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IONIZATION OF SYSTEMS BOUND BY SHORT-RANGE FORCES BY THE FIELD OF AN ELECTROMAGNETIC WAVE

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Probabilities were obtained for the splitting of neutral and charged systems, bound by short-range forces, by the field of a plane electromagnetic wave. The angular and energy distributions of the outgoing particles, distributions over the number of absorbed photons, and the dependence of the probability on the polarization of the electromagnetic wave and polarization of the bound system are examined. Conditions are given under which the internal structure of the system affects the dependence of splitting probability on the field intensity and frequency of the wave. The results obtained are applicable in particular to the description of multiquantum splitting of negative and molecular ions.

1. INTRODUCTION

A number of studies devoted to the ionization of atoms by the field of an electromagnetic wave have recently appeared[1,2]. From the theoretical standpoint, the process of ionization of atoms, i.e., systems of charged particles bound by long-range Coulomb forces, is very complex because of the necessity of precisely calculating the simultaneous effect of both the wave field and the Coulomb field on the electron. In addition, many common properties of the process of ionization by the field of an electromagnetic wave can be found by examining a simpler problem, the ionization of a system bound by short-range forces. In this case, when the range of the forces is sufficiently small, the effect of the wave field and that of the short-range forces can be considered separately. This substantially simplifies the problem and makes it possible to find both the ionization probability and the exact range of applicability of the results obtained.

The problem under consideration is applicable to many real physical systems. These include primarily negative ions, such as the hydrogen ion H^-, iodide ion I^-, etc., in which the Coulomb forces acting on the outermost electron are screened at distances greater than atomic and are manifested as short-range forces. Experiments on the splitting of such ions are already being carried out[3]. This pertains also to molecular ions (for example, $H_2^+ \to H + p$). If electromagnetic waves of very high intensity are considered, one can also include such strongly bound systems as the deuteron, etc. In addition, the solution of this problem permits one to complicate it further by introducing Coulomb forces and to find the changes produced in the ionization formulas by these forces. This is what we are proposing to do in the present study.

The contents of the work can be divided into two parts. In the first part (Secs. 2 and 3) we examine the ionization of the simplest system, the bound state of a spinless particle moving in a field of short-range forces. The total probability of ionization by the wave field depends on the field amplitude B and frequency ω through the dimensionless parameters[1)]

$$\xi = ea/\eta = eB/\eta\omega, \quad B/B_0,$$

where a is the potential amplitude, $\eta = \sqrt{m^2 - \epsilon_0^2}$, ϵ_0 is the energy of the particle, $B_0 = \eta^3/em$ characterizes the field intensity in the bound system. An important special case of the system under consideration is the limiting case of zero frequency $\omega \to 0$ or $\xi \to \infty$, when the process is reduced to the ionization by a constant crossed field. In this case, the ionization probability can be found by a quantum-mechanical method which permits one to draw a number of important physical conclusions about the process. In particular, comparison with the known calculations of Oppenheimer and Lanczos[5,6]

[1)]Let us note that parameter ξ, which determines the relative probability of absorption of several photons, differs from the corresponding parameter x in [4] in that m is replaced by η.

and Landau and Lifshitz (see[7], page 327) for the ionization of atoms by a constant electric field makes it possible to separate the effect of Coulomb forces in the ionization by a constant field. The result obtained for a constant crossed field is subsequently treated as a check on the calculation of the ionization by the wave field. We obtain the total ionization probabilities in the field of linearly and circularly polarized waves, and also the distributions of ionization probability over the angles of emission of the charged particles and over the number of photons absorbed from the field.

Conditions are examined under which the internal structure of the interaction is essential (effective radius of interaction r_0, potential depth U_0). It is shown that in the wave field at

$$\eta r_0\left(\frac{B\sqrt{1+\xi^2}}{B_0\xi}\right)^{1/2}\ll 1$$

the structure of the interaction is manifested in the form of a constant coefficient and affects the functional dependence of the ionization probability on the frequency and intensity of the field only when $B\sqrt{1+\xi^2}/\xi \gtrsim B_0/(\eta r_0)^2$.

The second part (Sec. 4) discusses a relativistically and gauge invariant model describing the splitting of a neutral or charged system into two particles of arbitrary masses. Conditions are given under which the splitting probability in a weak field substantially depends on the polarization of the initial system.

2. QUANTUM-MECHANICAL CALCULATION OF IONIZATION IN A CONSTANT CROSSED FIELD

We shall consider the problem of the "drawing out" of a charged particle by a constant crossed field from a spherical potential well formed by short-range forces. In the absence of an external field, the bound state of the particle is described by the wave function $\psi_i(x)$, [2] which outside the range of the forces has the universal form

$$\psi_i(x)=\frac{N}{r}\exp(-\eta r-i\varepsilon_0 t), \qquad (1)$$

where $\eta=\sqrt{m^2-\epsilon_0^2}$, and m and ϵ_0 are the mass and energy of the particle. The constant N depends on the structure of the potential. Thus, for a spherical potential well of radius r_0 and depth U_0 we have

$$N=[\eta/4\pi\varepsilon_0(1+\eta r_0)]^{1/2}e^{\eta r_0}\sin\left[(\varepsilon_0+U_0)^2-m^2\right]^{1/2}r_0.$$

The stationary solution of the Klein-Gordon equation for a charged particle in a crossed field with potential $A_\mu=-k_\mu(ax)$, $k^2=(ak)=0$, is

$$\psi_{Ep_2\gamma}(x)=\exp[-iEt+ip_2x_2+i(E-\gamma)x_3]\begin{cases}2iv(y)\\ w(y)\end{cases},$$

$$y=\left[\frac{m^2-E^2+p_2^2+(E-\gamma)^2}{2eB\gamma}-x_1\right](2eB\gamma)^{1/3}. \qquad (2)$$

In the coordinate system chosen here $a_0=0$, **a** is directed along axis 1, and **k** along axis 3. In this system, the electric field is directed along axis 1, and the magnetic field along axis 2. The non-invariant notation will henceforth always be made in this "special" coordinate system. In formula (2), E, p_2 and γ are the eigenvalues of the invariant operators $i\partial/\partial t$, $-i\partial/\partial x_2$ and $i\partial/\partial t+i\partial/\partial x_3$, which characterize the state of the particle, B is the absolute value of the field amplitude, and v(y) and w(y) are Airy functions defined by the integral (see[7,8])

$$\frac{i}{\sqrt{\pi}}\int_{C_{v,w}}dz\exp\left[i\left(yz+\frac{z^3}{3}\right)\right]=\begin{cases}2iv(y)\\ w(y),\end{cases}$$

$$v(y)|_{y\gg1}=\tfrac{1}{2}y^{-1/4}\exp(-2y^{3/2}/3),$$

$$w(y)|_{y\gg1}=y^{-1/4}\exp(2y^{3/2}/3)+\tfrac{1}{2}iy^{-1/4}\exp(-2y^{3/2}/3) \qquad (3)$$

in which the integration contours C_v and C_w for functions v and w are as shown in Fig. 1. For real values of y, function v(y) is real, and function w(y) is complex, so that only the second of the two solutions (2) has a flux different from zero in the direction of axis 1; it is equal to $j_1=2(2eB\gamma)^{1/3}$, since $ww'^*-w'w^*=2i$.

