double Compton effect, that is, we are dealing with the next higher order ($\sim e^3$) of perturbation theory.) An analogous effect can easily be traced also in the model of meson-nucleon interaction with weak coupling.

We note that in the renormalized theory the result is similar to that obtained by Ginzburg:[3] during the decay of the nonequilibrium state (2.5) into an equilibrium electron Φ^e and a packet of free photons (with recoil electrons) Φ', the energy E^e contained in the additional term in Φ^e is equal to the energy E^f of the packet. Indeed, if H_0 is the energy operator of the noninteracting electron and photon fields, then, using (2.6) and (2.7), we have

$$E^f = (\Phi^f, H_0\Phi^f) = \frac{e^2}{(2\pi)^3}\int d^3k \sum_j (\varepsilon_{\mathbf{p}_2-\mathbf{k}} + k)\, T_{\mathbf{k}}^2\, |M'|^2 \quad (4.1)$$

$$= ((\Phi^e - a_{\nu_2}^{+}(\mathbf{p}_2)\,|0\rangle),\quad H_0\,(\Phi^e - a_{\nu_2}^{+}(\mathbf{p}_2)\,|0\rangle)) = E^e.$$

5. CONNECTION WITH OTHER PHENOMENA

It is easy to note that the effect under consideration will have a close bearing on inelastic diffraction processes and the transition radiation of relativistic particles (see, for example, [4]). Many electromagnetic phenomena occurring at high energies are based on the fact that the emission of a photon by an electron is a process that takes place on a long effective path

$$l_{eff} \sim |\mathbf{p} - \mathbf{p}' - \mathbf{k}|^{-1}, \quad (5.1)$$

where $\mathbf{k}$, $\mathbf{p}$, and $\mathbf{p}'$ are the momenta of the photon and the electron before and after radiation. If there are N scattering centers (a crystal) along the line of motion of the electron in this path, then we obtain coherent bremsstrahlung immediately from N centers.[5] If, to the contrary, strong multiple scattering takes place on the path l_{eff}, then the bremsstrahlung act will be suppressed,[6] etc.

At high energies l_{eff} practically coincides with T_k. From the point of view of deductions concerning the time of formation of an equilibrium shell, the forementioned diffractive inelastic processes can be understood in the following way. In order for the photon to be emitted, it must have time to build up in the electron state (2.6). To this end, the time required in the rest system of the electron is $\sim 1/k_0$. In the laboratory system, for a relativistic particle, it is larger by a factor ϵ_p/m. During that time the electron covers a path l_{eff}. There will be no radiation if the electron experiences along this path strong scattering that returns it to a nonequilibrium state $a_\nu^+(\mathbf{p})|0\rangle$, in which it lacks the quantum necessary for the radiation (the effect of Landau and Pomeranchuk[6]). To the contrary, if the electron experiences during that time the coherent scattering action of N centers, then the radiation becomes amplified[5] (similar reasoning is used in [7] for the diffraction splitting of a deuteron). In the analysis used in this article we deal with a different aspect of the question: the state in which the electron has a quantitatively reduced and a qualitatively distorted radiating ability is characterized by a certain autonomy and can be regarded to a considerable degree separately from the process of formation of this state.

This naturally raises the question of extending the results to strong interactions. We have assumed that the duration of the regeneration of the equilibrium state does not depend on the magnitude of the coupling constant. This is also explained by the fact that the greater this constant the faster the growth of the additional term in (2.6), the equilibrium level of the term itself. The two factors cancel each other. However, we cannot, of course, transfer this deduction literally to strong interactions. Nevertheless, we can present qualitative arguments to show that similar phenomena can occur here, too.