The wave function of the particle outside the well may be written in the form of a superposition of the solutions (2) with $E=\epsilon_0$:

$$\psi(x)=e^{-i\varepsilon_0 t}\int dp_2\,d\gamma\exp[ip_2x_2+i(\varepsilon_0-\gamma)x_3]$$

$$\times\begin{cases}2iv(y)f_-(p_2\gamma), & x_1\leqslant -r_0\\ w(y)f_+(p_2\gamma), & x_1\geqslant r_0\end{cases}. \qquad (4)$$

The probability of ionization per unit time W is equal to the total flux through the plane perpen-

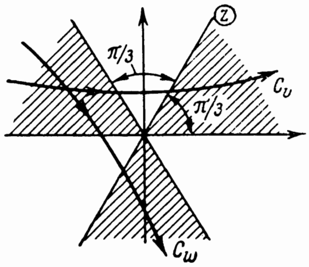

FIG. 1

[2]For simplicity, states with zero orbital angular momentum are considered.

dicular to axis 1, i.e.,

$$W = -i\int dx_2dx_3\left(\psi^+\frac{\partial\psi}{\partial x_1} - \frac{\partial\psi^+}{\partial x_1}\psi\right)_{x_1\geqslant r_0}$$

$$= 8\pi^2\int_{-\infty}^{\infty}dp_2\int_0^{\infty}d\gamma\,(2eB\gamma)^{1/3}|f_+(p_2\gamma)|^2. \quad (5)$$

Thus, it is necessary to find $f_+(p_2\gamma)$.

From (4) we find by means of a Fourier transform

$$f_+(p_2\gamma) = \frac{1}{(2\pi)^2 w(y)}\int dx_2dx_3\,\psi(x)$$
$$\times\exp[i\varepsilon_0 t - ip_2x_2 - i(\varepsilon_0 - \gamma)x_3],$$
$$x_1 \geqslant r_0. \quad (6)$$

The characteristic intensity of the field in the problem under consideration is $B_0 = \eta^3/em$. If the field is weak ($B/B_0 \ll 1$), wave function $\psi(x)$ differs little from wave function $\psi_i(x)$ of the bound state at distances not much in excess of $1/\eta$, namely, as shown by calculation, when $|x_1| \ll \sqrt{B_0/B}/\eta$. Therefore in (6) we can set

$$\psi(x) \approx (N/r)\exp(-\eta r - i\varepsilon_0 t)$$

and choose $x_1 = r_0$, as a result of which

$$f_+(p_2\gamma) = \frac{N\exp(-x_1\eta\sqrt{\zeta})}{2\pi\eta\sqrt{\zeta}\,w(y)}\Bigg|_{x_1=r_0}$$

$$= \frac{N}{2\pi\eta\zeta^{1/4}}\left(\frac{\eta^3}{2eB\gamma}\right)^{1/6}\exp\left(-\frac{\eta^3\zeta^{3/2}}{3eB\gamma}\right). \quad (7)$$

Here and further on $\zeta = 1 + [p_2^2 + (\gamma - \epsilon_0)^2]/\eta^2$. Substituting this function into (5) and integrating by the method of steepest descent, which utilizes the smallness of B/B_0, we obtain

$$W = 2\pi N^2\eta\frac{B\gamma_0}{B_0 m[\zeta_0(2\zeta_0 - 1)]^{1/2}}\exp\left(-\frac{2B_0 m\zeta_0^{3/2}}{3B\gamma_0}\right), \quad (8)$$

where $\gamma_0 = [\epsilon_0 + (9\epsilon_0^2 + 8\eta^2)^{1/2}]/4$ and $\zeta_0 = 1 + (\gamma_0 - \epsilon_0)^2/\eta^2$ are the values of γ and ζ at the saddle point. We shall recall that (7) and (8) are valid for $B/B_0 \ll 1$, $(\eta r_0)^2 B/B_0 \ll 1$. In the nonrelativistic case $\eta/m \to 0$, so that $\gamma_0 \to m$, $(\epsilon_0 - \gamma_0)/\eta \to 0$, and

$$W_{\text{nonrel}} = 2\pi N^2\eta\frac{B}{B_0}e^{-2B_0/3B}. \quad (8')$$

A characteristic feature of formulas (8) and (8′) is that the preexponential factor is proportional to the field. Let us note that in the ionization probability of the ground state of the hydrogen atom the preexponential factor is inversely proportional to the field[7]. Thus, the introduction of Coulomb forces increases the probability of ionization by the factor $\sim(B_0/B)^2$, which is associated with an increased probability that the particle will remain at great distances in the Coulomb field. Indeed, the wave function of a particle moving in the field of short-range and Coulomb forces at great distances is

$$\psi_i \sim r^{-1+\alpha m/\eta}e^{-\eta r},$$

where $\alpha = e^2/4\pi = 1/137$. On the other hand, the ionization probability is determined by the total current in the direction of the electric field at barrier distances $r_b \sim I/eB \sim B_0/B\eta$ ($I = m - \epsilon_0$ is the ionization energy). Since the total current $\sim|\psi_i|^2 vS$, and the effective velocity v and area S at the barrier distances are $v \sim \eta/m$ and $S \sim B_0/B\eta^2$, then

$$W \sim (B/B_0)^{1-2\alpha m/\eta}\exp(-CB_0/B).$$

In the absence of Coulomb forces, $\alpha = 0$ and (8) is qualitatively reproduced. In a pure Coulomb field $\alpha m/\eta = n$, where n is the principal quantum number; case n = 1 corresponds to the result of Landau and Lifshitz[7].

These considerations pertain to states with an orbital angular momentum equal to zero. If $l \neq 0$, then in the nonrelativistic case in the expression for the ionization probability in a weak field there appears an additional factor $\sim(B/B_0)^{|m_l|}$, where m_l is the projection of the orbital momentum on the direction of the electric field.

3. IONIZATION BY THE WAVE FIELD

Ionization by the wave field is calculated most simply and conveniently by the diagram method. Generally, in the final state it is necessary to consider the simultaneous action of the external wave field and of the short-range forces. However, it is physically clear that allowances for the short-range potential will change the wave function of the final state at distances $\sim r_0$, and if distances $r \gg r_0$ are effective in the problem, then the effect of these forces on the final state can be neglected. The ionization process can then be described by the diagram of Fig. 2, where the line with crosses represents the bound state, and the line with dashes represents the state in the wave field. This diagram corresponds to the matrix element

$$M_0 = \int d^4x\,\psi_p^+(x)(\Box - m^2)\psi_i(x), \quad (9)$$

where $\psi_i(x)$ is the wave function of the bound state in the absence of an external field, and $\psi_p(x)$ is the wave function of the particle in the wave field with potential

$$A_\mu = A_\mu(\varphi), \quad \varphi = (kx), \quad k^2 = (kA) = 0.$$

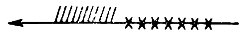

FIG. 2

The quantum numbers p_μ form a 4-vector with $p^2 = -m^2$. In the "special" coordinate system, where $k_\mu = (0, 0, \omega, i\omega)$, the components p_1, p_2, $p_0 - p_3 \equiv \gamma$ are the eigenvalues of the operators $-i\partial/\partial x_1$, $-i\partial/\partial x_2$, and $i\partial/\partial t + i\partial/\partial x_3$.

Matrix element (9) is not gauge-invariant, since it does not allow for the contribution of the diagrams obtained from the diagram of Fig. 2 by a permutation of the vertices (i.e., crosses and dashes). However, if large distances are effective in the problem ($r \gg r_0$), then a suitable choice of the gauge of the wave potential can make the contribution of these diagrams sufficiently small, and thus the use of the non-gauge-invariant matrix element M_0 will be justified.

To elucidate these problems, we shall use the diagram method to examine the ionization by a constant crossed field.

A. Constant crossed field. For a potential $A_\mu = a_\mu(kx)$, $k^2 = (ak) = 0$, the wave function is

$$\psi_p(x) = \frac{1}{\sqrt{2p_0}} \exp\left[-i\left(\frac{\alpha(kx)^2}{2} - \frac{4\beta(kx)^3}{3}\right) + i(px)\right],$$

$$\alpha = -e\frac{(ap)}{(kp)}, \qquad \beta = -\frac{e^2a^2}{8(kp)}. \qquad (10)$$

Substituting (10) into (9) and performing the calculations, we obtain

$$M_0 = -\frac{2\sqrt{\pi}}{\sqrt{2p_0}k_0(4\beta)^{1/3}} \exp\left[-is\frac{\alpha}{8\beta} + i\frac{8\beta}{3}\left(\frac{\alpha}{8\beta}\right)^3\right]$$

$$\times v(y)(\mathbf{Q}^2 + \eta^2)\phi(\mathbf{Q}), \qquad (11)$$

where $v(y)$ is the Airy function (3) with argument

$$y = (4\beta)^{2/3}\left[\frac{s}{4\beta} - \left(\frac{\alpha}{8\beta}\right)^2\right]$$

$$= \left(\frac{\eta^3}{2eB\gamma}\right)^{2/3}\left[1 + \frac{p_2^2 + (\gamma - \varepsilon_0)^2}{\eta^2}\right]; \qquad (12)$$

$\phi(\mathbf{Q})$ is the Fourier transform of function $\psi_i(x)$:

$$\phi(\mathbf{Q}) = \int d^3x \exp(-i\mathbf{Q}\mathbf{x} + i\varepsilon_0 t)\psi_i(x) \qquad (13)$$

and

$$\mathbf{Q} = \mathbf{p} - s\mathbf{k}, \qquad \varepsilon_0 = p_0 - sk_0,$$

$$\mathbf{Q}^2 = p_1^2 + p_2^2 + (\gamma - \varepsilon_0)^2.$$

If $(\mathbf{Q}^2 + \eta^2)\phi(\mathbf{Q})$ is not a constant, then $|M_0|^2$ depends on p_1, as it should not from general physical considerations. For this reason, the matrix element (11) needs to be supplemented with other diagrams in this case. If, however,

$$(\mathbf{Q}^2 + \eta^2)\phi(\mathbf{Q}) = \text{const} = 4\pi N_0,$$

then the Fourier transform $\phi(\mathbf{Q})$ and the corresponding coordinate function

$$\psi_0(x) = (N_0/r)\exp(-\eta r - i\varepsilon_0 t)$$

describe the bound state with zero interaction radius, and $N_0 = \sqrt{\eta/4\pi\epsilon_0}$. In this case (9) and (11) do indeed yield a gauge-invariant result valid for a field of arbitrary intensity:

$$W = \int \frac{|M_0|^2}{T}\frac{d^3p}{(2\pi)^3} = \frac{8N_0^2}{\eta}\int_{-\infty}^{\infty} dp_2 \int_0^{\infty} d\gamma \left(\frac{\eta^3}{2eB\gamma}\right)^{1/3} v^2(y). \qquad (14)$$

It is essential here that $|M_0|^2$ is independent of p_1; the resultant infinite integral with respect to p_1 is equal to eBT, so that the ionization probability per unit time is finite.

If the crossed field is described by another potential $A'_\mu = -k_\mu(ax)$, related to the preceding potential by the transformation

$$A_\mu' = A_\mu + \partial f/\partial x_\mu, \qquad f = -(ax)(kx),$$

then the wave function of the particle in the crossed field will differ from (10) by an additional phase factor e^{ief}, and the matrix element corresponding to the diagram of Fig. 2 will be

$$M_0' = -\frac{1}{(2p_0)^{1/2}k_0(4\beta)^{1/3}} \exp\left[-is\frac{\alpha}{8\beta} + i\frac{8\beta}{3}\left(\frac{\alpha}{8\beta}\right)^3\right]$$

$$\times \int_{-\infty}^{\infty} dz \exp\left[i\left(yz + \frac{z^3}{3}\right)\right](\mathbf{Q}^2 + \eta^2)\phi(\mathbf{Q}), \qquad (11')$$

and in contrast to (11),

$$\mathbf{Q} = \mathbf{p} - s\mathbf{k} - e\mathbf{a}\varphi, \qquad \varphi = \alpha/8\beta - z/(4\beta)^{1/3},$$

$$\mathbf{Q}^2 = \eta^2(2eB\gamma/\eta^3)^{2/3}z^2 + p_2^2 + (\gamma - \varepsilon_0)^2.$$

The remaining quantities are the same as in (11). The square of the matrix element $|M_0'|^2$ is independent of p_1 and for a zero effective radius of the forces, when

$$(\mathbf{Q}^2 + \eta^2)\phi(\mathbf{Q}) = 4\pi N_0,$$

M_0' coincides with M_0.

For $r_0 \neq 0$, the function $(\mathbf{Q}^2 + \eta^2)\phi(\mathbf{Q})$ in the complex plane $Q \equiv \sqrt{\mathbf{Q}^2}$ is smooth in a vicinity, $\Delta Q = Q - i\eta \ll r_0^{-1}$ at the pole $Q = i\eta$ of the function $\phi(\mathbf{Q})$. If the field is weak, then the integral (11′) can be calculated by the method of steepest descent, and the saddle point $z = i\sqrt{y}$ coincides with the pole $Q = i\eta$ of the function $\phi(\mathbf{Q})$. By the same token, M_0' (in contrast to M_0) correctly reflects the physical situation whereby in a weak field large distances are effective to which the vicinity of the pole of the wave function $\phi(\mathbf{Q})$ is known to correspond in the Q plane. In the integral with respect to z, the interval $\Delta z = z - i\sqrt{y} \sim y^{-1/4}$ is effective, i.e.,

$$\Delta Q = Q - i\eta \sim \eta(2eB\gamma\sqrt{\zeta}/\eta^3)^{1/2} \sim \eta(B/B_0)^{1/2}.$$

If $\eta\sqrt{B/B_0} \ll r_0^{-1}$, the function $(\mathbf{Q}^2+\eta^2)\phi(\mathbf{Q})$ changes little in this interval ΔQ and can be replaced by

$$(\mathbf{Q}^2+\eta^2)\phi(\mathbf{Q})|_{Q=i\eta} = 4\pi N,$$

where N is the coefficient in the asymptotic form of (1). As a result, the matrix element (11′) yields expression (8) for the ionization probability with the same conditions of applicability

$$B/B_0 \ll 1, \qquad \eta r_0\sqrt{B/B_0} \ll 1.$$

The entire dependence on the structure of the potential is contained in coefficient N.