We can consider separately two stages of the interaction process, for example for fast nucleons. During the time of the nucleon collision a perturbation can occur in their meson cloud and can have the same order of magnitude as the time required for the given Fourier component of the meson cloud to traverse the relativistically compressed cloud. In the rest system, these dimensions are $1/k^0$, and in the c.m.s. of the colliding nucleons they are contracted by a factor ϵ_p/M, where $\epsilon = (\mathbf{p}^2 + M^2)^{1/2}$ is the energy of the nucleon of mass M and momentum $\mathbf{p}$. Thus, the time of formation of the nonequilibrium state for the component k^0 is

$$T_{k^0}^{form} \sim \frac{M}{\varepsilon_p}\frac{1}{k^0} \tag{5.2}$$

(in the electrodynamic case considered above, it coincided with the travel time of the packet τ_L (1.15)). The time of regeneration of the k^0-component of the equilibrium cloud in the nucleon rest system can be assumed to be of the order $\omega_{k^0}^{-1} = (\mathbf{k}^{0^2} + \mu^2)^{-1}$, where μ is the meson mass. In the c.m.s. of the colliding nucleons it is increased by a factor ϵ_p/M, that is

$$T_{k^0}^{regen} \equiv T_{k^0} \sim \frac{\varepsilon_p}{M}\frac{1}{(\mathbf{k}^{02} + \mu^2)^{1/2}}. \tag{5.3}$$

Thus, T_{k0}^{regen} can be large in absolute magnitude. Furthermore, it is $\sim(\epsilon_p/M)^2$ times larger than T_{k0}^{form} and the equilibrium state can again be regarded as relatively autonomous and long-lived.

If such considerations are valid, then they can find important applications in high-energy physics. Thus, for example, in connection with some still unconfirmed experiments with cosmic rays, a rather fantastic scheme has been proposed for nucleon-nucleus collisions. Namely, Zatsepin[8] has proposed that the nucleon incident on the nucleus can lose its meson cloud by collision with a surface nucleon of the nucleus, and has no time to recover it during the succeeding collisions inside the nucleus. As a result, the "undressed" nucleon can pass through the nucleus and experience fewer new collisions, or none at all, and lose no energy.

We see that such a concept can be even formulated theoretically in a consistent fashion. Zatsepin[8] justified the slowness of regeneration of the cloud by assuming a weak interaction between the nucleon and the meson. It was shown above that actually in the case of weak coupling the value of the interaction constant does not influence T_k^{regen}. However, although for strong coupling we cannot make any such statements with the same degree of assurance as for electrodynamics, the hypothesis outlined above gains a certain degree of support in our analysis. The stretching out of the time of regeneration of the equilibrium state has to a considerable degree a kinematic nature and should apparently take place for any particle that interacts with its field. It would be exceedingly interesting to investigate experimentally this possibility in nucleon physics at very high energies.

In conclusion I am sincerely grateful to my friends in the theoretical division of the Physics Institute for a very interesting and useful critical discussion of the results.

[1] N. N. Bogolyubov and D. V. Shirkov, Vvedenie v teoriyu kvantovannykh poleĭ (Introduction to the Theory of Quantized Fields), 1957.

[2] F. Bloch and A. Nordsieck, Phys. Rev. **52**, 54 (1937); A. Nordsieck, Phys. Rev. **52**, 59 (1937).

[3] V. L. Ginzburg, DAN SSSR **23**, 775 and 899 (1939) and **24**, 131 (1939).

[4] E. L. Feĭnberg and I. Pomeranchuk, Nuovo Cimento Suppl. **3**, 652 (1956).
[5] M. L. Ter-Mikaelyan, JETP **25**, 289 and 296 (1953).
[6] L. D. Landau and I. Ya. Pomeranchuk, DAN SSSR **92**, 535 and 735 (1953).
[7] E. L. Feĭnberg, JETP **29**, 115 (1955), Soviet Phys. JETP **2**, 58 (1956).
[8] G. T. Zatsepin, Izv AN SSSR ser. fiz. **26**, 674 (1962), transl. Bull. Acad. Sci. Phys. Ser. p. 673.

Translated by J. G. Adashko
28