Let us note that the nonstationary solution $\psi_p(x)$ (or $\psi_p'(x)$) used here for the Klein-Gordon equation is connected with the stationary solution (2) by the relation

$$\psi_{Ep_2\gamma}(x) = \int \psi_p'(x) f(p_1, E)\, dp_1$$

$$= \exp[i(p_2x_2 + (E-\gamma)x_3 - Et)]$$

$$\times \frac{i}{\sqrt{\pi}} \int_{C_{v,w}} dz \exp\left[i\left(yz + \frac{z^3}{3}\right)\right] \qquad (15)$$

with the transformation function

$$f(p_1, E) = \frac{i(2p_0)^{1/2}}{\sqrt{\pi}(2eB\gamma)^{1/3}} \exp\left[-i\frac{(p_0-E)p_1}{eB} + \frac{ip_1^3}{3eB\gamma}\right],$$

$$z = [eB(x_3 - t) - p_1](2eB\gamma)^{-1/3}.$$

A characteristic feature of this relation is that in order to obtain $\psi_{Ep_2\gamma}$ with a nonzero flux along the electric field, complex-plane integration with respect to p_1 or z must be performed (see C_W, Fig. 1). Let us also note that the matrix element M_0' (11′) corresponds to the function $f_+(p_2\gamma)$ in (4), whereas the function $f_-(p_2\gamma)$ corresponds to the same matrix element (11′) but with integration with respect to z not along C_V, but along C_W.

B. Linearly polarized waves. The potential is $A_\mu = a_\mu \cos(kx)$ or $A'_\mu = k_\mu(ax)\sin(kx)$. To calculate the ionization in the wave field we shall use the same matrix element (9) in which, however, the wave function $\psi_p(x)$ in the wave field is equal to

$$\psi_p(x) = (2q_0)^{-1/2}\exp\{-i[\alpha\sin(kx) - \beta\sin 2(kx) - (qx)]\},$$

$$\psi_p'(x) = \psi_p(x)\exp[-ie(ax)\cos(kx)], \qquad (16)$$

where $q_\mu = p_\mu - k_\mu e^2a^2/4(kp)$ is the average momentum of the particle in the field; α and β are the same as in (10). For the potential A'_μ

$$M_0' = -\frac{1}{\sqrt{2q_0}}\sum_s \delta(sk_0 + \varepsilon_0 - q_0)$$

$$\times\int_{-\pi}^{\pi} d\varphi \exp[i(\alpha\sin\varphi - \beta\sin 2\varphi + s\varphi)](\mathbf{Q}^2+\eta^2)\phi(Q), \qquad (17)$$

where

$$\mathbf{Q} = \mathbf{q} - s\mathbf{k} - e\mathbf{a}\cos\varphi,$$

$$\mathbf{Q}^2 = (q_1 - ea\cos\varphi)^2 + q_2^2 + (\gamma - \varepsilon_0)^2.$$

For the potential A_μ the matrix element M_0 is given by the same expression (17), but with $\mathbf{Q} = \mathbf{q} - s\mathbf{k}$.

For a zero interaction radius $(\mathbf{Q}^2+\eta^2)\phi(\mathbf{Q}) = 4\pi N_0$ and both gauges yield the same expression for the ionization probability per unit time:

$$W = 2N_0^2\sum_{s>s_0} q\int d\Omega\, A_0^2(s\alpha\beta),$$

$$A_0(s\alpha\beta) = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\varphi \exp[i(\alpha\sin\varphi - \beta\sin 2\varphi - s\varphi)], \qquad (18)$$

which is valid for a wave with an arbitrary intensity and frequency. The summation is performed over integral s values greater than $s_0 = I_*/\omega$, $I_* = m_* - \epsilon_0$, $m_* = (m^2 + e^2a^2/2)^{1/2}$, and the integration is performed along the directions of the vector $\mathbf{q}$. The integration in (18) can be performed only in special cases.

For $r_0 \neq 0$, the matrix element M_0' has a greater range of applicability than M_0. In particular, when ηr_0, $Qr_0 \ll 1$, it leads to formula (18), and for sufficiently large s, to the same expression, but with the replacement of N_0 by N.

Indeed, if $s \gg 1$, the integral in (17) can be calculated by the method of steepest descent, and the saddle points

$$\cos\varphi_{1,2} = \alpha/8\beta \pm [(\alpha/8\beta)^2 + 1/2 - s/4\beta]^{1/2}$$

$$= (q_1 \pm i\eta\sqrt{\zeta})/ea$$

coincide with the pole $Q = i\eta$ of the function $\phi(\mathbf{Q})$. This means that large distances are essential. When the inequality $\eta r_0(2eB\gamma\sqrt{\zeta}|\sin\varphi_{1,2}|/\eta^3)^{1/2} \ll 1$ is fulfilled (for the effective values of q_2, γ and s the left-hand side of the inequality is $\sim\eta r_0(2B\sqrt{1+\xi}/B_0\xi)^{1/2}$), the function $(\mathbf{Q}^2+\eta^2)\phi(\mathbf{Q})$ changes little in the effective region of integration and can be replaced by its value at the saddle point, which is equal to $4\pi N$. Then the integral in (17) becomes equal to $4\pi N \cdot 2\pi A_0$, where

$$A_0 = \mathrm{Re}\frac{2}{\sqrt{\pi}}\left(\frac{2}{|f'''|}\right)^{1/3}\exp\left(f + \frac{f''^3}{3f'''^2} + i\eta'\right)v(y),$$

$$y = i\frac{f''^2}{2f'''}\left(\frac{2}{|f'''|}\right)^{1/3} e^{i\eta'}, \qquad (19)$$

and the ionization probability is given by formula (18) with the replacement of N_0 by N. In (19), f, f″ and f‴ are the values of function $f(\varphi) = i(\alpha \sin \varphi - \beta \sin 2\varphi - s\varphi)$ and its derivatives at the saddle point. These values were found in[4] (formula (B.10); see also the meaning of phase η' in the same paper) and are expressed via the variables ψ and ϵ, which are related to α, β, and s by

$$\left.\begin{matrix}2\cos\psi\\ 2\,\mathrm{ch}\,\varepsilon\end{matrix}\right\} = \left(\frac{1}{2}+\frac{s}{4\beta}+\frac{\alpha}{4\beta}\right)^{1/2} \mp \left(\frac{1}{2}+\frac{s}{4\beta}-\frac{\alpha}{4\beta}\right)^{1/2}. \qquad (20)^*$$

In our case

$$s/4\beta = 2Bs(q_0 - q\sin\theta'\cos\varphi')/B_0\xi^3 m,$$

$$\alpha/4\beta = 2q\cos\theta'/\eta\xi,$$

where θ' is the angle between **q** and **a**, and φ' is the angle between the planes (**a**, **q**) and (**a**, **k**).

We shall assume further that $|y| \gg 1$ (for $s \gg 1$, $y \gg 1$ if $\xi \lesssim 1$; if however $\xi \gg 1$, then $|y| \gg 1$ for a weak field $B/B_0 \ll 1$). Then

$$A_0(s\alpha\beta) = \left(\frac{2}{\pi|f''|}\right)^{1/2} e^{\mathrm{Re} f} \sin\left(\mathrm{Im} f - \frac{1}{2}\arg f''\right), \qquad (19')$$

and the ionization probability is

$$W = \frac{2N^2}{\pi}\sum_{s>s_0} q\int d\Omega \frac{e^{2\mathrm{Re} f}}{|f''|}[1-\cos(2\,\mathrm{Im} f - \arg f'')]. \qquad (21)$$

Integration in (21) along the directions of vector **q** remains difficult. We shall assume therefore that $\eta/m \ll 1$ (nonrelativistic case). Then $s/4\beta \to 2Bs/B_0\xi^3$ and the dependence in (21) on the angle φ' vanishes. In the sum (21) the significant values of s are those for which

$$s - s_0 \lesssim (1+\xi^2)^{3/2}, \qquad (s-s_0)/s_0 \ll 1.$$

Expanding f and f″ in the effectively small parameter $\delta \equiv [(s - s_0)/2s_0]^{1/2}$, we obtain

$$2\,\mathrm{Re} f = 2s\left[\frac{t}{1+t^2} - \mathrm{Arth}\, t + 2\delta^2 t\left(\frac{t^2}{1+t^2} - \sin^2\theta'\right) + \ldots\right],$$

$$2\,\mathrm{Im} f = 2s\left[\frac{\pi}{2} - \frac{4\delta\cos\theta'}{\sqrt{1+t^2}} + \ldots\right],$$

$$|f''| = \frac{4st}{1+t^2} + \ldots, \qquad \arg f'' = \pi + \delta t\sqrt{1+t^2}\cos\theta' + \ldots, \qquad (22)^\dagger$$

where $t = \tanh\epsilon|_{s=s_0} = 1/\sqrt{1+\xi^2}$, and the dots stand for terms $\sim\delta^2$ smaller than the smallest of those which are written out. In the expansion of 2 Re f in the effective region of s and θ' we can confine ourselves to the terms which have been written out, since the remaining ones are $\sim s\delta^4 t^3 \sim \delta^2 \ll 1$. In 2 Im f this can be done only if $s\delta^3 \ll 1$, which is not fulfilled for $\xi^3 \gtrsim \sqrt{B_0/B} \gg 1$. For this reason, we shall keep 2 Im f unexpanded. Then

$$W = N^2\eta\frac{(2+\xi^2)^{3/2}}{(1+\xi^2)^{1/4} s_0^{3/2}}\sum_{s>s_0}\exp\left[-2s\left(\mathrm{Arsh}\frac{1}{\xi} - \frac{\sqrt{1+\xi^2}}{2+\xi^2}\right) + \frac{2(s-s_0)}{\sqrt{1+\xi^2}(2+\xi^2)}\right] F_s(\xi, s_0),$$

$$F_s(\xi, s_0) = e^{-z^2}\int_0^z dz' e^{z'^2}[1+\cos 2\,\mathrm{Im} f]. \qquad (23)^*$$

Here

$$z = [2(s-s_0)/\sqrt{1+\xi^2}]^{1/2} \quad z' = z\cos\theta',$$

$$s_0 = B_0\xi(1+\xi^2/2)/2B = I(1+\xi^2/2)/\omega.$$

The oscillating term cos 2 Im f is essential for the differential distribution relative to the angle θ'. Its contribution to $F_s(\xi, s_0)$ is negligibly small, with the exception of s, which is very close to the threshold: $s\delta \lesssim 1$, when its inclusion leads to the correct threshold singularities $(s - s_0)^{1/2}$ and $(s - s_0)^{3/2}$ for even and odd s. The differential distribution with respect to cos θ' oscillates rapidly with a frequency $\approx 8s\delta/\sqrt{1+t^2}$ about the mean value $(2N^2 q/\pi|f''|)e^{2\,\mathrm{Re} f}$, which decreases smoothly and exponentially from the points $\cos\theta' = \pm 1$ and is significant in the interval $\Delta\cos\theta' \sim 1/8s\delta^2 t \approx \sqrt{1+\xi^2}/4(s-s_0)$. Formula (23) is applicable when

$$s_0 \gg 1, \quad \eta r_0((2+\xi^2)\sqrt{1+\xi^2}/s_0)^{1/2} \ll 1,$$

$$(s-s_0)^2/(1+\xi^2)^{3/2}s_0 \ll 1.$$

If $\xi \gg 1$, the main contribution to (23) is made by the terms with $s - s_0 \sim \xi^3$. In this case, the summation over s can be replaced by an integration. Then

$$W = \int_{s_0} ds W_s = 2\sqrt{3\pi}N^2\eta\left(\frac{B}{B_0}\right)^{3/2} \times \exp\left[-\frac{2B_0}{3B}\left(1 - \frac{1}{10\xi^2}\right)\right]. \qquad (23')$$

The probability distribution with respect to s for $s - s_0 \sim \xi^3$ is very simple:

$$W_s = W\left(\frac{2}{3\pi\xi^3}\right)^{1/2}\frac{\exp[-2(s-s_0)/3\xi^3]}{(s-s_0)^{1/2}}. \qquad (23'')$$

It has its greatest value near the threshold and decreases exponentially with increasing s, the distri-

*ch ≡ cosh.

†Arth ≡ $\tanh^{-1}$.

*Arsh ≡ $\sinh^{-1}$.

bution width being given by $(s - s_0)_{eff} \sim \xi^3$ or $(s - s_0)_{eff}/s_0 \sim B/B_0 \ll 1$. Thus, in units of s_0, the distribution over s is "compressed" against the threshold. Let us note that near the threshold, when $s - s_0 \sim 1 \ll \xi^3$, and at very large s, when $s - s_0 \gg \xi^3$, formula (23″) does not apply and it is necessary to use the more accurate formulas (23) or (21).

The probability (23′) at $\xi = \infty$ is related to the probability (8′) in the constant field by a phase averaging, i.e., if $B \sin\psi$ instead of B is introduced into (8′) and averaging over ψ is performed, we obtain (23′) for $\xi = \infty$.

C. Circularly polarized wave. The potential is $A_\mu = a_{1\mu}\cos(kx) + a_{2\mu}\sin(kx)$ or $A'_\mu = k_\mu(a_1x) \times \sin(kx) - k_\mu(a_2x)\cos(kx)$, and $k^2 = (a_1a_2) = (a_1k) = (a_2k) = 0$, $a_1^2 = a_2^2 = a^2$; in the "special" system $a_{1\mu} = (a, 0, 0, 0)$, $a_{2\mu} = (0, a, 0, 0)$, $k_\mu = (0, 0, \omega, i\omega)$. For the second gauge we have

$$M_0' = -\frac{1}{\sqrt{2q_0}}\sum_s \delta(sk_0 + \varepsilon_0 - q_0) \times \int_{-\pi}^{\pi} d\varphi \exp[i(\alpha_1\sin\varphi - \alpha_2\cos\varphi - s\varphi)](Q^2 + \eta^2)\phi(Q), \tag{24}$$

$$q_\mu = p_\mu - k_\mu e^2a^2/2(kp),$$

$$\mathbf{Q} = \mathbf{q} - s\mathbf{k} - e\mathbf{a}_1\cos\varphi - e\mathbf{a}_2\sin\varphi,$$

$$Q^2 = (q_1 - ea\cos\varphi)^2 + (q_2 - ea\sin\varphi)^2 + (\gamma - \varepsilon_0)^2.$$

The matrix element M_0 has for the first gauge the same form, but $\mathbf{Q} = \mathbf{q} - s\mathbf{k}$.

For a zero interaction radius we have

$$W = 4\pi N_0^2 \sum_{s>s_0} q \int_0^\pi d\theta \sin\theta J_s^2(z), \quad z = \frac{ea}{\omega}\frac{q\sin\theta}{q_0 - q\cos\theta}. \tag{25}$$

Here θ is the angle between $\mathbf{q}$ and $\mathbf{k}$, and $q_0 = s\omega + \epsilon_0$.

For $r_0 \neq 0$, formula (25) remains valid when ηr_0 and $Qr_0 \ll 1$. If, however, $s \gg 1$ and $r_0\omega\gamma\sqrt{s}\tanh\alpha/\eta \ll 1$ (for effective values of θ, s the left-hand side $\sim r_0\eta(B\sqrt{1+\xi^2}/B_0\xi)^{1/2}$), this formula can be used by substituting N for N_0 and using instead of the Bessel function the following asymptotic expression:

$$J_s(z) = \frac{1}{\sqrt{\pi}}\left(\frac{2}{s}\right)^{1/2}\exp\left[-s\left(\alpha - \text{th}\,\alpha - \frac{1}{3}\text{th}^3\alpha\right)\right]v(y),$$
$$\text{ch}\,\alpha = s/z, \quad y = (s/2)^{2/3}\text{th}^2\alpha. \tag{26*}$$

If $y \gg 1$ (for $s \gg 1$ the condition $y \gg 1$ is fulfilled when $\xi \lesssim 1$, and when $\xi \gg 1$ it will be fulfilled for a weak field $B/B_0 \ll 1$), formula (26) becomes simplified:

$$J_s(z) = \frac{\exp[-s(\alpha - \text{th}\,\alpha)]}{(2\pi s\,\text{th}\,\alpha)^{1/2}}. \tag{27}$$

Assuming the conditions $s \gg 1$ and $y \gg 1$ to be fulfilled and using relation (27), we can perform the integration in (25) by the method of steepest descent; the saddle point is determined by the condition $\cos\theta = q/q_0$ and then

$$W = 2\sqrt{\pi}N^2\sum_{s>s_0} q\frac{\exp[-2s(\alpha - \text{th}\,\alpha)]}{(q_0/m_*)^2(s\,\text{th}\,\alpha)^{3/2}};$$
$$\text{ch}\,\alpha = s/z = s\omega m_*/eaq, \quad q_0 = s\omega + \varepsilon_0. \tag{28}$$

In the nonrelativistic case

$$\text{ch}\,\alpha = \frac{s}{2\xi}\left[\frac{1+\xi^2}{s_0(s-s_0)}\right]^{1/2}, \quad q = \eta\left[\frac{(s-s_0)(1+\xi^2)}{s_0}\right]^{1/2},$$
$$s_0 = \frac{I(1+\xi^2)}{\omega}.$$

The probabilities W_s have characteristic threshold singularities $(s - s_0)^{s+1/2}$.

If $\xi \gg 1$, the summation in (28) can be replaced by integration, and the method of steepest descent can be employed. The saddle point $s = s_c$ is found from the conditions

$$\alpha/\text{th}\,\alpha = s_c\omega q_0/q^2, \quad \text{ch}\,\alpha = s_c\omega m_*/eaq.$$

As a result,

$$W = 2\pi N^2\eta\frac{\xi\exp[-{}^2/_3 s_c\,\text{th}^3\alpha(1 + {}^3/_5\text{th}^2\alpha)]}{s_c\,\text{th}^2\alpha(1 + q_0^2\,\text{th}^2\alpha/9m_*^2)^{1/2}}$$
$$= 2\pi N^2\eta\frac{B\gamma_0}{B_0 m[\zeta_0(2\zeta_0 - 1)]^{1/2}} \times \exp\left[-\frac{2B_0 m\zeta_0^{3/2}}{3B\gamma_0}\left(1 - \frac{\zeta_0}{15\xi^2}\right)\right]; \tag{29}$$

$$\gamma_0 = [\varepsilon_0 + (9\varepsilon_0^2 + 8\eta^2)^{1/2}]/4, \quad \zeta_0 = 1 + (\gamma_0 - \varepsilon_0)^2/\eta^2.$$

In the nonrelativistic case (cf. formula (23′)),

$$W_{\text{nonrel}} = 2\pi N^2\eta\frac{B}{B_0}\exp\left[-\frac{2B_0}{3B}\left(1 - \frac{1}{15\xi^2}\right)\right]. \tag{29'}$$

The probability distribution over s has for $s - s_c \sim \sqrt{s_c\xi}$ a Gaussian character

$$W_s = \frac{W}{\sqrt{\pi s_c\xi}}\exp\left[-\frac{(s-s_c)^2}{s_c\xi}\right], \tag{29''}$$

with a maximum at $s = s_c = 2s_0(1 - 1/3\xi^2 + \cdots)$ and a width $(s - s_c)_{eff} \sim \sqrt{s_c\xi}$ or $(s - s_c)_{eff}/s_0 \sim \sqrt{B/B_0}/\xi$. Thus, the distribution over s in units of s_0 has the form of a narrow peak located (in the nonrelativistic case) at the doubled threshold

*th ≡ tanh.

value $s_c = 2s_0$. Formula (29″) should not be used if $s - s_c \ll s_c$ or $s - s_c \gg \sqrt{s_c\xi}$.

The marked difference in the distributions over s for linearly and circularly polarized waves is due to the fact that in a linearly polarized wave the photons do not have a definite angular momentum projection on the **k** axis, whereas in a circularly polarized wave, for each photon this projection is equal to +1 or −1, for right-handed or left-handed wave polarization, respectively. Therefore, upon absorption of s photons from the circularly polarized wave, the particle should have an angular momentum s. On the other hand, in a circularly polarized wave, the particle should be located at effective distances[9]

$$r_{\text{eff}} = ea / m_*\omega|_{\text{nonrel}} = B_0\xi^2 / B\eta$$

and hence, have a moment $l(s) = r_{\text{eff}}q = 2[s_0(s - s_0)]^{1/2}$, since its momentum is $q = \eta[2B(s - s_0)/B_0\xi]^{1/2}$. This means that states with angular momenta greater than $l(s)$ will be suppressed, and hence, the absorption of a number s of photons such that $s > l(s)$ will be suppressed. Function $s - l(s)$ has a minimum (equal to zero) at $s = s_c = 2s_0$, which explains the position of the peak in the ionization probability distribution over s.

4. SPLITTING OF THE SYSTEM INTO TWO PARTICLES WITH ARBITRARY MASSES. RELATIVISTICALLY AND GAUGE INVARIANT MODEL.

Let us consider the splitting of a neutral system, bound by short-range forces, by the field of a linearly polarized wave. If the field does not change the interaction between the neutral system and the products of its disintegration, then the splitting can be described by the matrix element

$$M = f\int d^4x\, \psi_p{}^+\psi_{p'}{}^+\psi_l = \sum_s \frac{f}{(8q_0q_0'l_0)^{1/2}} \times A_0(s\alpha\beta)(2\pi)^4\delta(sk + l - q - q'), \qquad (30)$$

where $\psi_l(x) = (2l_0)^{-1/2}e^{i(lx)}$ is the wave function of the neutral system and $\psi_p(x)$ and $\psi_p'(x)$ are the wave functions of the charged particles, defined by formula (16); l, q, and q′ are the 4-quasimomenta of the corresponding particles; $\alpha = e((ap)/(kp) - (ap')/(kp'))$, $8\beta = -e^2a^2(1/(kp) + 1/(kp'))$; f is the constant of the three-particle interaction.

The probability of ionization calculated per unit volume and unit time is a function of the two invariant parameters $x = ea/m$ and $\chi = -x(kl)/m^2 = e\sqrt{(F_{\mu\nu}l_\nu)^2}/m^3$ and is equal to

$$W(\varkappa, x) = \frac{n}{(2\pi)^2 l_0}\sum_{s>s_0}\int_0^{2\pi} d\varphi \int_{\rho_1}^{\rho_2} d\rho\, w, \quad w = \frac{f^2}{8}A_0^2(s\alpha\beta), \qquad (31)$$

where

$$\rho = (kq')/(kl),$$

$\rho_{1,2} = (E_s^2 + m'^2 - m^2)/2E_s^2 \mp [(E_s^2 + m'^2 - m^2)^2/4E_s^4 - m_*'^2/E_s^2]^{1/2}$, $E^2 = -(sk + l)^2 = M^2 + 2s\kappa m^2/x$, $s_0 = x[(m_* + m_*')^2 - M^2]/2m^2\kappa$, φ is the angle between planes (**k**, **q**′) and (**k**, **a**) in the center-of-mass system, M is the mass of the neutral particle, and n is the density of the number of neutral particles. The variables α, β are related to ρ, φ by the expressions

$$\alpha = z\cos\varphi, \qquad z = \frac{E_s x^2}{m\varkappa\rho(1-\rho)}[(\rho_2 - \rho)(\rho - \rho_1)]^{1/2},$$

$$\beta = \frac{x^3}{8\varkappa\rho(1-\rho)}. \qquad (32)$$

The expression for the probability of ionization by a circularly polarized wave differs from (31) in that the function A_0 has been replaced by Bessel function $J_s(z)$, where z is defined by (32) and is independent of φ.

When $x \to \infty$, it follows from (31) that

$$W(\varkappa) = \frac{2}{\pi}\int_0^{\pi/2} d\psi\, F(\varkappa\sin\psi),$$

$$F(\varkappa) = \frac{f^2 n}{4\pi^2 l_0}\int_0^1 d\rho \int_0^\infty d\tau \frac{v^2(y)}{[2\varkappa\rho(1-\rho)]^{1/3}}, \qquad (33)$$

where $F(\kappa)$ is the probability of ionization by a constant crossed field,

$$y = \frac{\sigma}{[2\varkappa\rho(1-\rho)]^{2/3}}, \quad \sigma = \frac{m'^2}{m^2} - \frac{M^2 + m'^2 - m^2}{m^2}\rho + \frac{M^2}{m^2}\rho^2 + \tau^2,$$

$$\tau = \frac{\gamma q_2' - \gamma' q_2}{m(l_0 - l_3)} = \frac{eF_{\mu\nu}{}^* q_\mu q_\nu'}{m^4\varkappa}.$$

In the case of a weak field, when values $y \gg 1$ are effective, the Airy function v(y) can be replaced by its asymptotic expression, and the integration can be performed in (33) by the method of steepest descent. The saddle point is determined by coordinates $\tau = 0$ and $\rho = \rho_0$, where ρ_0 is the root of the equation

$$\rho_0^3 - \frac{m^2 - m'^2 + 3M^2}{2M^2}\rho_0^2 - \frac{m^2 + 3m'^2 - M^2}{2M^2}\rho_0 + \frac{m'^2}{M^2} = 0, \qquad (34)$$

lying in the range $0 < \rho_0 < 1$. Then

$$F(\varkappa) = \sqrt{3}\, f^2 nm\rho_0^{3/2}(1-\rho_0)^{3/2}\varkappa\exp[-2\sigma_0^{3/2}/3\varkappa\rho_0(1-\rho_0)] \times \Big[32\pi l_0 M\sigma_0\Big(\frac{m^2 + 3m'^2 - M^2}{2M^2} + \frac{m^2 - m'^2 + 3M^2}{M^2}\rho_0 - 3\rho_0^2\Big)^{1/2}\Big]^{-1}, \qquad (33')$$

where σ_0 is the value of σ at the saddle point $\tau = 0$, $\rho = \rho_0$. The condition for applicability of this formula is $y_0 \equiv [2\kappa\rho_0(1-\rho_0)]^{-2/3}\sigma_0 \gg 1$.

The cubic equation (34) has simple solutions only in special cases. We shall examine some of them.

1. If $m = m'$, then $\rho_0 = 1/2$.

2. If $\delta \equiv I/m' \ll 1$, where $I = m + m' - M$ and m' is the smaller of the masses m or m′, then accurate to the terms $\sim\delta^2$

$$\rho_0 = \frac{m'}{m+m'}\left\{1 - \delta\frac{m-m'}{3(m+m')} + \ldots\right\}.$$

In this case (33′) gives

$$F(\varkappa) = \frac{f^2 n m^3 \varkappa}{64\pi l_0 m'(m+m')^2\delta}$$

$$\times \exp\left(-\frac{4m'^2}{3\varkappa m^2}\sqrt{2\delta^3\frac{m+m'}{m}}\right). \qquad (33'')$$

We note that when $I/m' \sim \alpha^2$ where $\alpha = 1/137$, the Coulomb interaction of the scattering particles becomes essential. For this reason, formula (33″) is valid when

$$\alpha^2 \ll I/m' \ll 1, \qquad y_0 \gg 1, \qquad y_0^{3/2}\delta \ll 1.$$

3. If m'/m and $m'/M \ll 1$, then accurate to $(m'/m)^2$

$$\rho_0 = \frac{m'}{4m}\left[\frac{M-m}{m'} + \sqrt{8 + \left(\frac{M-m}{m'}\right)^2}\right] + \ldots$$

If we introduce $\epsilon_0' = M - m$ and $\gamma_0' = (\epsilon_0' + \sqrt{\epsilon_0'^2 + 8m'^2})/4$, then $\rho_0 = \gamma_0'/m + \ldots$ and, accurate to within the substitution of the primed quantities by the unprimed ones, (8) follows from (33), and $f^2/32\pi l_0 m$ corresponds to the constant $2\pi N^2$.[3)]

In the case of a very intense field, $F(\kappa)$ tends to a constant value equal to $f^2 n/48\pi l_0$.

Let us now examine the role of spin effects by taking as an example the conversion of a neutral particle with spin 1 into a pair of charged scalar particles. For the interaction

$$ig(\psi^+\partial\psi/\partial x_\mu - \psi\partial\psi^+/\partial x_\mu - 2ieA_\mu\psi^+\psi)\varphi_\mu$$

(φ_μ is the wave function of the neutral vector particle), the splitting probability is given by formula (31) with the differential probabilities

$$w_1 = \frac{g^2m^2}{2}[-\sigma A_0^2 + x^2(A_1^2 - A_0A_2)],$$

$$w_2 = \frac{g^2m^2}{2}\tau^2 A_0^2 \qquad w_3 = \frac{g^2M^2}{2}\left(\rho - \frac{1}{2}\right)^2 A_0^2, \quad (35)$$

corresponding to the polarization of the particle along axes 1, 2, 3 of the "special" coordinate system; $\sigma = 1 + \tau^2 - M^2\rho(1-\rho)/m^2$; for the functions A_i see[4].

In a constant crossed field, we obtain instead of (33)

$$F_{1,2,3}(\varkappa) = \frac{g^2m^2n}{\pi^2 l_0}\int_0^1 d\rho\int_0 d\tau\,[2\varkappa\rho(1-\rho)]^{-1/3}$$

$$\times\left\{\frac{\sigma v'^2(y)}{y},\ \tau^2v^2(y),\ \frac{M^2}{m^2}\left(\rho-\frac{1}{2}\right)^2 v^2(y)\right\}. \qquad (36)$$

For a weak field this formula gives

$$F_{1,2,3}(\varkappa) = \sqrt{\frac{3}{2}}\frac{g^2m^2n\varkappa\exp(-8\sigma_0^{3/2}/3\varkappa)}{64\pi l_0(1+M^2/8m^2)^{1/2}}$$

$$\times\left\{1,\ \frac{\varkappa}{8\sigma_0^{3/2}},\ \frac{3M^2\varkappa}{64m^2(1+M^2/8m^2)\sigma_0^{3/2}}\right\}. \qquad (36')$$

Here $\sigma_0 = 1 - M^2/4m^2$. In a weak field the splitting reaches the optimum value only for polarization of the particle along the electric field **E** (the projection of the spin on the direction of **E** is equal to zero), and for the other polarizations is suppressed by a factor $\sim y_0^{-3/2} \ll 1$. Such a suppression is due to the fact that in the weak field the particle scattering should occur primarily in the (1, 3) plane, and in the nonrelativistic case, along axis 1. On the other hand, however, polarization of the states forbids scattering in directions orthogonal to the polarization. The suppresion in the case of polarization along axis 3 in the relativistic case is due to the equality of the masses. When M = 0, formulas (35)—(36′) give the probability of production of a pair of scalar particles by the photon (g should then be replaced by e). In this case it is useful to compare $F_{1,2}$ with the probabilities $F_{\|,\perp}$ of production of an electron-position pair by the photon[4]. In contrast to $F_{1,2}$, the probabilities $F_{\|,\perp}$ in a weak field are of the same order, for owing to the spins of the electron and positron the polarization of the state does not forbid scattering in directions orthogonal to the polarization.

In conclusion, let us consider the splitting of a charged system with momentum q into a neutral and a charged particle with momenta l and q' (for example, $H_2^+ \to H + p$). The splitting probability is a function of

$$x = ea/m, \qquad \chi = -x(kq)/m^2 = e\sqrt{(F_{\mu\nu}q_\nu)^2}/m^3$$

and is equal to

3)The equation $f^2/32\pi m^2 = 2\pi N^2 = \eta/2m'$ defines the maximum value of the constant of interaction of particles via their masses (cf., for example, [10]).

$$W(\chi, x) = \frac{n}{(2\pi)^2 q_0} \sum_{s>s_0} \int_0^{2\pi} d\varphi \int_{\rho_1}^{\rho_2} \frac{d\rho}{(1+\rho)^2} w, \quad w = \frac{f^2}{8} A_0^2(s\alpha\beta), \qquad (37)$$

where

$$\rho = (kq')/(kl),$$

$$\rho_{1,2} = (E_s^2 - M^2 - m_*'^2)/2M^2 \mp [(E_s^2 - M^2 - m_*'^2)^2/4M^4 - m_*'^2/M^2]^{1/2},$$

$$E_s^2 = -(sk+q)^2 = m_*^2 + 2s\chi m^2/x,$$

$$s_0 = x[(m_*' + M)^2 - m_*^2]/2m^2\chi,$$

φ is the angle between planes (**k**, **q**′) and (**k**, **a**) in the c.m.s., and the variables α, β are related to ρ, φ by the expressions

$$\alpha = z\cos\varphi, \quad z = Mx^2[(\rho_2 - \rho)(\rho - \rho_1)]^{1/2}/m\chi\rho,$$

$$\beta = x^3/8\chi\rho. \qquad (38)$$

When $x \to \infty$, the probability W is related to the probability $F(\chi)$ of splitting in the constant crossed field as follows:

$$W(\chi) = \frac{2}{\pi} \int_0^{\pi/2} d\psi\, F(\chi \sin\psi),$$

$$F(\chi) = \frac{f^2 n}{4\pi^2 p_0} \int_0^\infty d\rho \int_0^\infty d\tau \frac{v^2(y)}{(1+\rho)^2 (2\chi\rho)^{1/3}}. \qquad (39)$$

Here τ is the same as in (33), and

$$y = (2\chi\rho)^{-2/3}\sigma, \quad \sigma = \frac{m'^2}{m^2} + \frac{M^2 + m'^2 - m^2}{m^2}\rho + \frac{M^2}{m^2}\rho^2 + \tau^2.$$

In a weak field

$$F(\chi) = \frac{\sqrt{3} f^2 n \rho_0^{3/2} \chi \exp(-2\sigma_0^{3/2}/3\chi\rho_0)}{32\pi p_0 (1+\rho_0)^2 \sigma_0 [(M^2 + m'^2 - m^2)^2/4m^4 + 8M^2 m'^2/m^4]^{1/4}}, \qquad (39')$$

where σ_0 is the value of σ at the saddle point $\tau = 0$, $\rho = \rho_0$, and

$$\rho_0 = -\frac{M^2 + m'^2 - m^2}{8M^2} + \left[\left(\frac{M^2 + m'^2 - m^2}{8M^2}\right)^2 + \frac{m'^2}{2M^2}\right]^{1/2}.$$

Formula (39′) applies when $y_0 \gg 1$. In the special case when m, $M \to \infty$, (8) follows from (39′) if we set $\epsilon_0' = m - M$, $\gamma_0' = [\epsilon_0' + (\epsilon_0'^2 + 8m'^2)^{1/2}]/4$; $f^2/32\pi p_0 m$ corresponds to the constant $2\pi N^2$.

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